Chapter I: Time series

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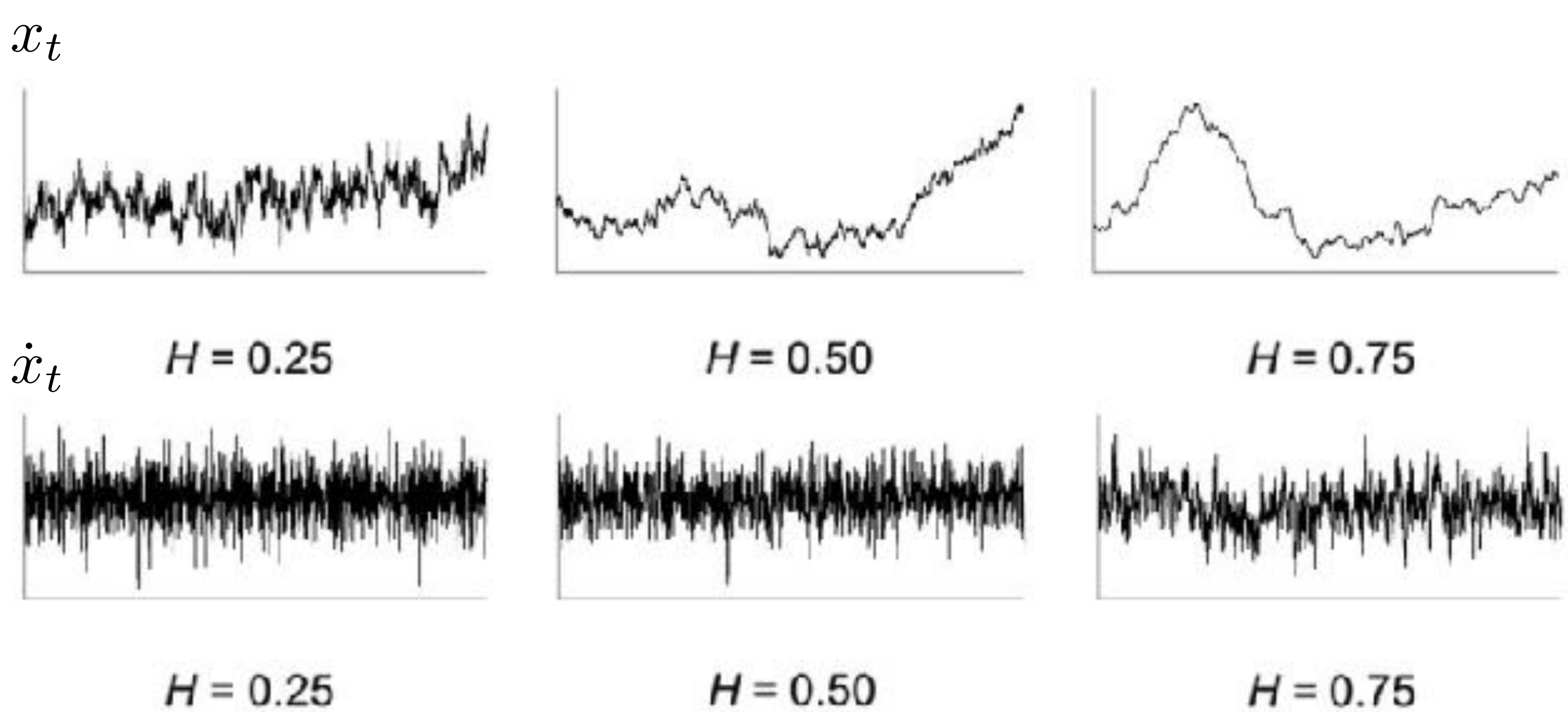
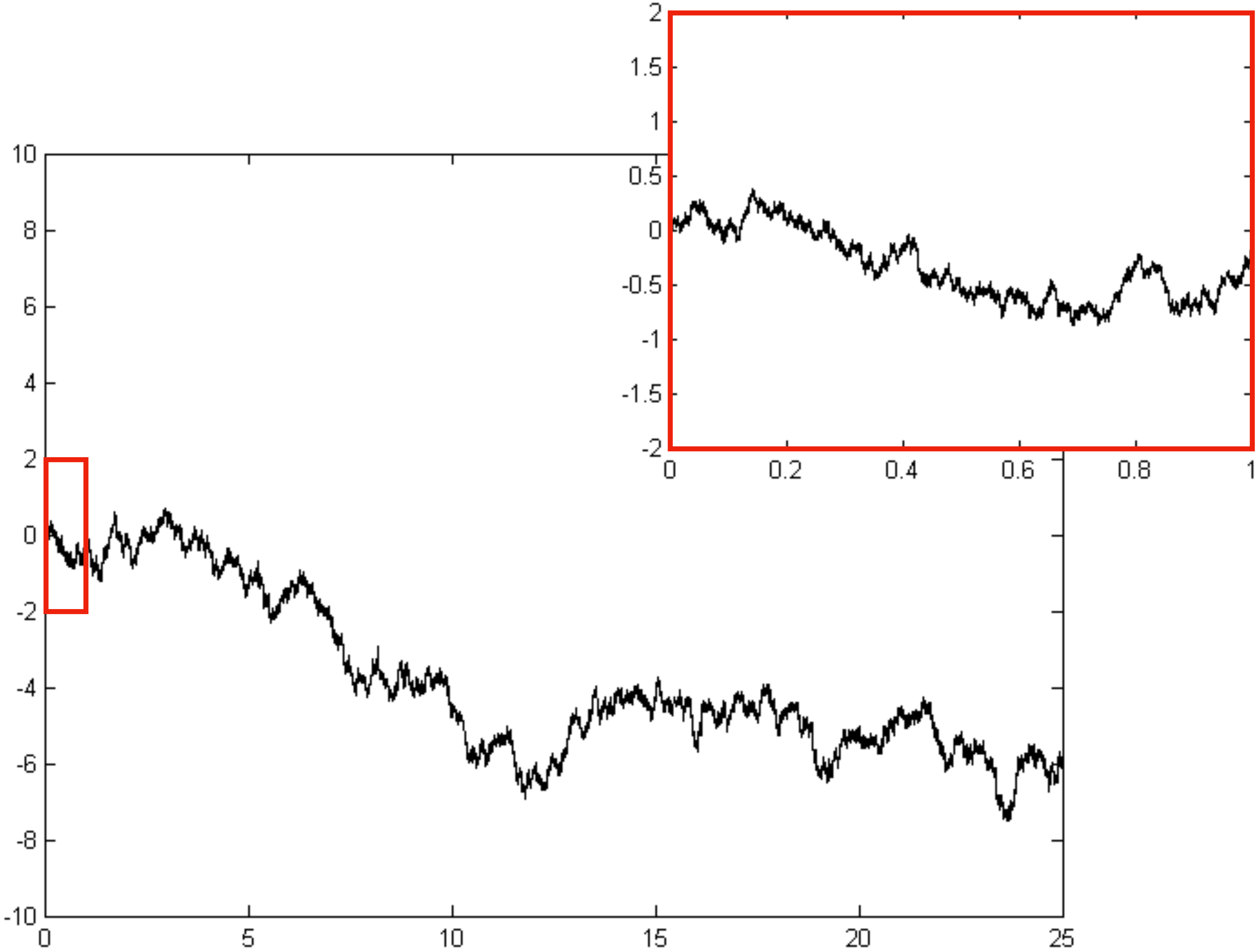
Fractional Brownian motion (fBm)

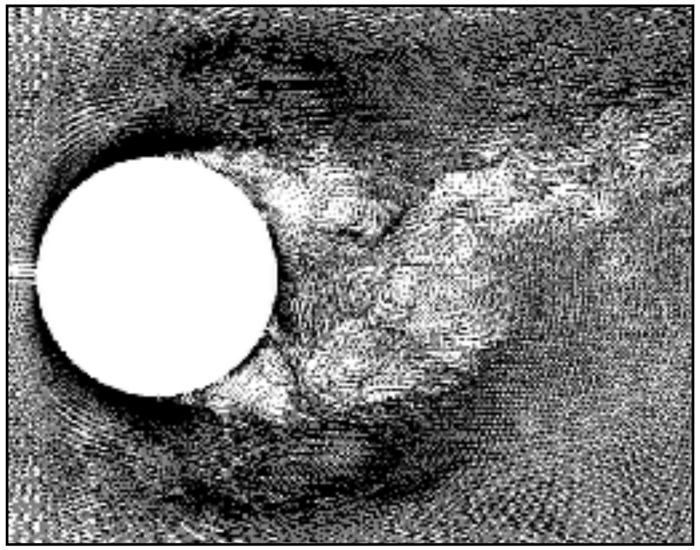


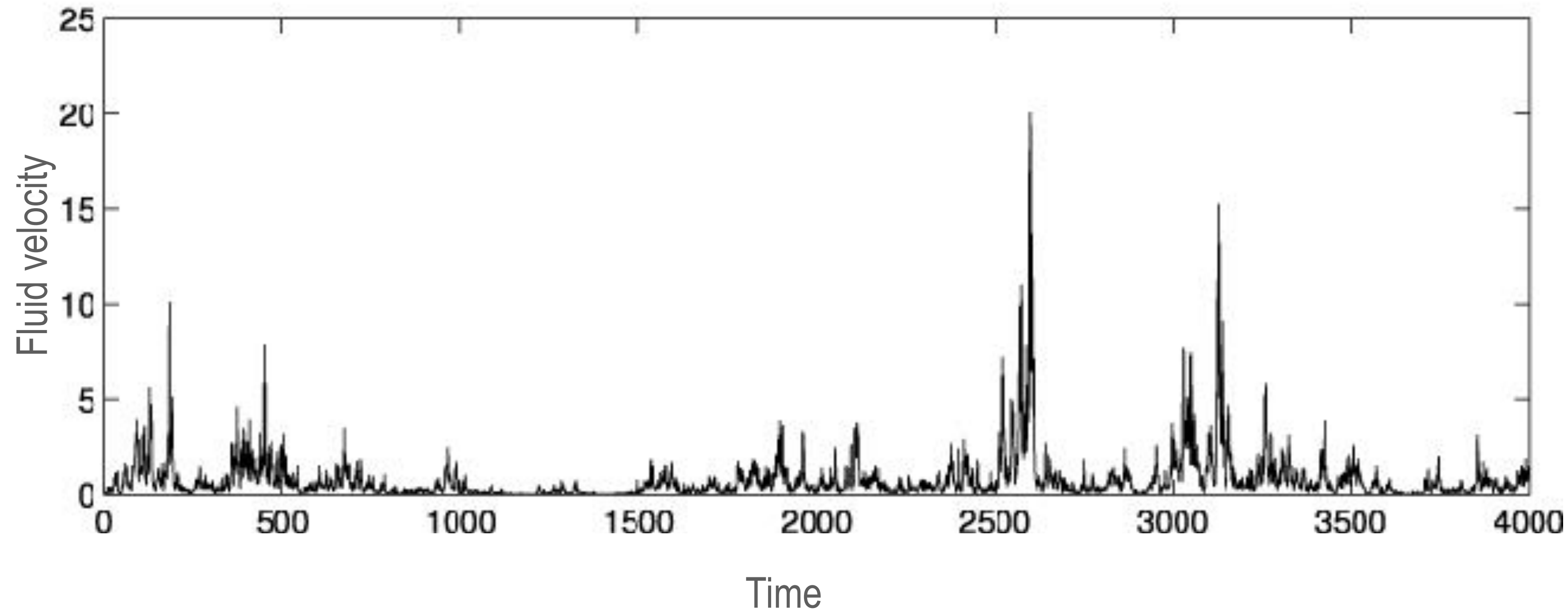
Scale invariance (random walk)



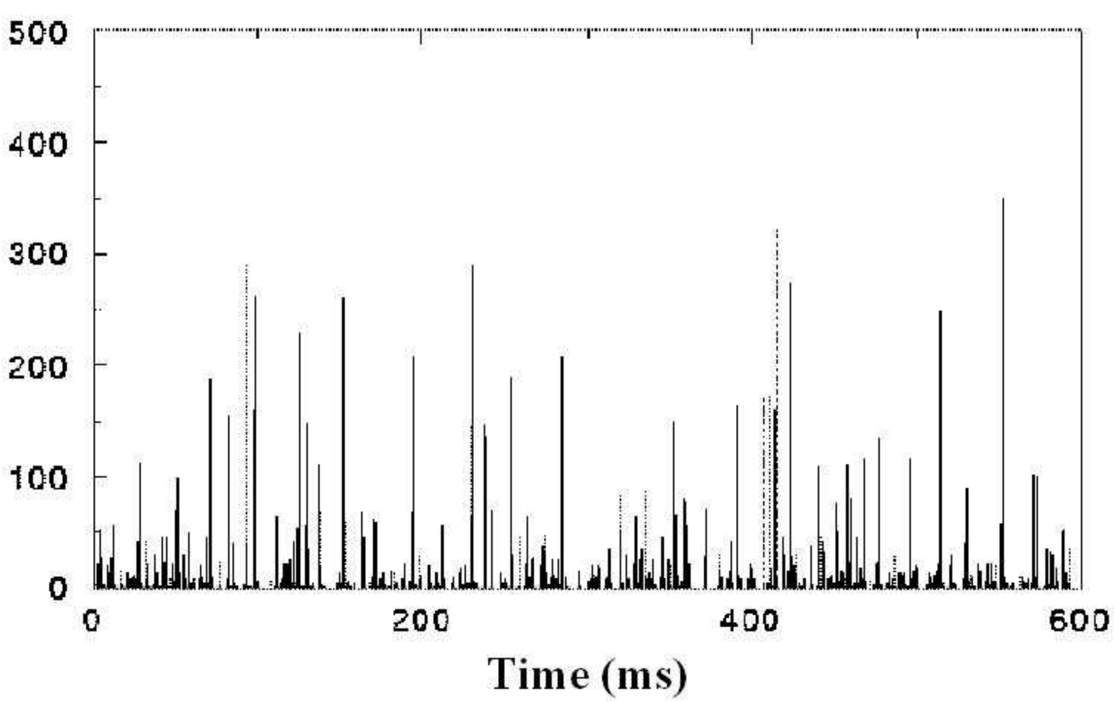
Intermittency and activity clustering

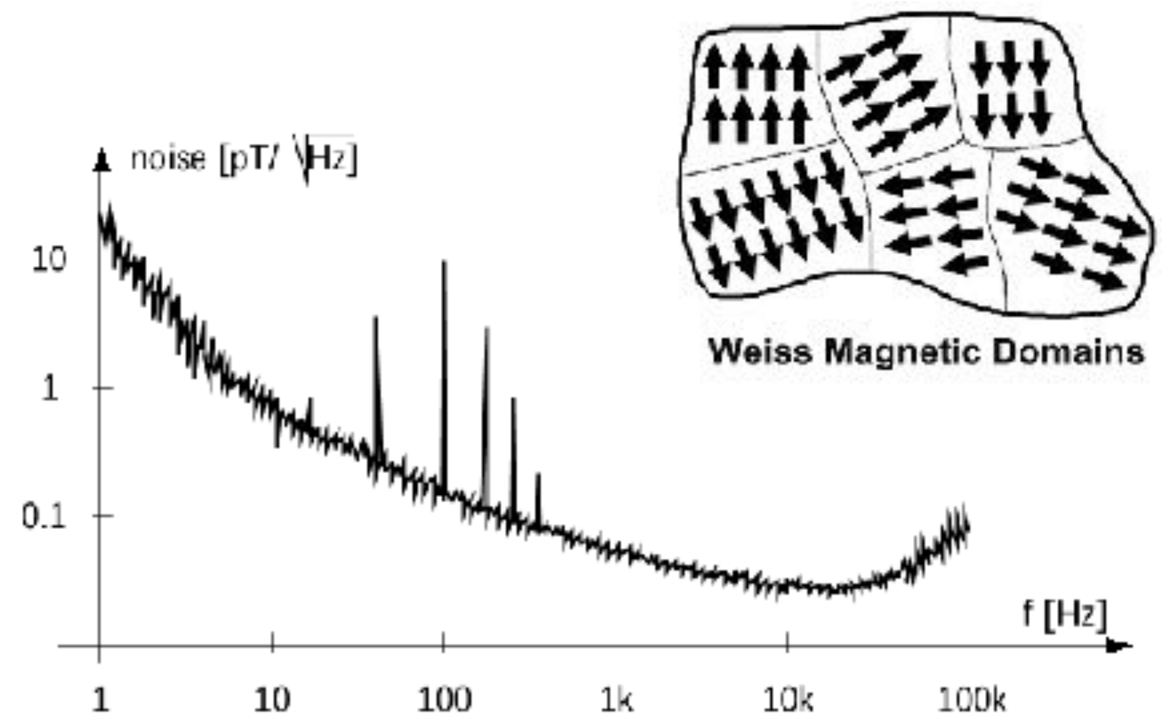
Turbulence (fluid mechanics)



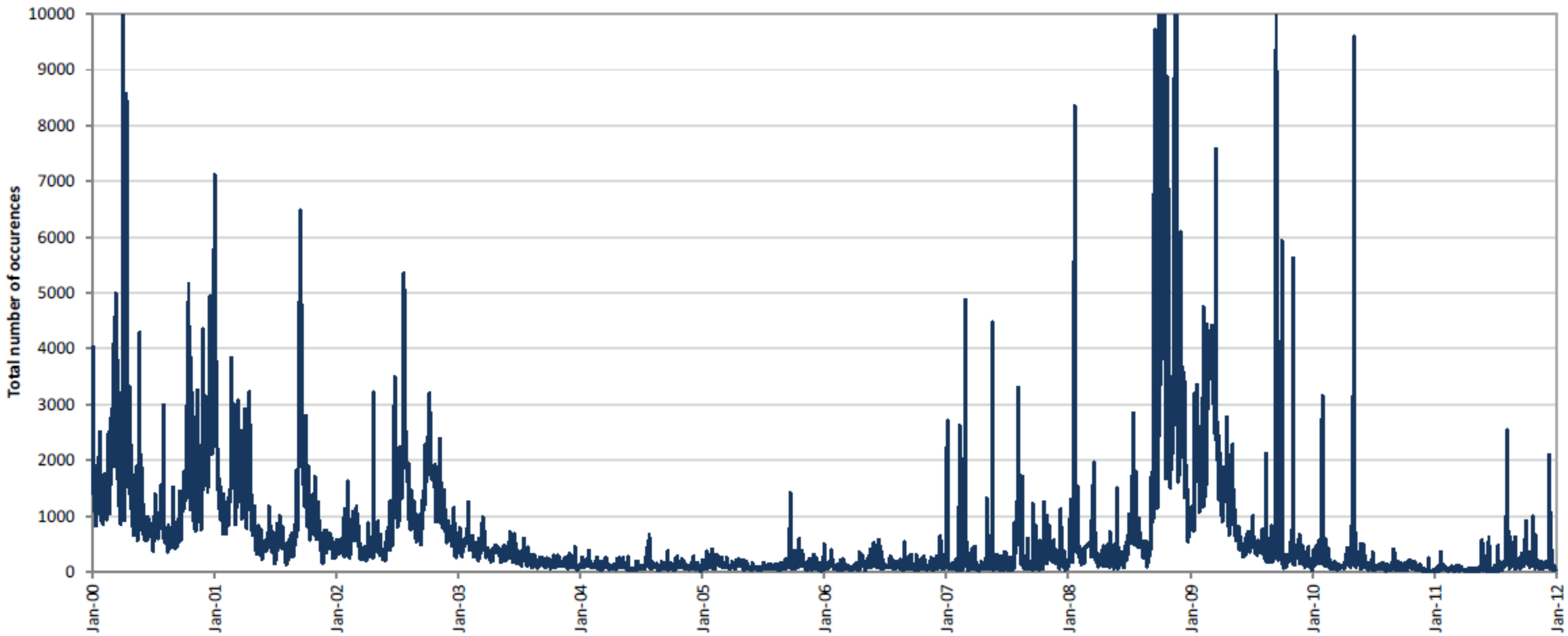


Barkhausen noise (electromagnetism)





Excess volatility (Number of weekly 1% jumps/min on S&P stocks)



Chapter II: Statistics of real prices

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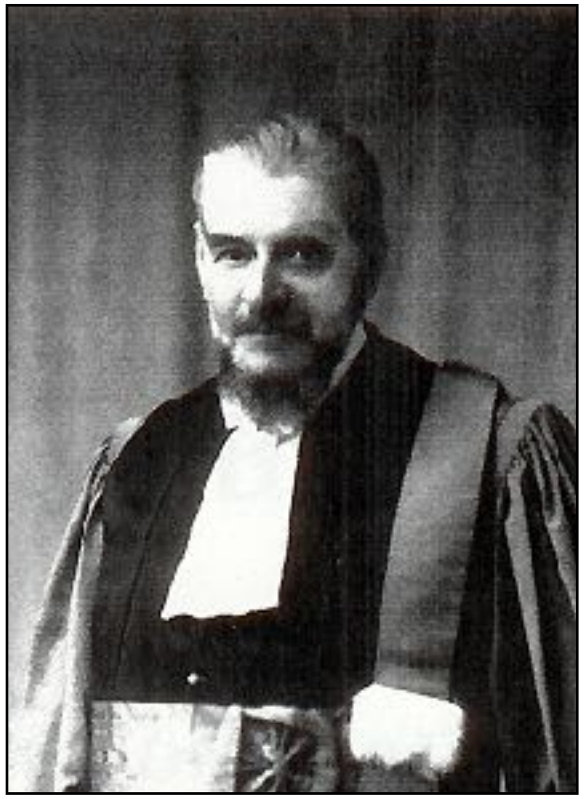
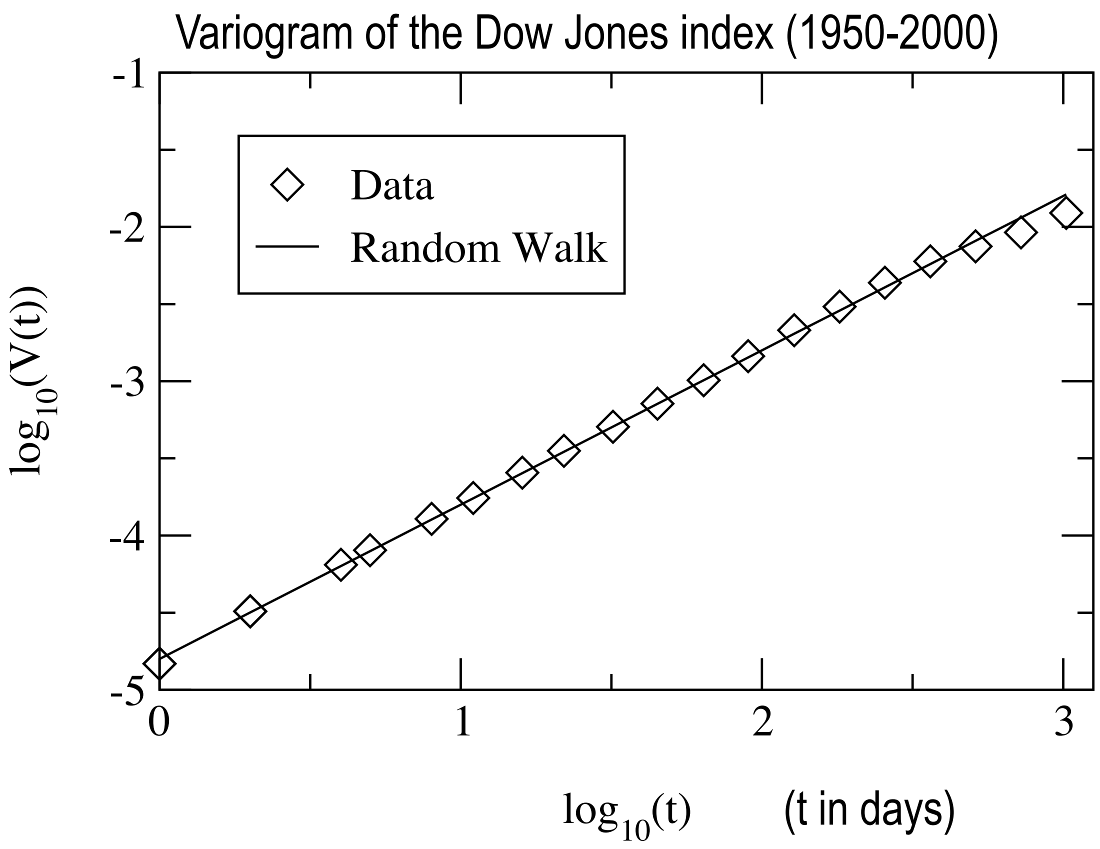
Bachelier's first law (1870-1946)

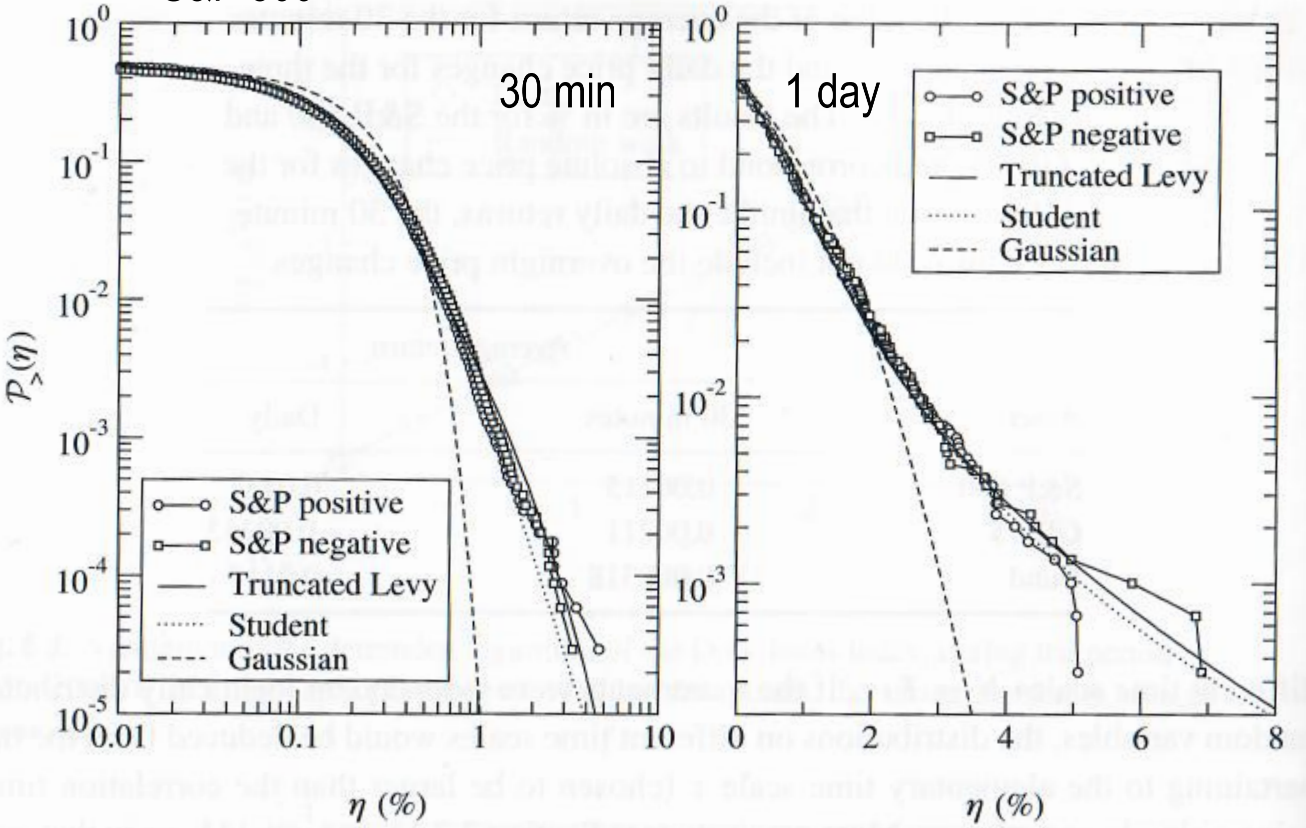
Bachelier à 15 ans



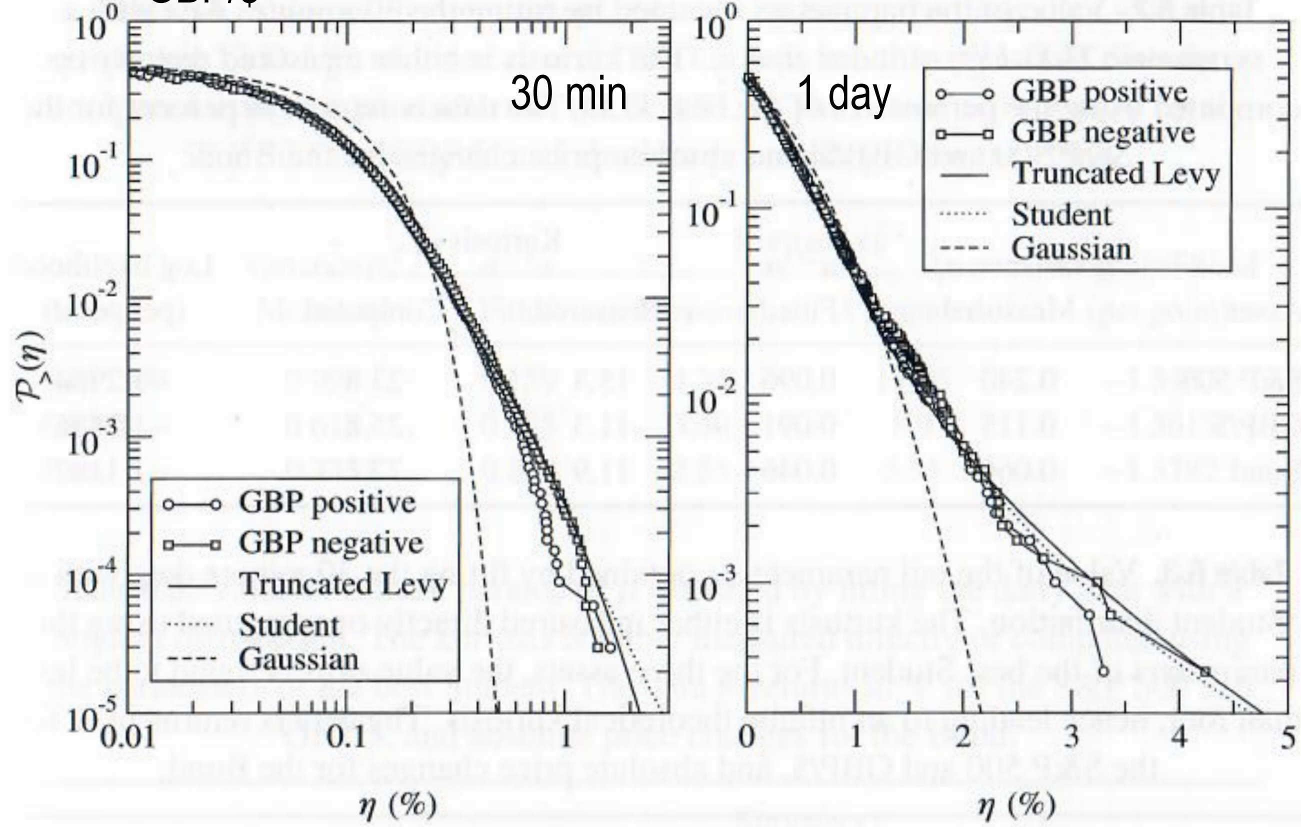


Elementary cumulative pdf of returns (30 min and daily) for different assets

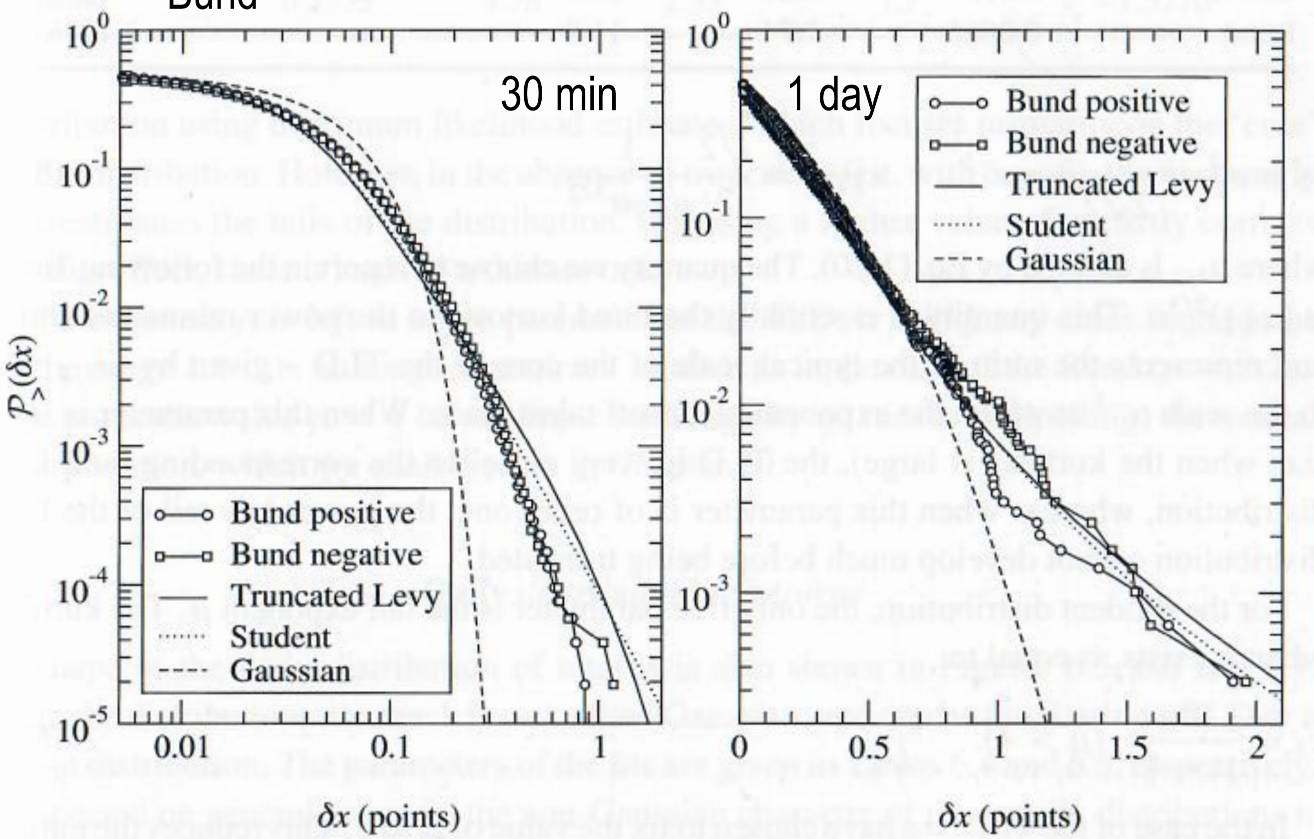
S&P 500



GBP/$

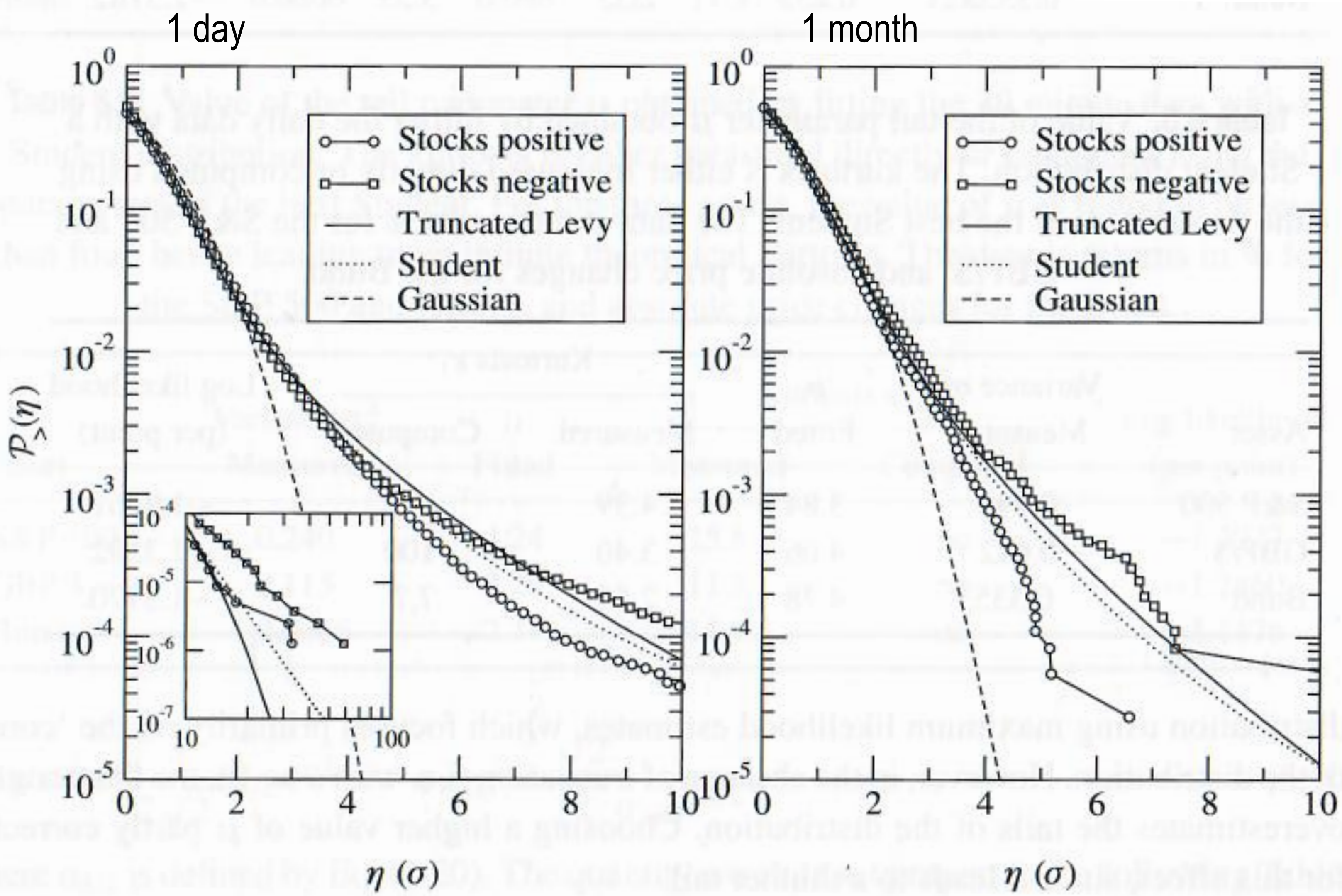


Bund



	30 min		1 day	
Asset	Variance σ_1^2 Measured	μ Fitted	Variance σ_1^2 Measured	μ Fitted
S&P 500	0.240	3.24	0.990	3.84
GBP/$	0.115	3.20	0.612	4.06
Bund	0.0666	2.74	0.3353	4.78

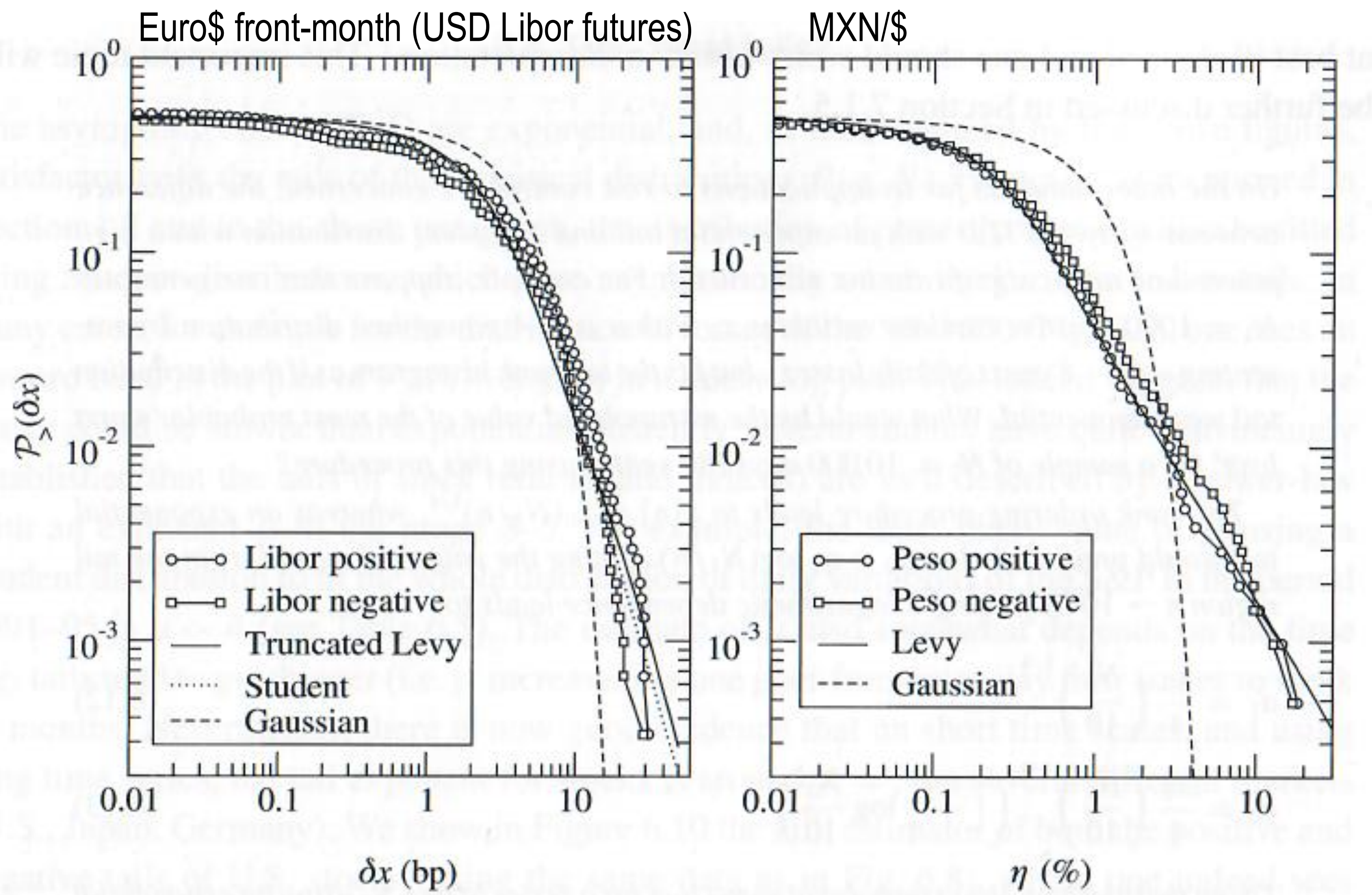
Elementary cumulative pdf of returns (daily and monthly) for a pool of 500 US stocks



negative tail: $\mu \approx 3$
positive tail: $\mu \approx 4.1$

negative tail: $\mu \approx 3$
positive tail: $\mu \approx 5.7$

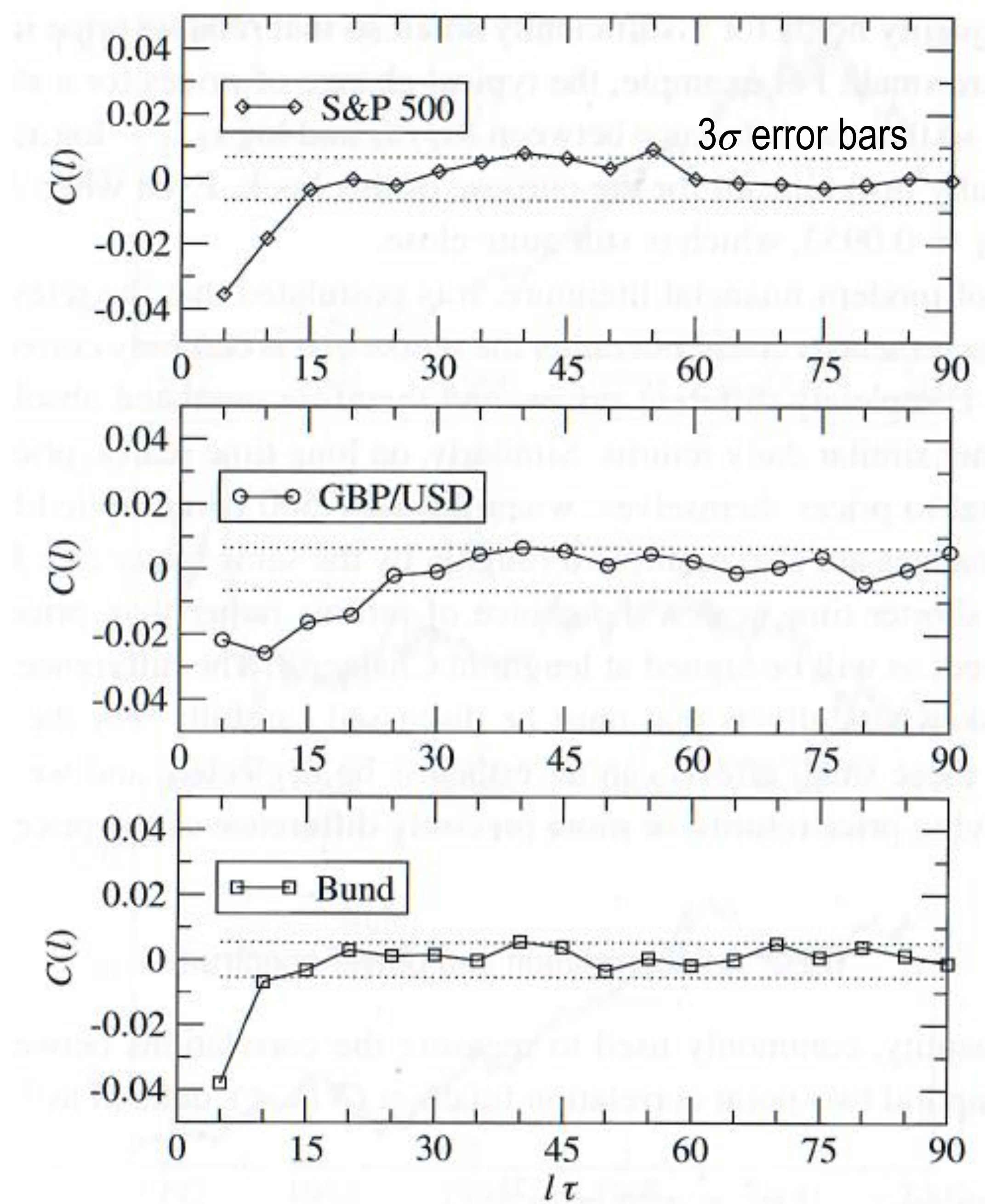
Elementary cumulative pdf of daily returns in extreme markets



$\mu \approx 2.64$

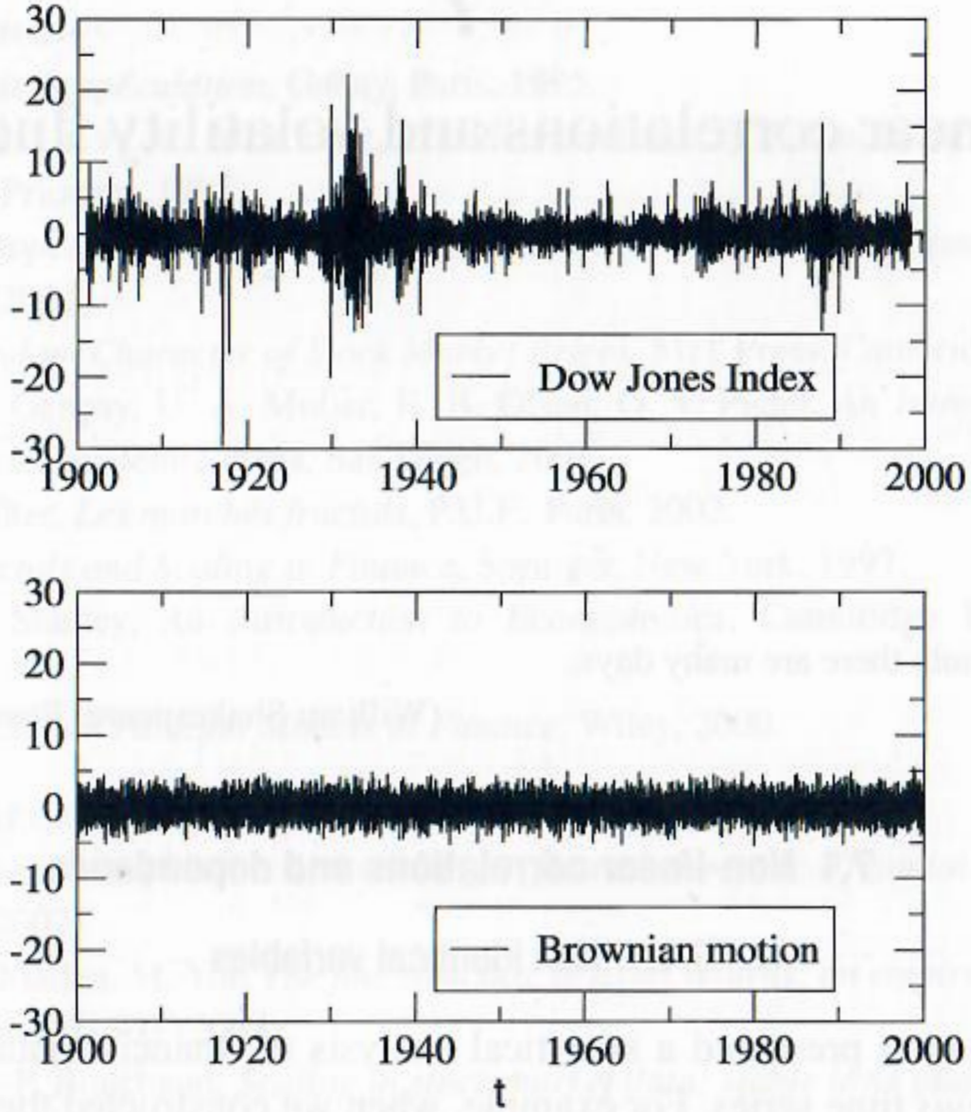
$\mu \approx 1.34$

Two point correlation function or returns

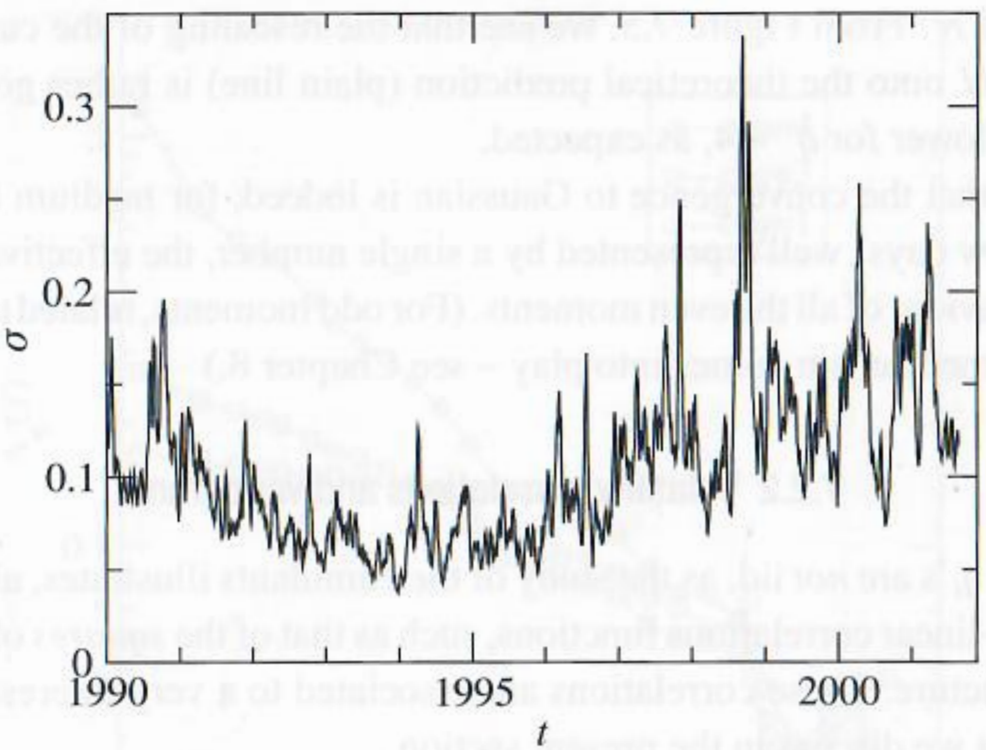


Intermittent nature of volatility fluctuations

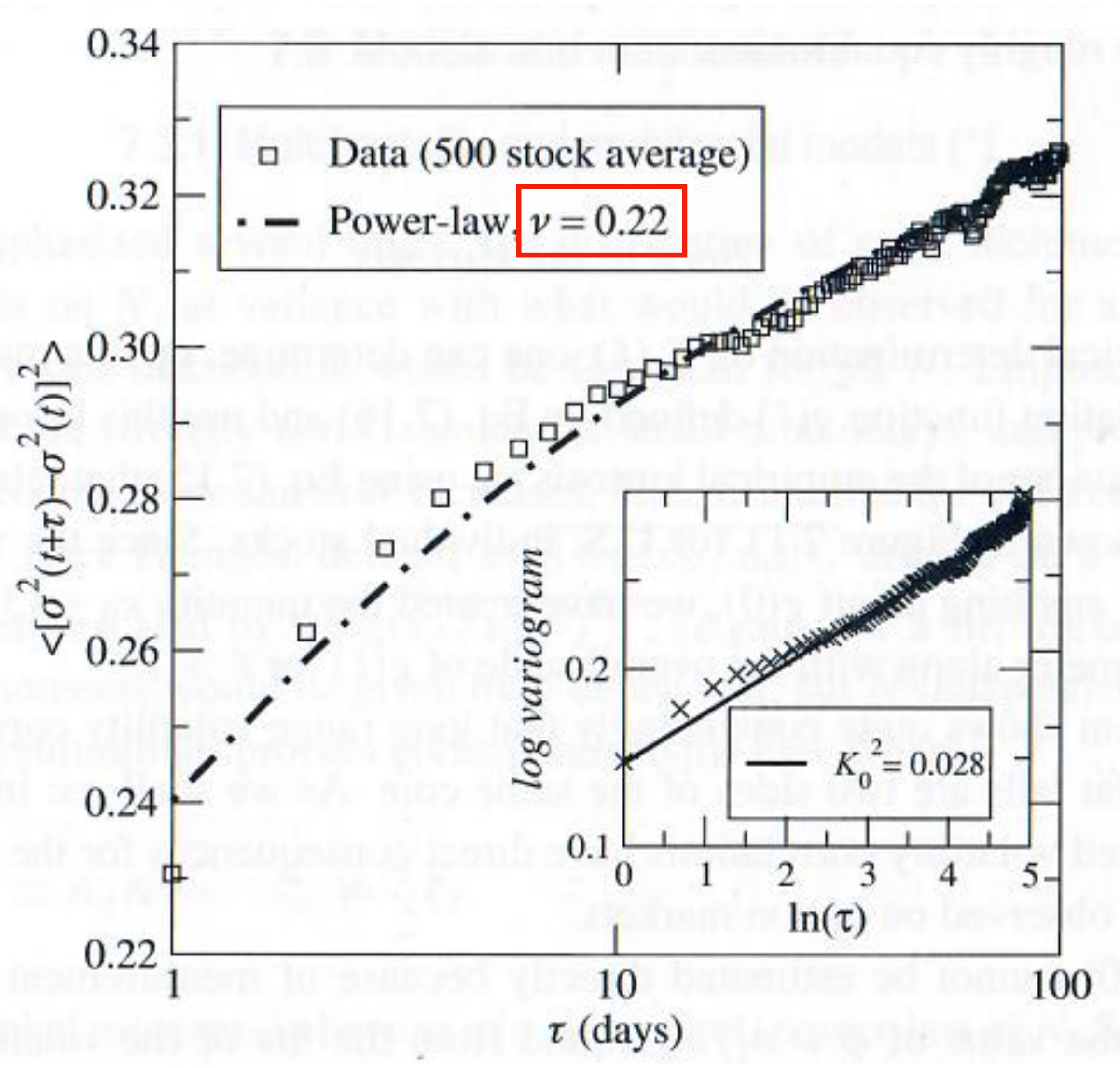
Daily returns



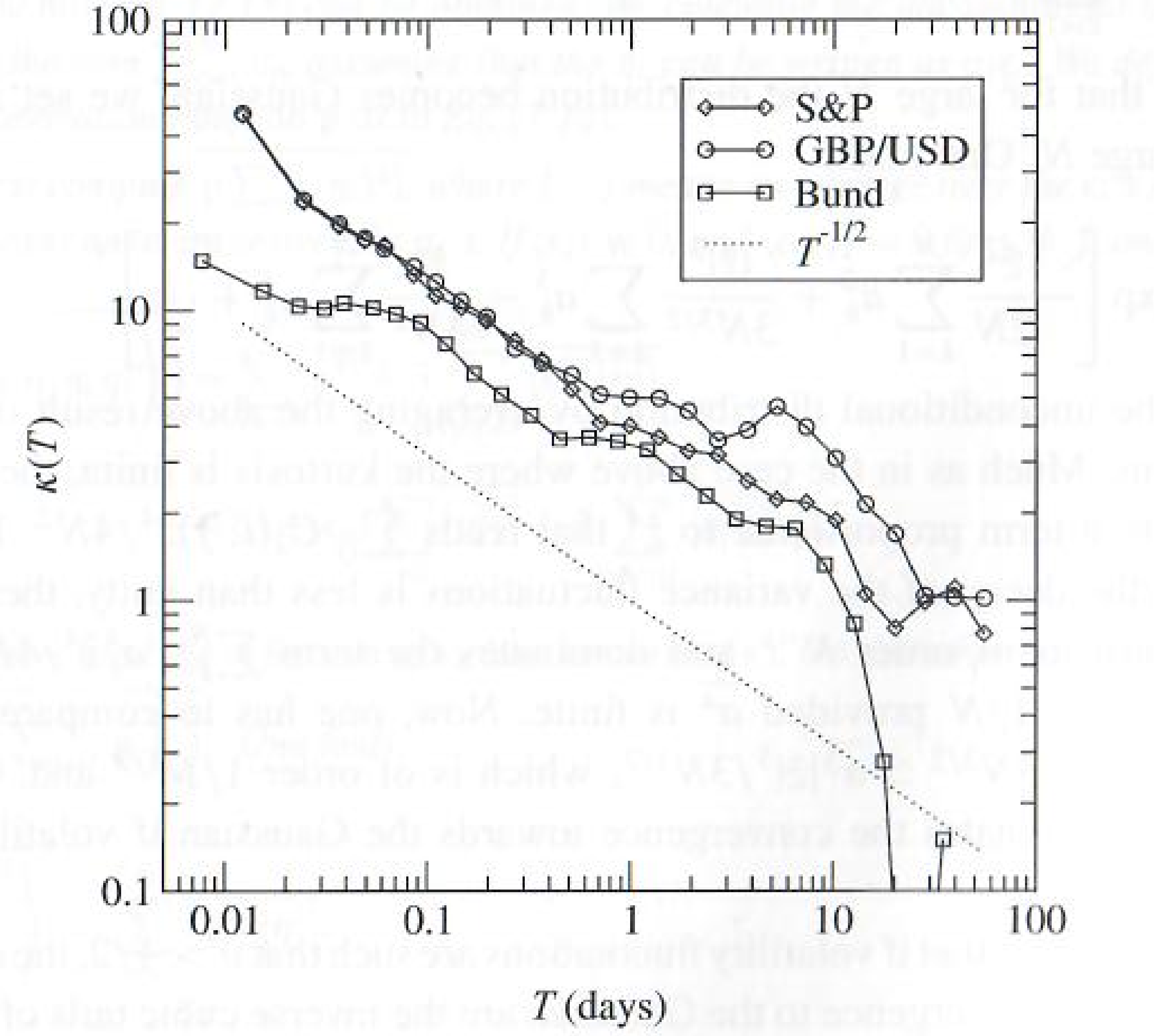
Daily volatility of the S&P 500



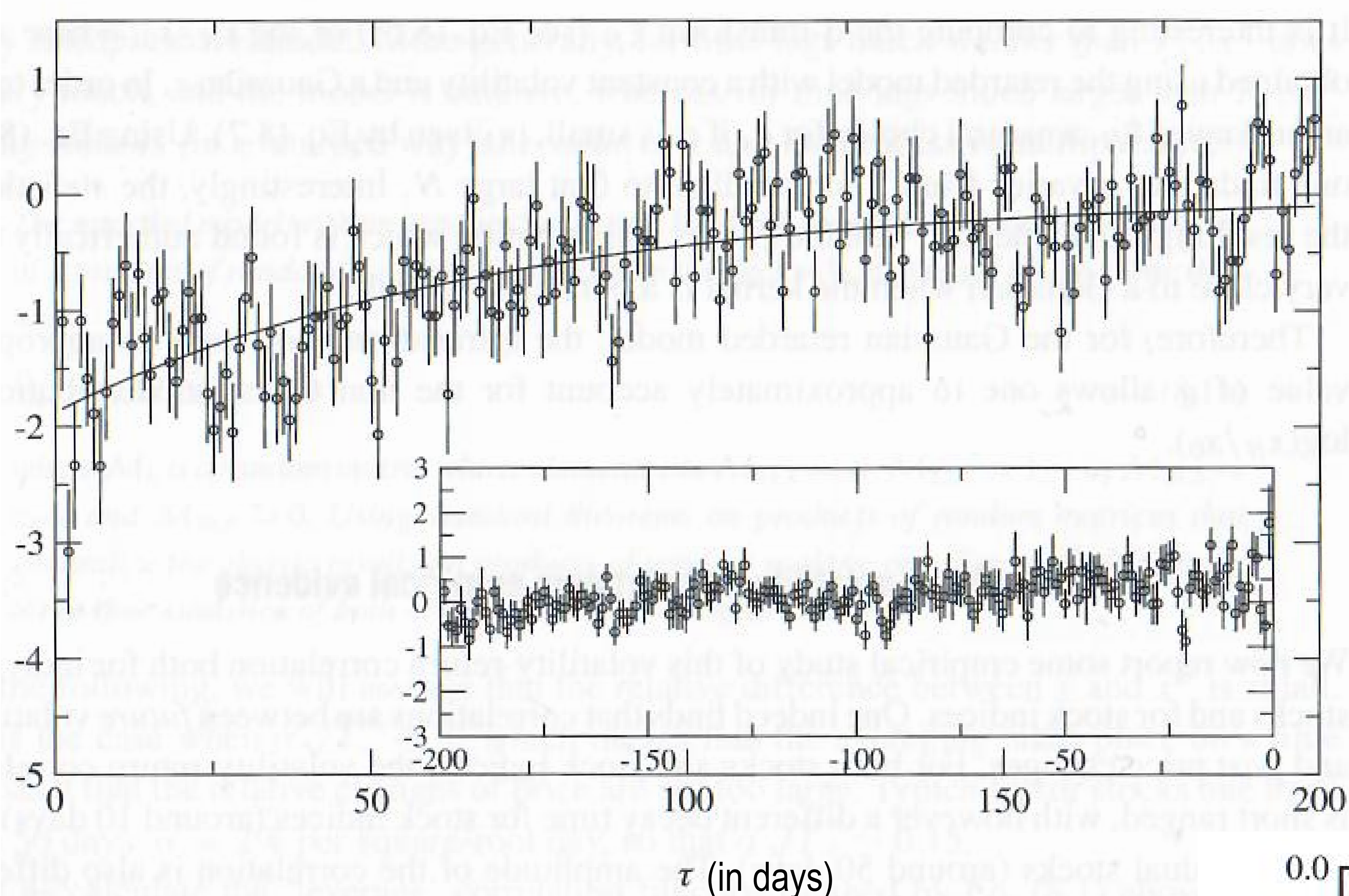
Variogram of the volatility process (500 stocks)

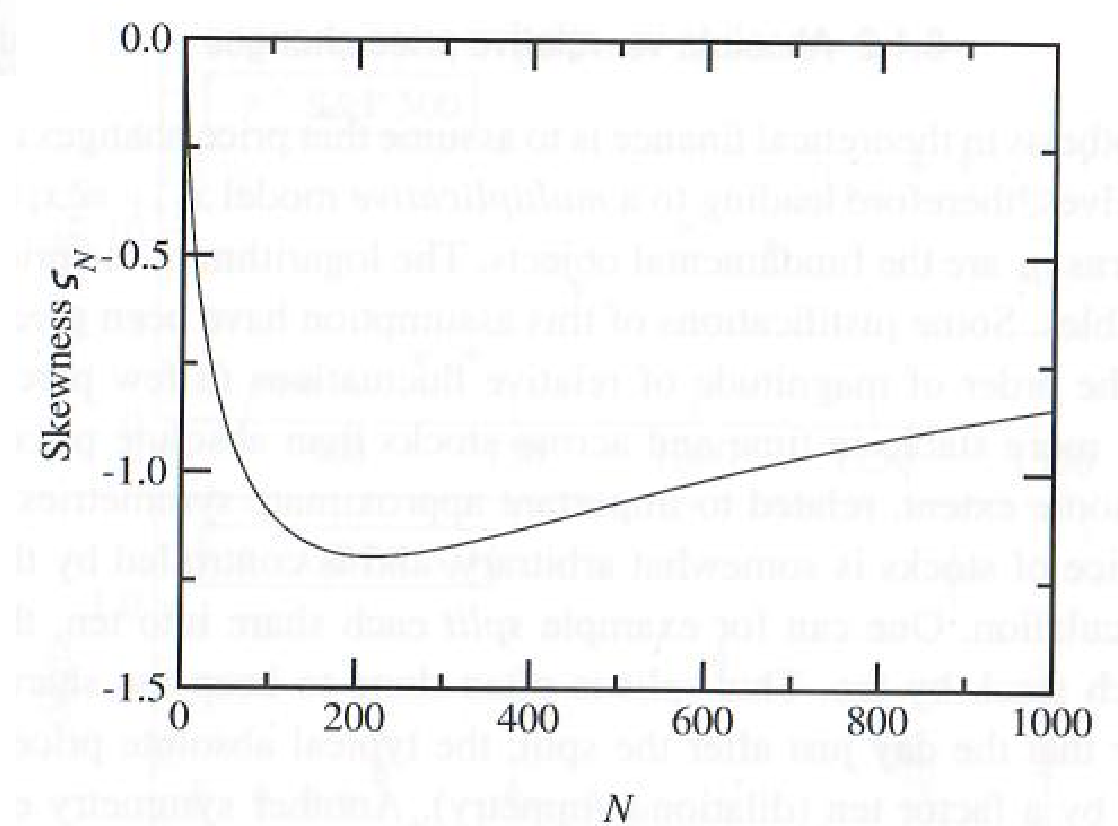


Kustosis of cumulated returns (slow decay)



Return-volatility correlations (Leverage effect) and skewness





Chapter III: Why do prices move?

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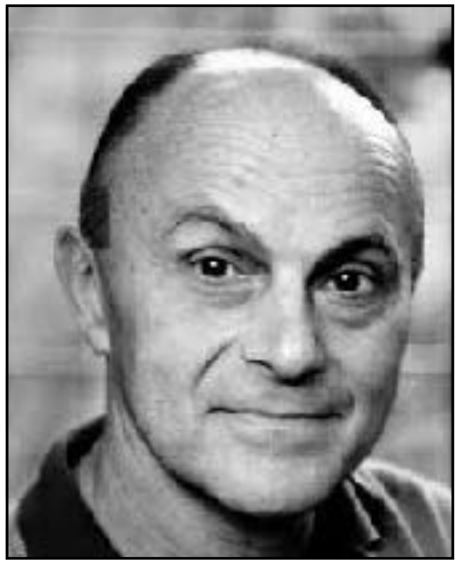
The Efficient Market Hypothesis (EMH)



I can't figure out why anyone invests in active management [...]. Since I think everything is appropriately priced, my advice would be to avoid high fees. So you can forget about hedge funds.

Eugene Fama

We might define an efficient market as one in which price is within a factor of 2 of (its fundamental) value [...]. By this definition, I think all markets are efficient almost all of the time. "Almost all" means at least 90%.

Fisher Black

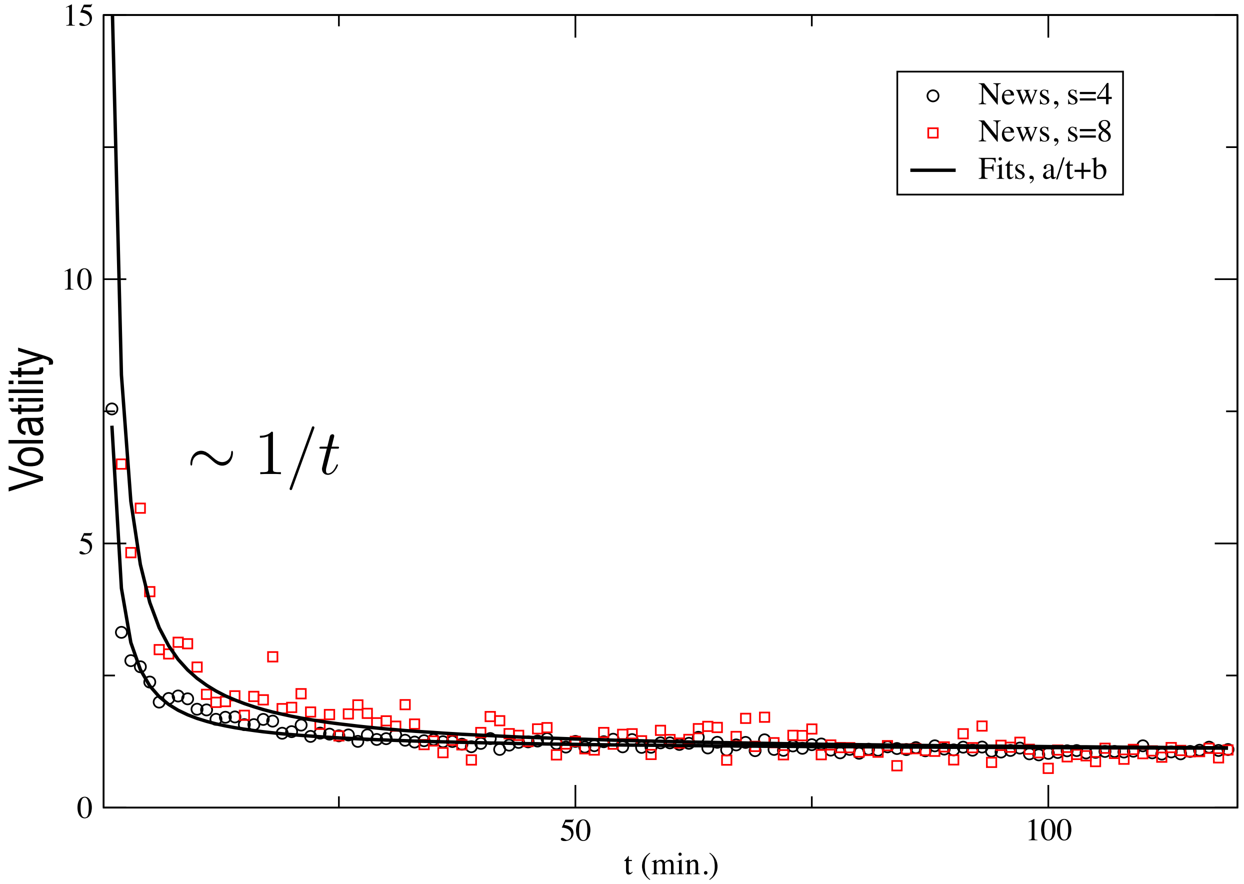
No, markets weren't "efficient" at finding the truth; they were just very efficient at converging on a conclusion – often the wrong conclusion.

Ben Horowitz

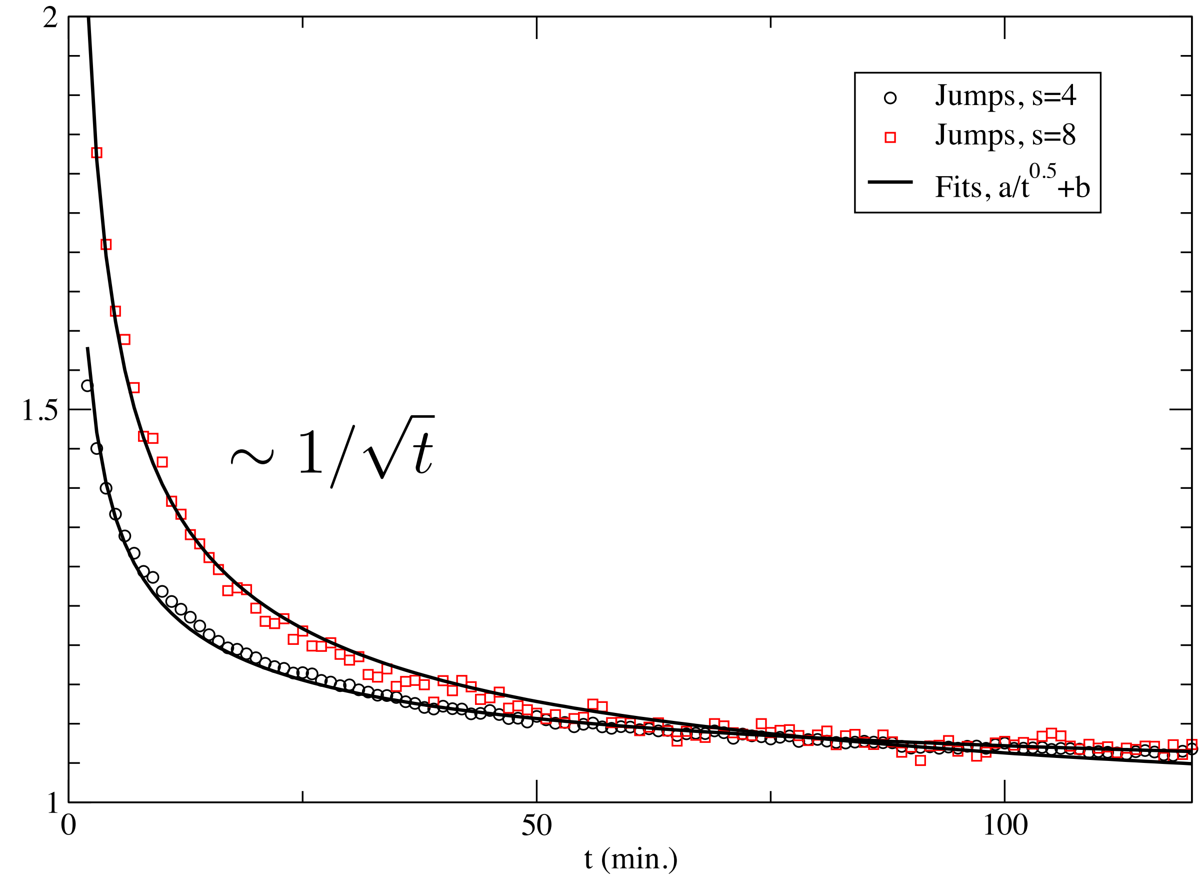
Inspired from J. Donier PhD thesis

News feeds and the Omori law

Exogenous (news-related) jumps

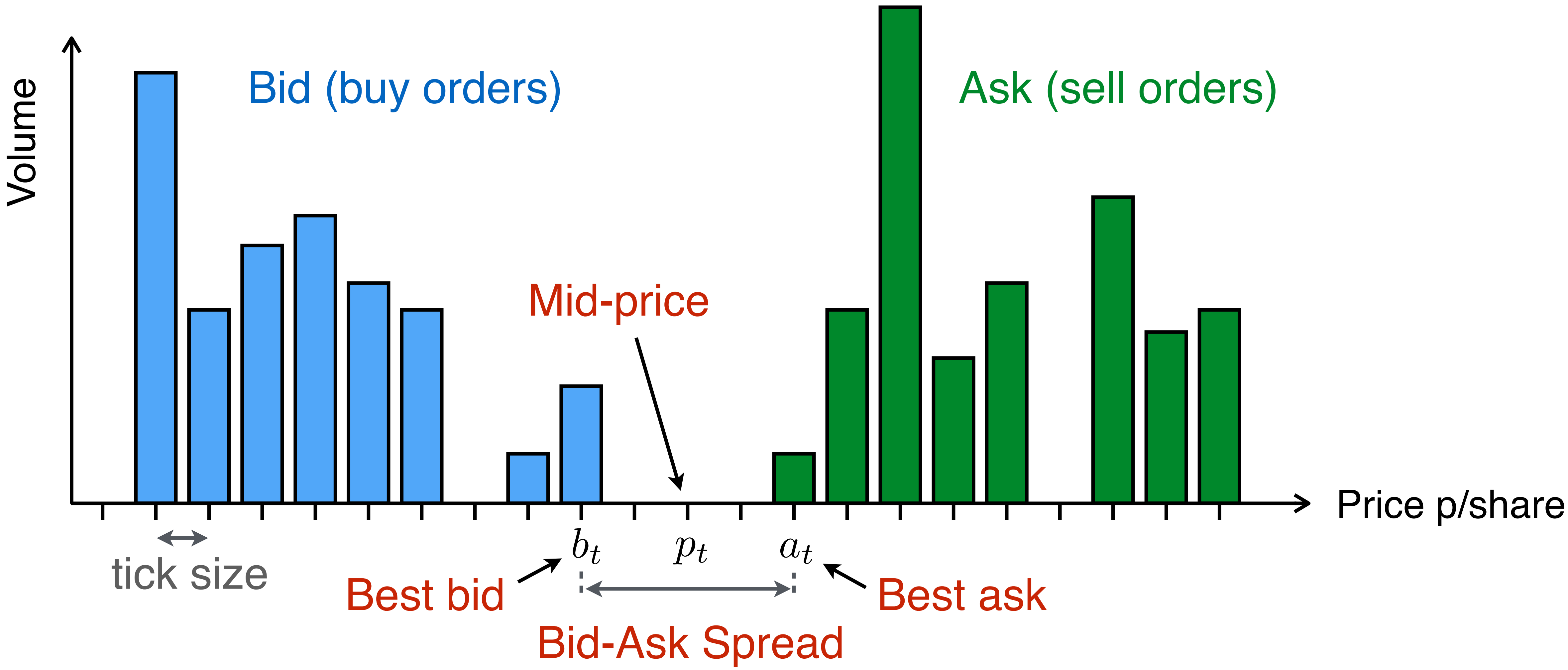


Endogenous jumps



Joulin, A., et al. "Stock price jumps: News and volume play a minor role." *Wilmott Magazine* (2008): 1-7.

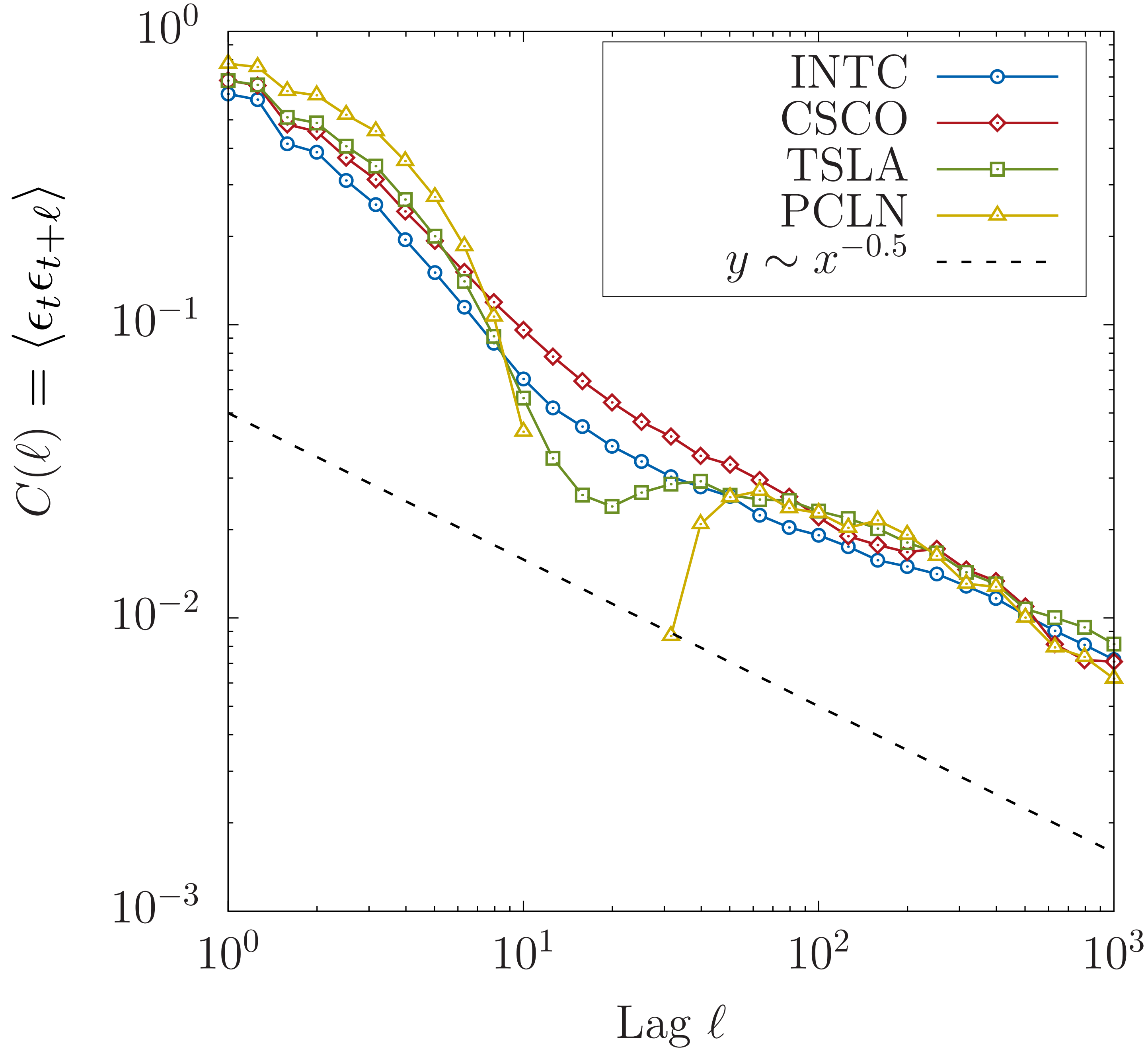
Continuous double auction markets and the limit order book



- **Limit order**: Buy or sell the item at its specified price.
- **Market order**: Buy or sell the item immediately, at the current best price.

Continuous double auction markets and the limit order book

Persistent order flow



Chapter IV: Econometric models for price changes

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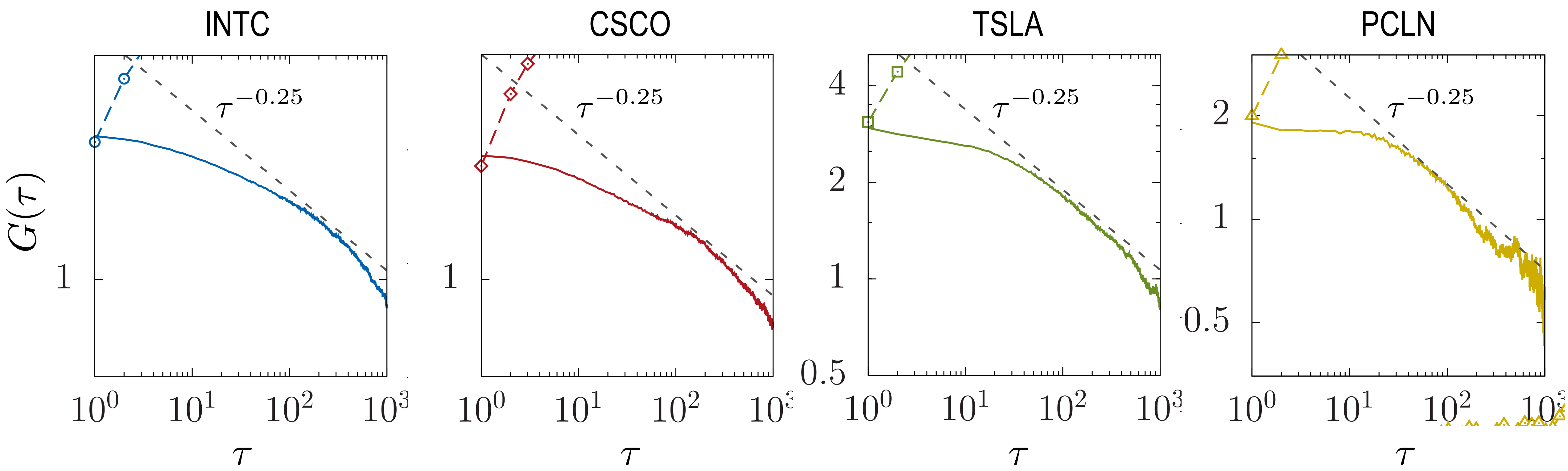
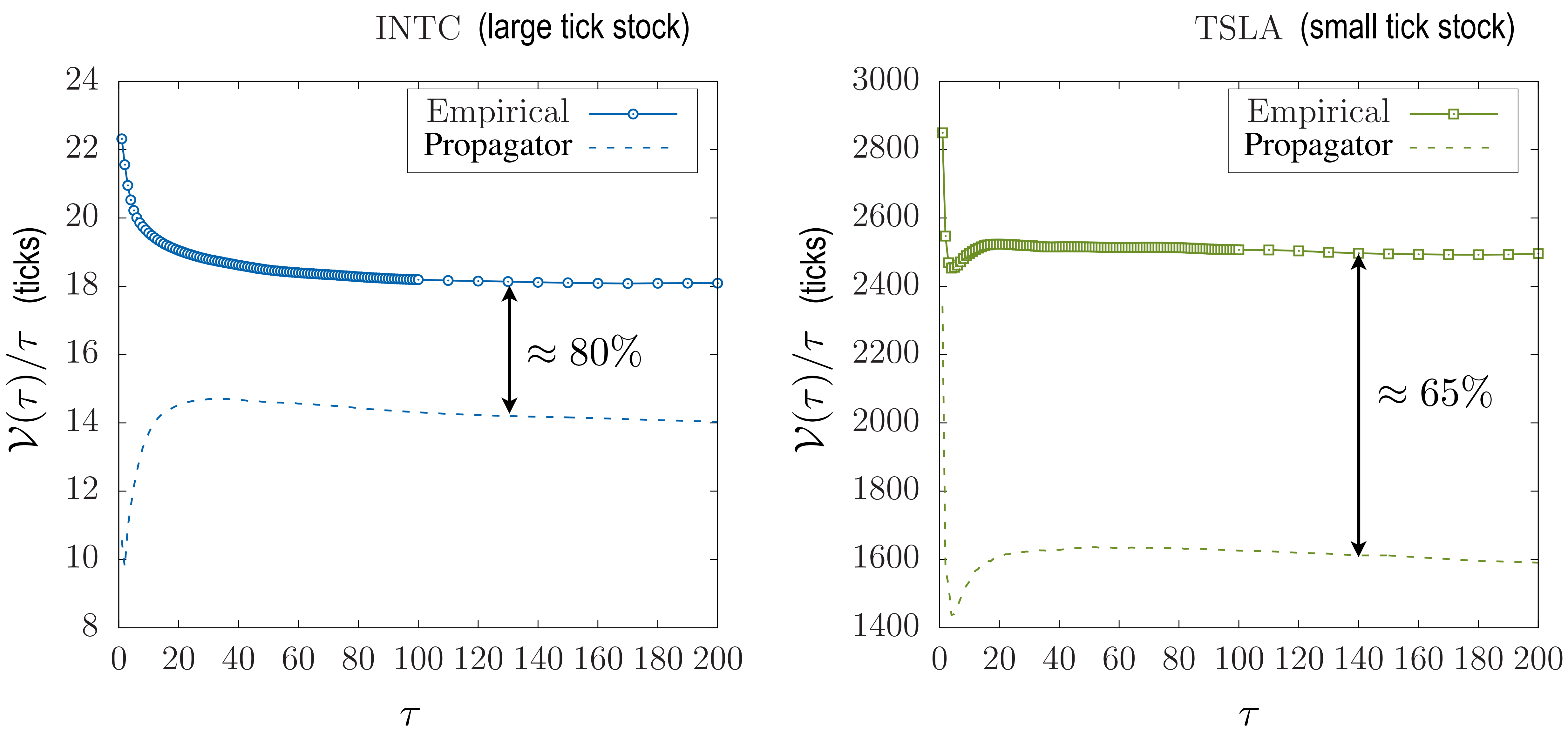
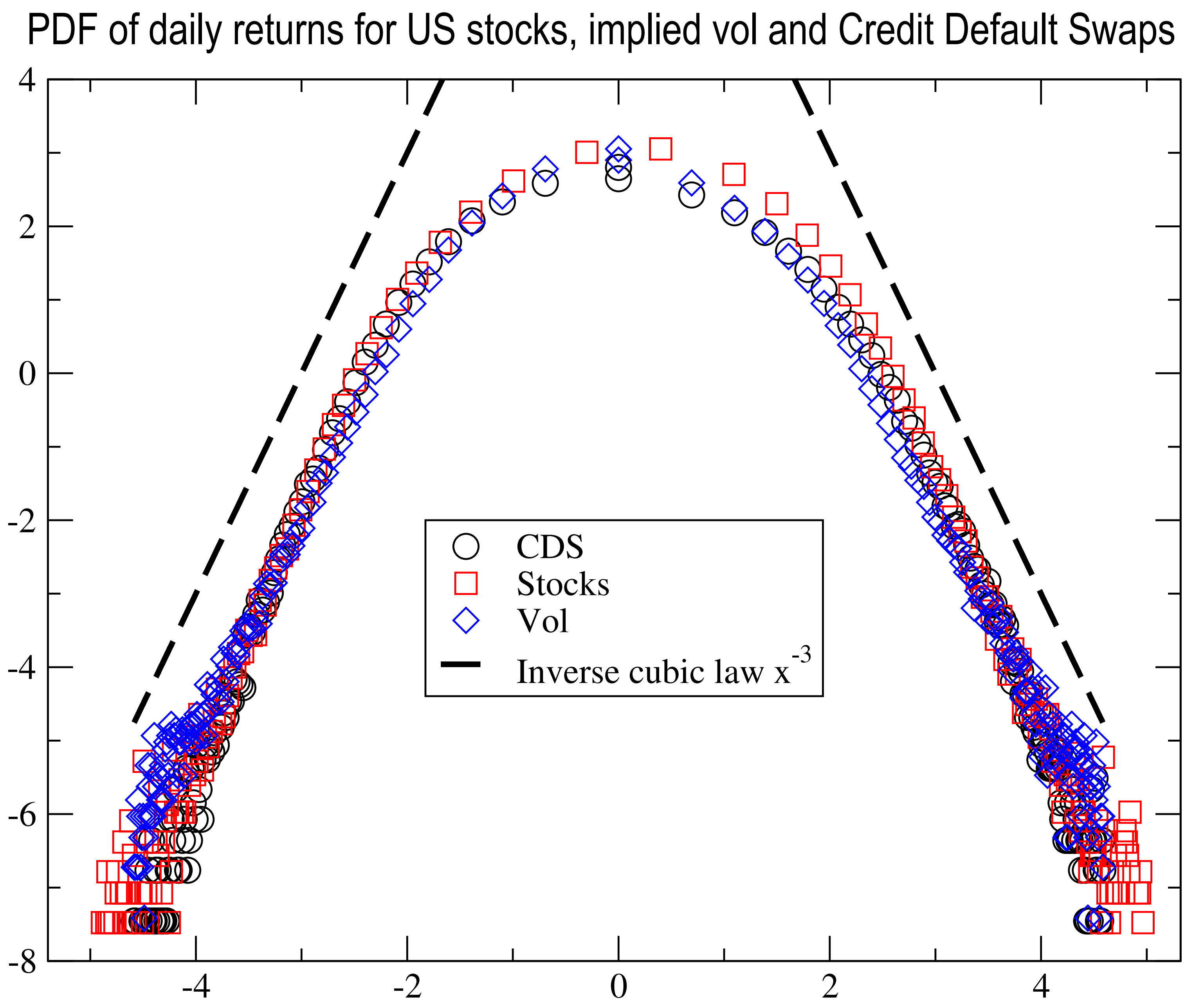
Propagator Calibration



Propagator (signature plot): how much volatility trades account for?



Pareto tails (power law)



Chapter V: Microscopic (agent based) models for price changes

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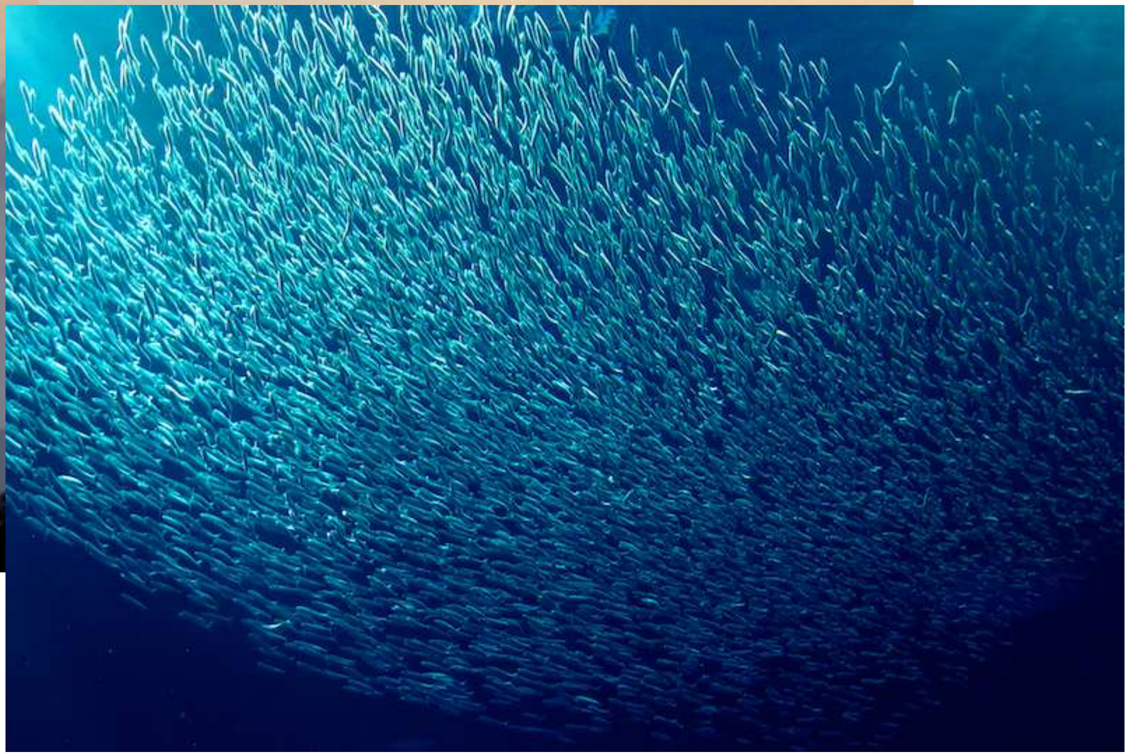
Examples from the animal world

Starlings in Rome



Fish schools

There is no leader, 10 000 birds can collectively change direction in less than 0.5 seconds

Examples from the animal world

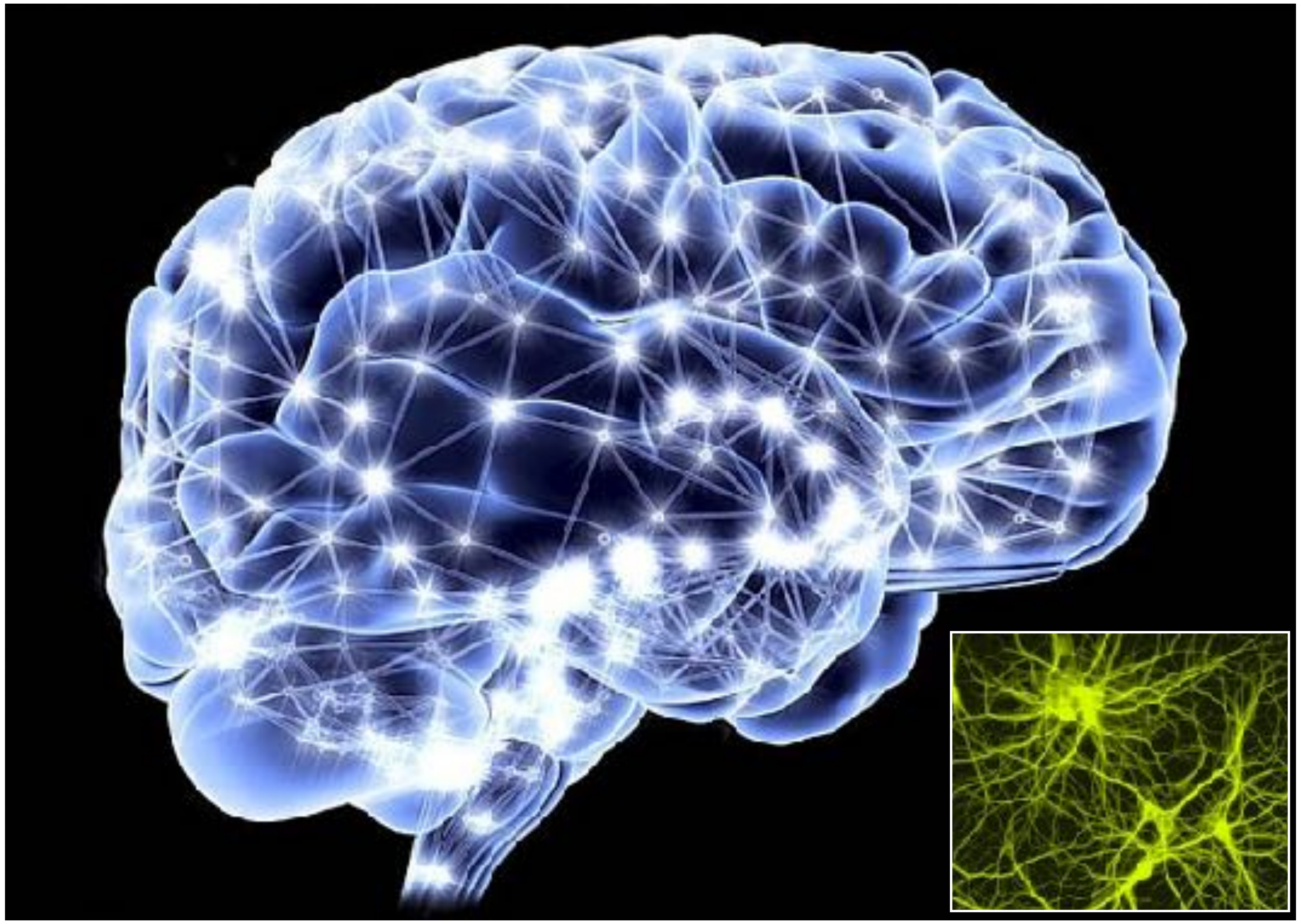
Synchronisation of fireflies

"Fireflies blinking on and off in unison (Southeast Asia), in displays that stretched for miles along the riverbanks. How could thousands of fireflies orchestrate their flashing so precisely and on such vast scales? For decades, no one could come up with a plausible theory. A few believed there must be **a maestro**, a firefly that cues all the rest. Only by the late 60's did the pieces of the puzzle begin to fall into place..."

S. H. Strogatz (Sync, The emerging science of spontaneous order)

Examples from the animal world

Neural networks



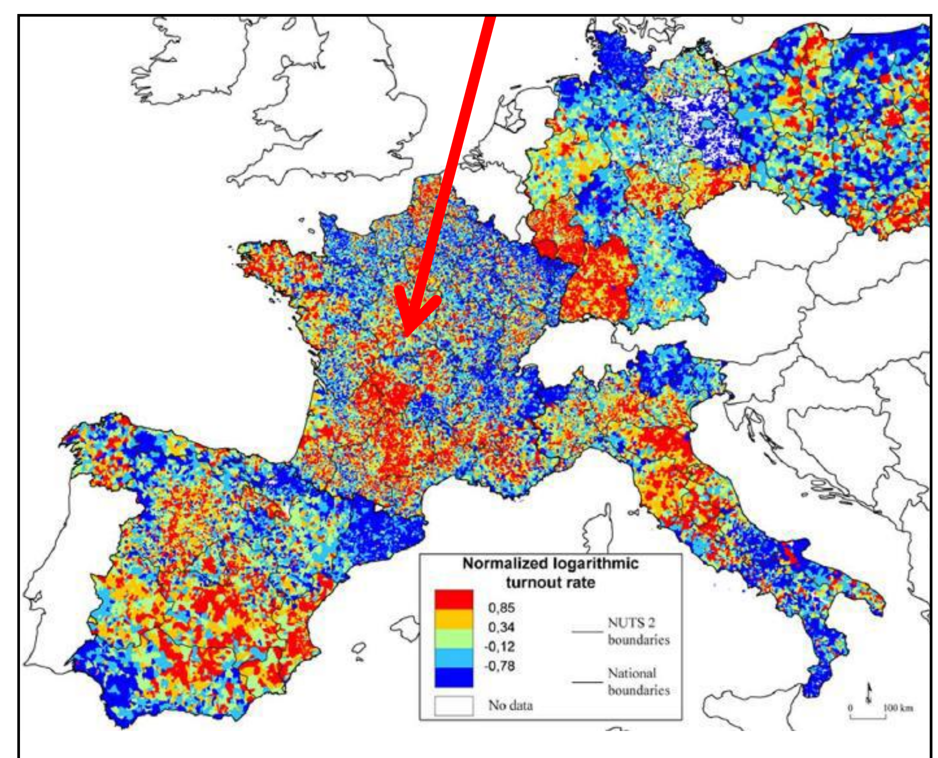
Conscience is clearly an *emerging* phenomenon, not « belonging » to any single neurone.

What about humans?

Mexican wave

Lovelocks

Voting patterns



"I can calculate the motion of heavenly bodies, but not the madness of people."

Isaac Newton

The behaviour of large assemblies of interacting individuals (particles) cannot be understood as a simple extrapolation of the properties of isolated individuals. Instead, entirely new, unanticipated behaviours may appear and their understanding requires new ideas and methods.

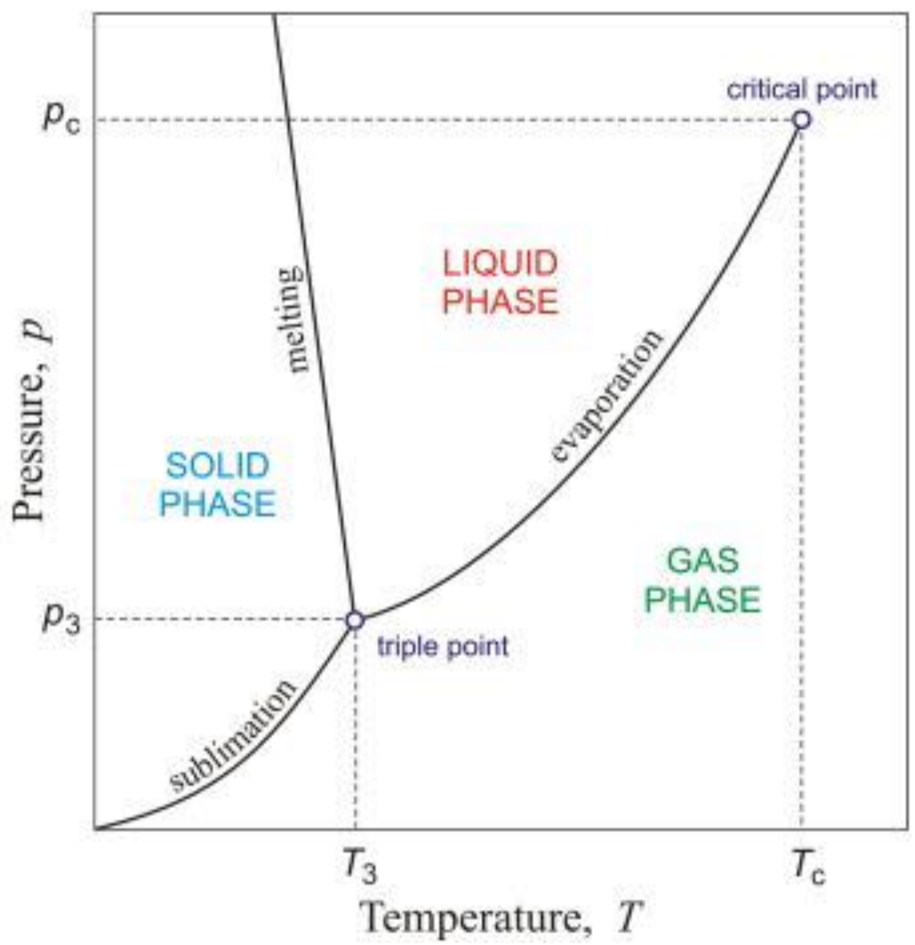
Statistical physics has developed tools to describe these “collective phenomena”, pertaining to crowds and not to any of its single constituents. Small changes at the individual level can trigger dramatic effects at the collective level - for the better or for the worse.

Adapted from Phil Anderson (Nobel prize 1977) « More is different » in Science (1972).

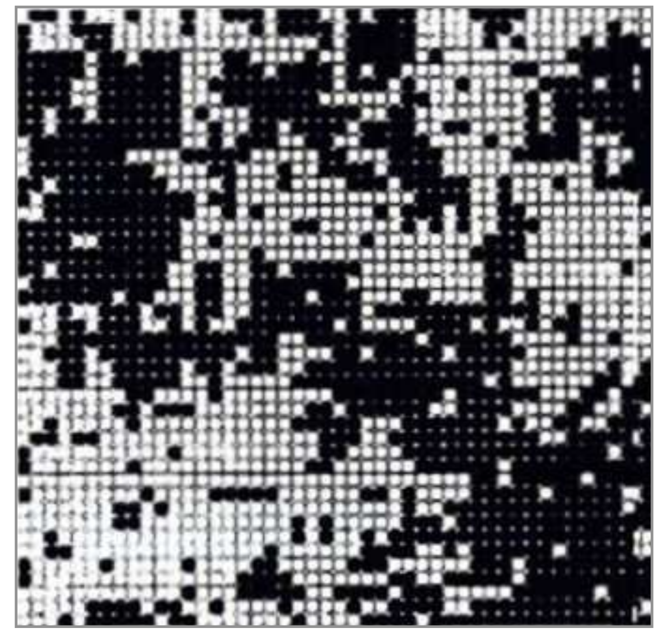
Critical phenomena

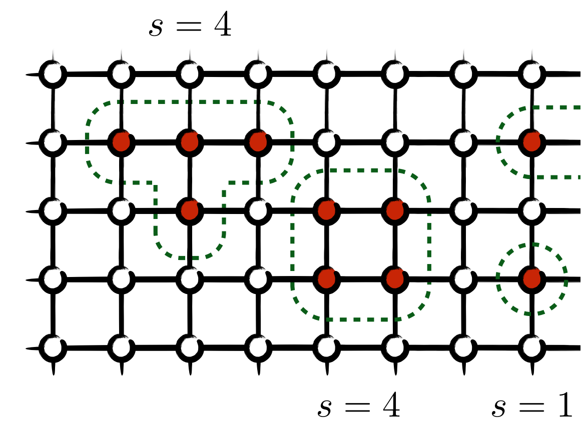
Liquid-gas coexistence at the critical point

Critical opalescence



Percolation network





Density of size s clusters

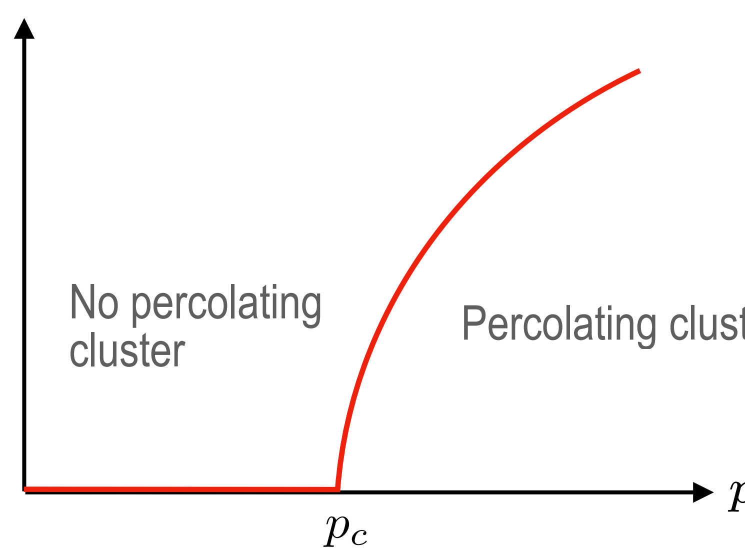
$$n(s) \sim s^{-\tau} e^{-4(p-p_c)^2 s}$$

At the critical point:

$$n_c(s) \sim s^{-\tau}$$

Fisher exponent

Probability that a randomly chosen site belongs to the percolating cluster

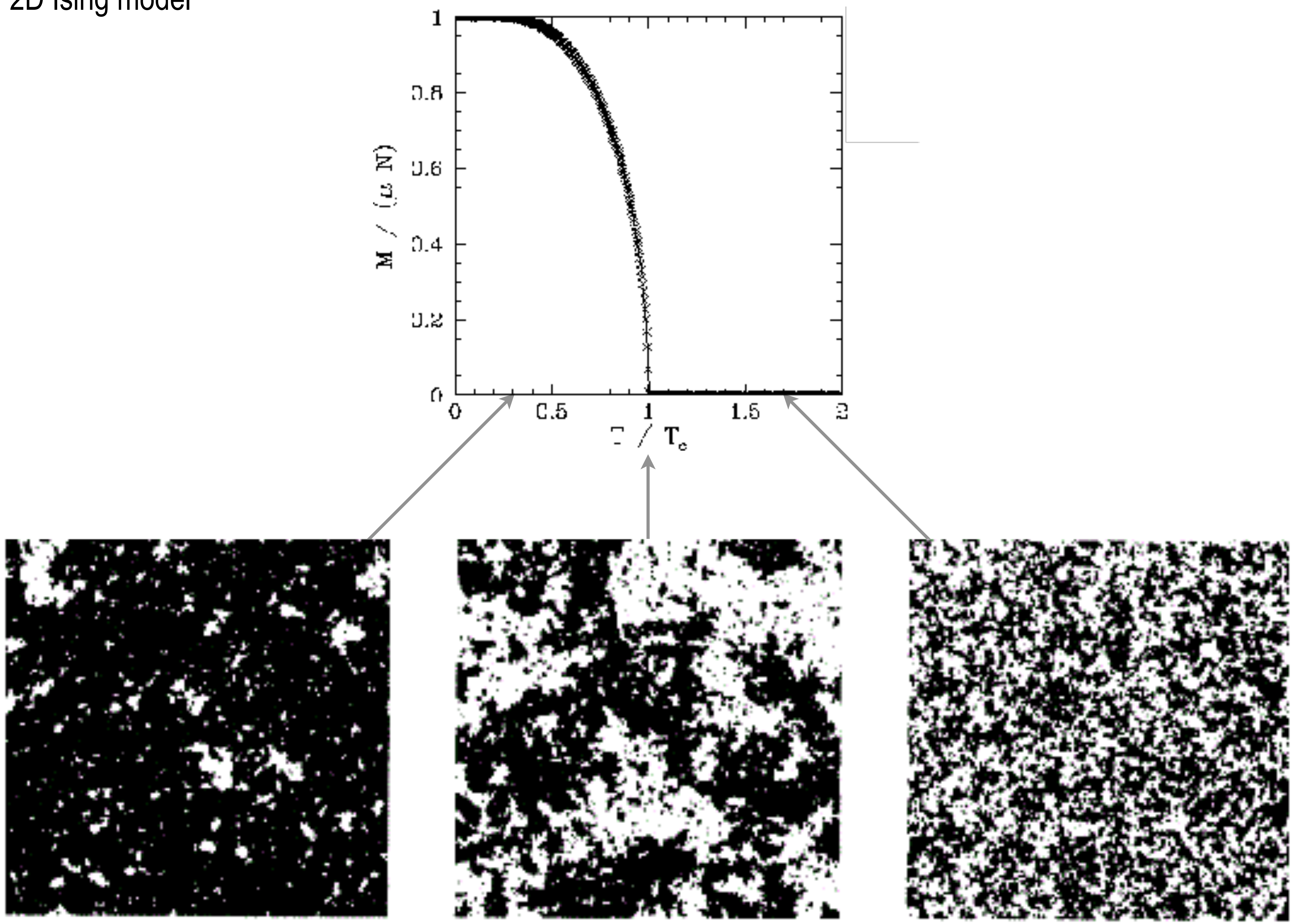


Percolating cluster exists

critical point: scale invariance etc.

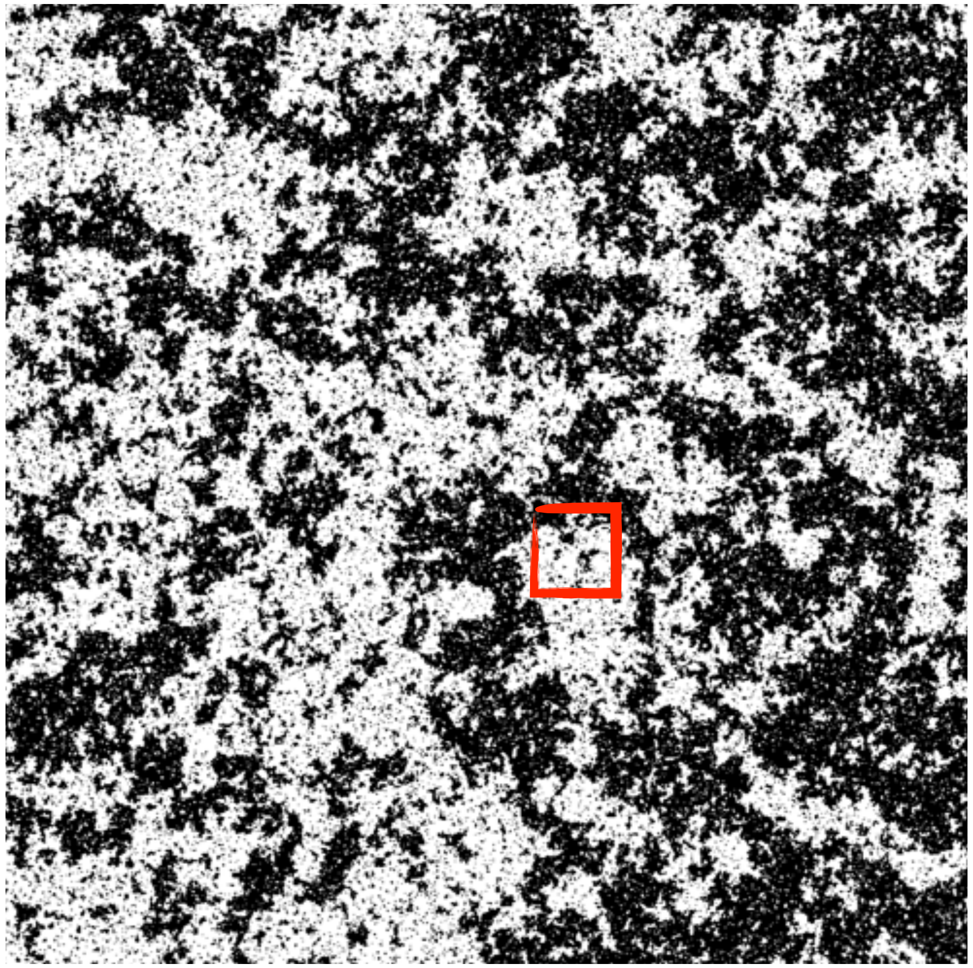
Critical phenomena

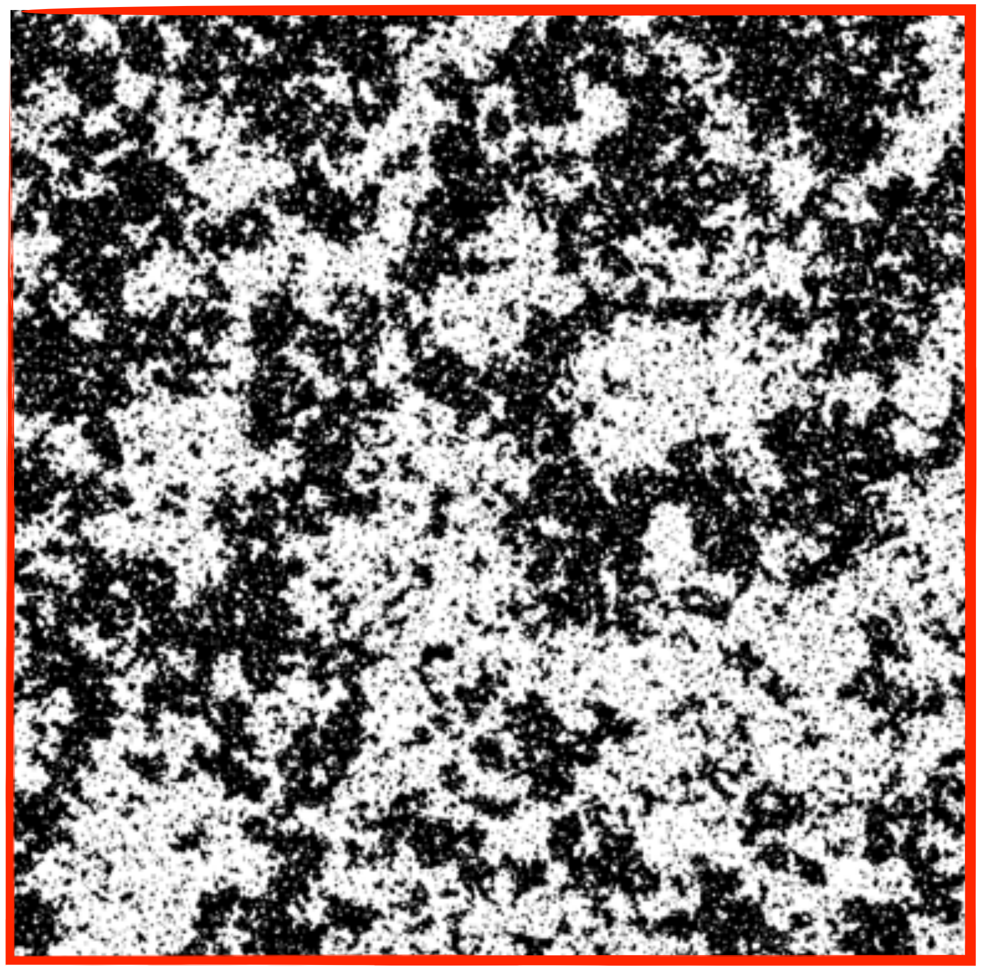
2D Ising model



Scale invariance at the critical point

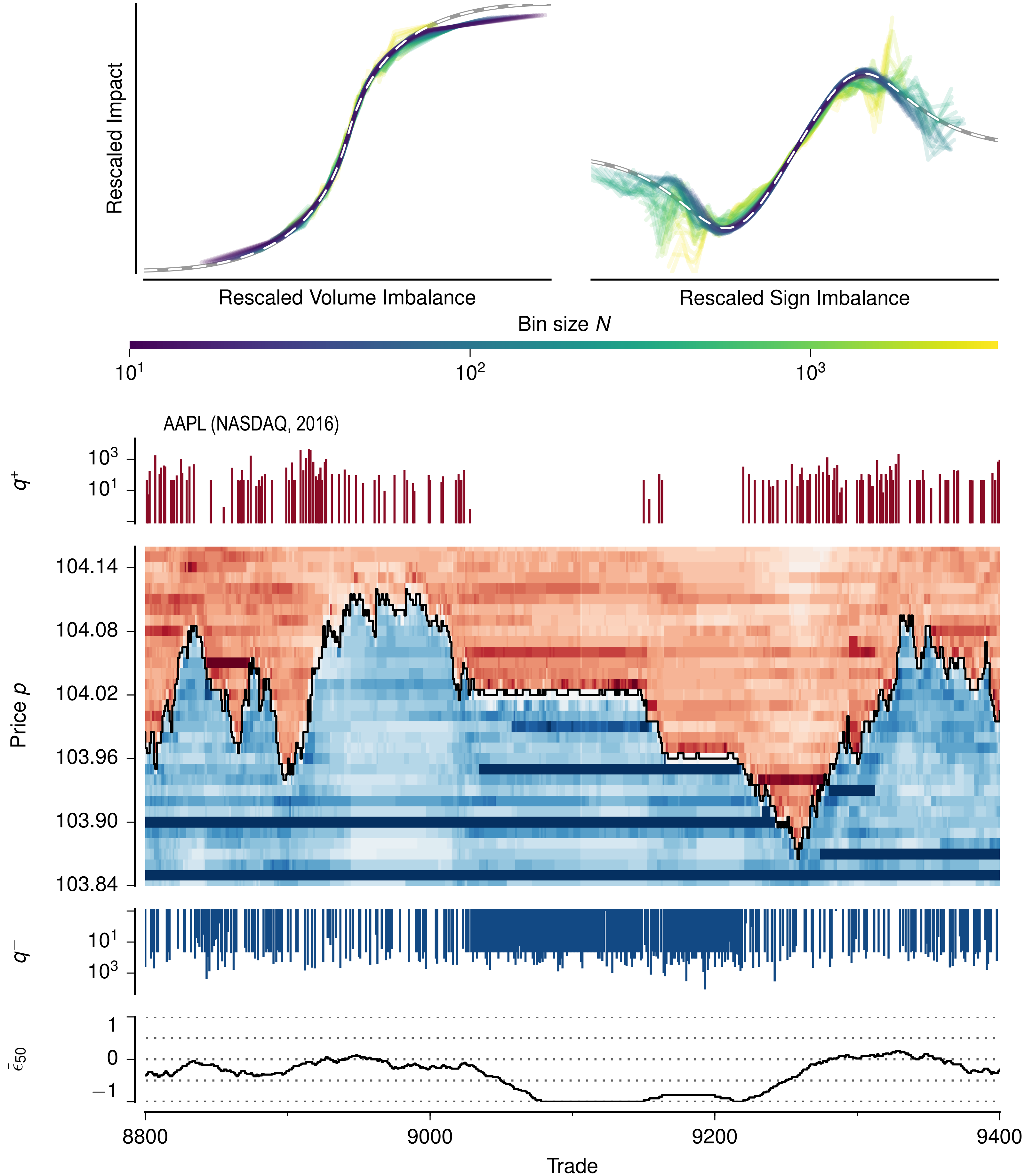
2D Ising model





Simulation de percolation au point critique $p = p_c$ à gauche, zoom $\times 10$ à droite.

Herding and percolation - aggregate impact



Aggregate-volume impact:

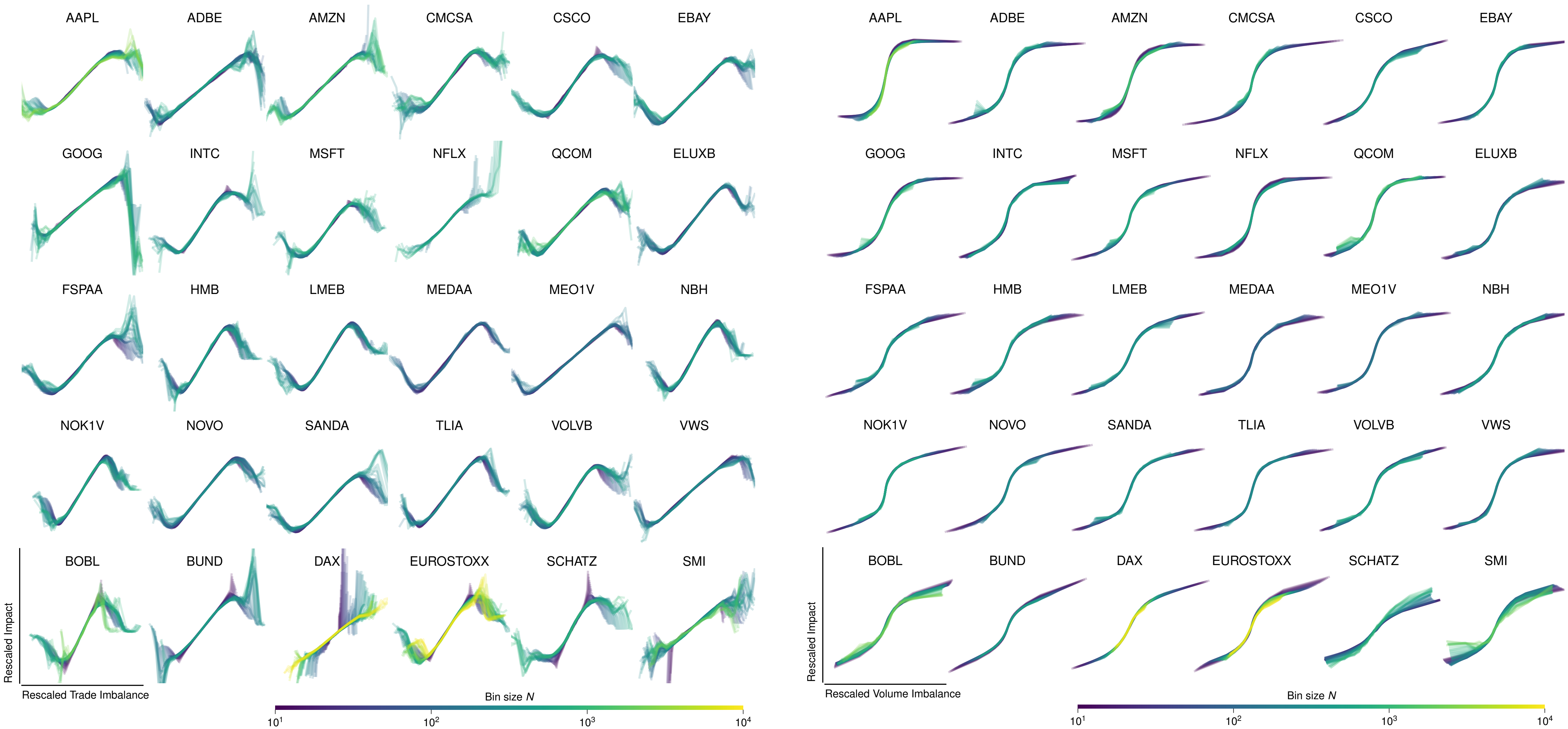
$$\mathcal{R}_N(\mathcal{Q}) := \left\langle \log m_{t+N} - \log m_t \,\middle|\, \mathcal{Q} = \sum_{i=0}^{N-1} q_{t+i} \right\rangle$$

Aggregate-sign impact:

$$\mathcal{R}_N(\mathcal{E}) := \left\langle \log m_{t+N} - \log m_t \,\middle|\, \mathcal{E} = \sum_{i=0}^{N-1} \epsilon_{t+i} \right\rangle$$

Patzelt & Bouchaud 2017

Herding and percolation - aggregate impact



Patzelt & Bouchaud 2017

2nd Schelling's model - Segregation

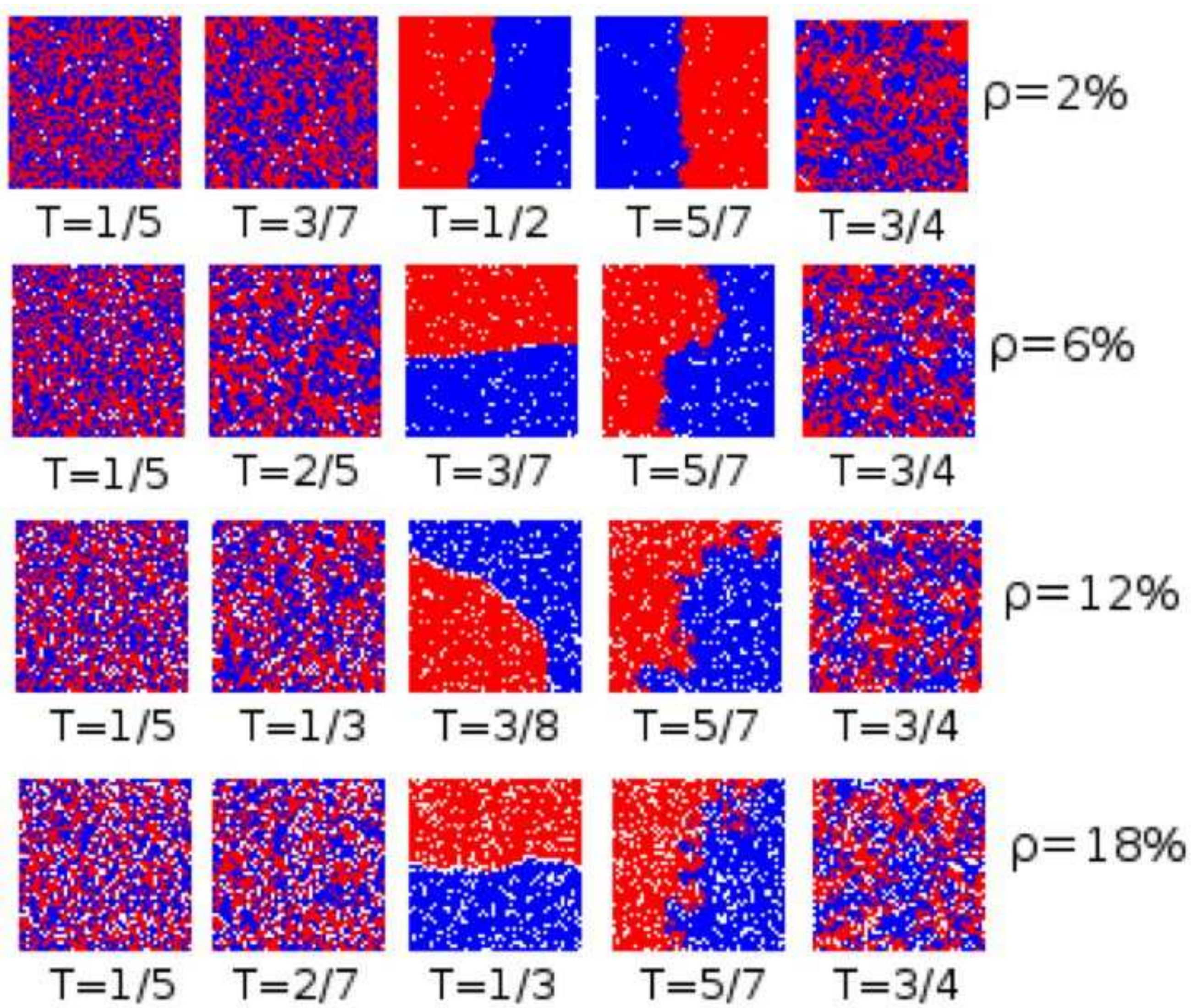


Figure 6.10: Typical configurations found in a simulation of Schelling's model (from Gauvin et al. (arXiv: 0903.4694v1)), starting from random initial conditions.

Chapter VI: Financial engineering and derivative pricing

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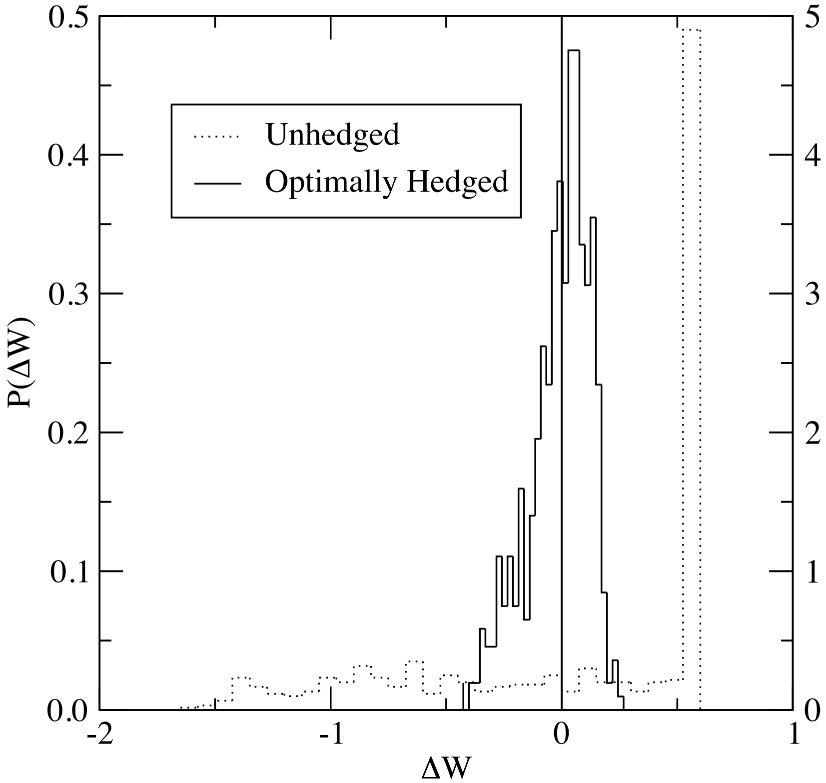
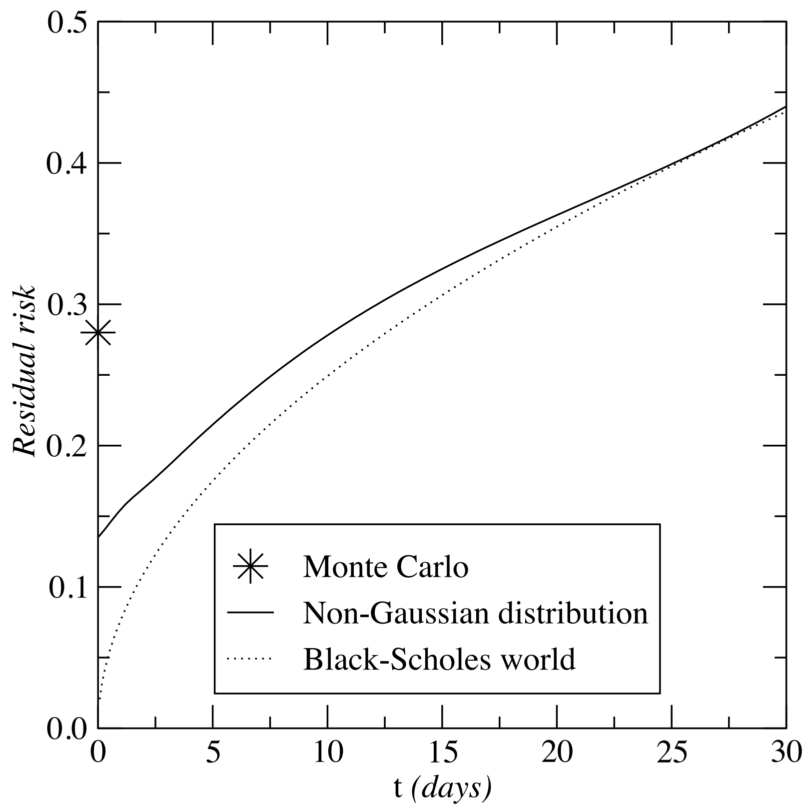
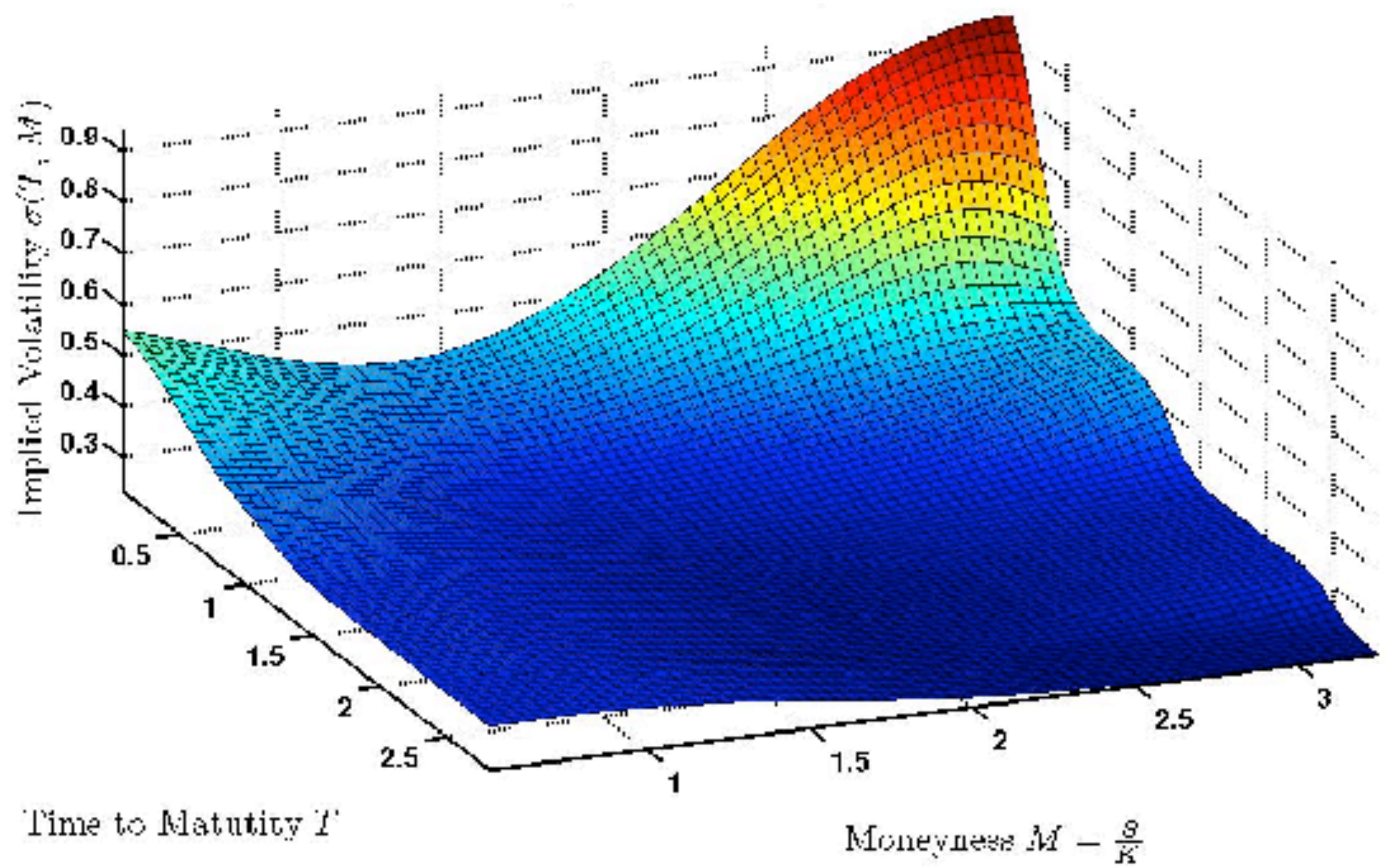
Hedging in the real world

Histogram of ΔW for an optimally hedged real option with τ = 30 min (top) and evolution of the residual risk $\langle \Delta W \rangle^2$ as a function of τ in a Gaussian world and in the real world (bottom).





The volatility surface (smile)



The modeler’s Hippocratic oath:

- I will remember that I didn’t make the world, and it doesn’t satisfy my equations.
- Though I will use models boldly to estimate value, I will not be overly impressed by mathematics.
- I will never sacrifice reality for elegance without explaining why I have done so.
- Nor will I give the people who use my model false comfort about its accuracy. Instead, I will make explicit its assumptions and oversights.
- I understand that my work may have enormous effects on society and the economy, many of them beyond my comprehension.

Emanuel Derman & Paul Wilmott, 2009

The Financial Modelers' Manifesto was a proposal for more responsibility in risk management and quantitative finance written by financial engineers Emanuel Derman and Paul Wilmott. The Manifesto and Oath were written in response to the Financial crisis of 2007-2008 with the collapse of subprime mortgages. Note that both authors had written extensively about the risks related to financial models for several years before the crisis; for example:

Emanuel Derman in 1996: "There are always implicit assumptions behind a model and its solution method. But human beings have limited foresight and great imagination, so that, inevitably, a model will be used in ways its creator never intended. This is especially true in trading environments… but it’s also a matter of principle: you just cannot foresee everything. So, even a “correct” model, “correctly” solved, can lead to problems. The more complex the model, the greater this possibility."

Paul Wilmott in 2000: "Unfortunately, as the mathematics of finance reaches higher levels so the level of common sense seems to drop. There have been some well publicised cases of large losses sustained by companies because of their lack of understanding of financial instruments…. It is clear that a major rethink is desperately required if the world is to avoid a mathematician-led market meltdown."

Chapter VII: Market impact of metaorders

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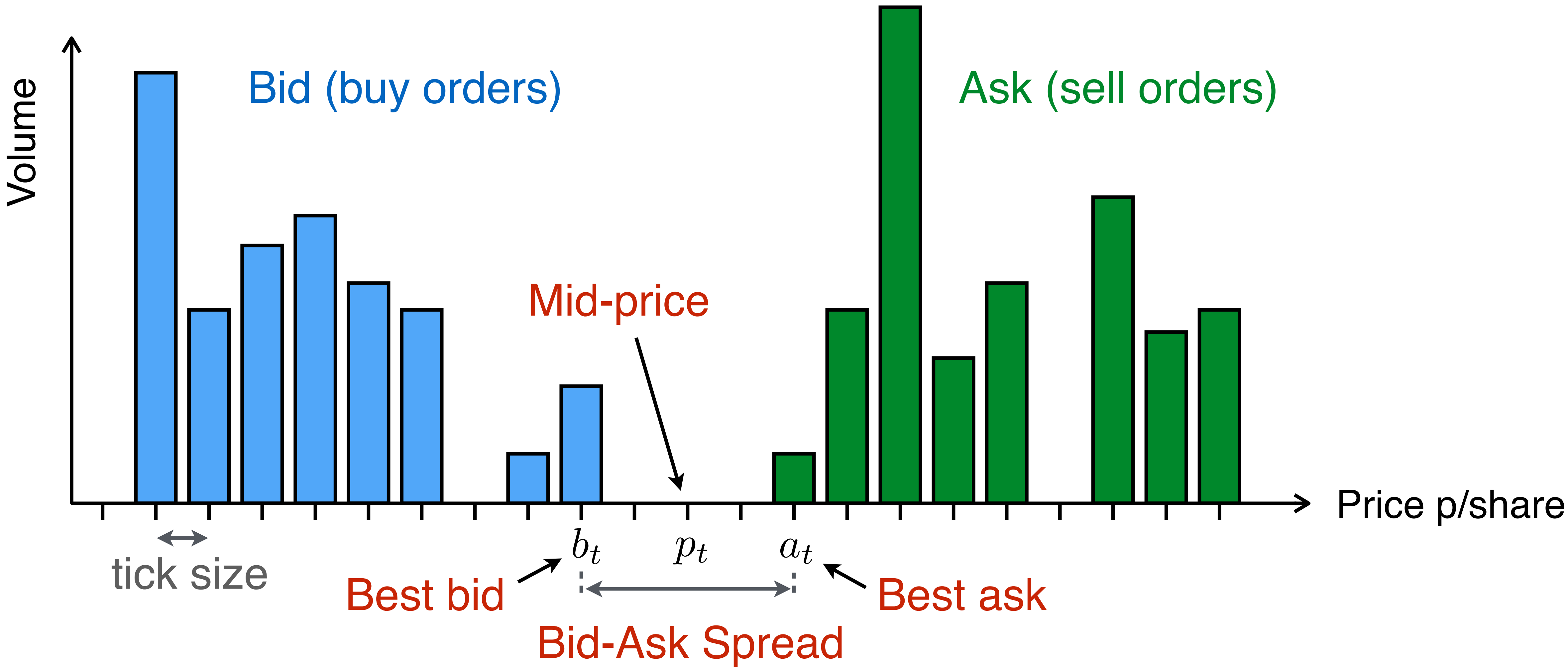
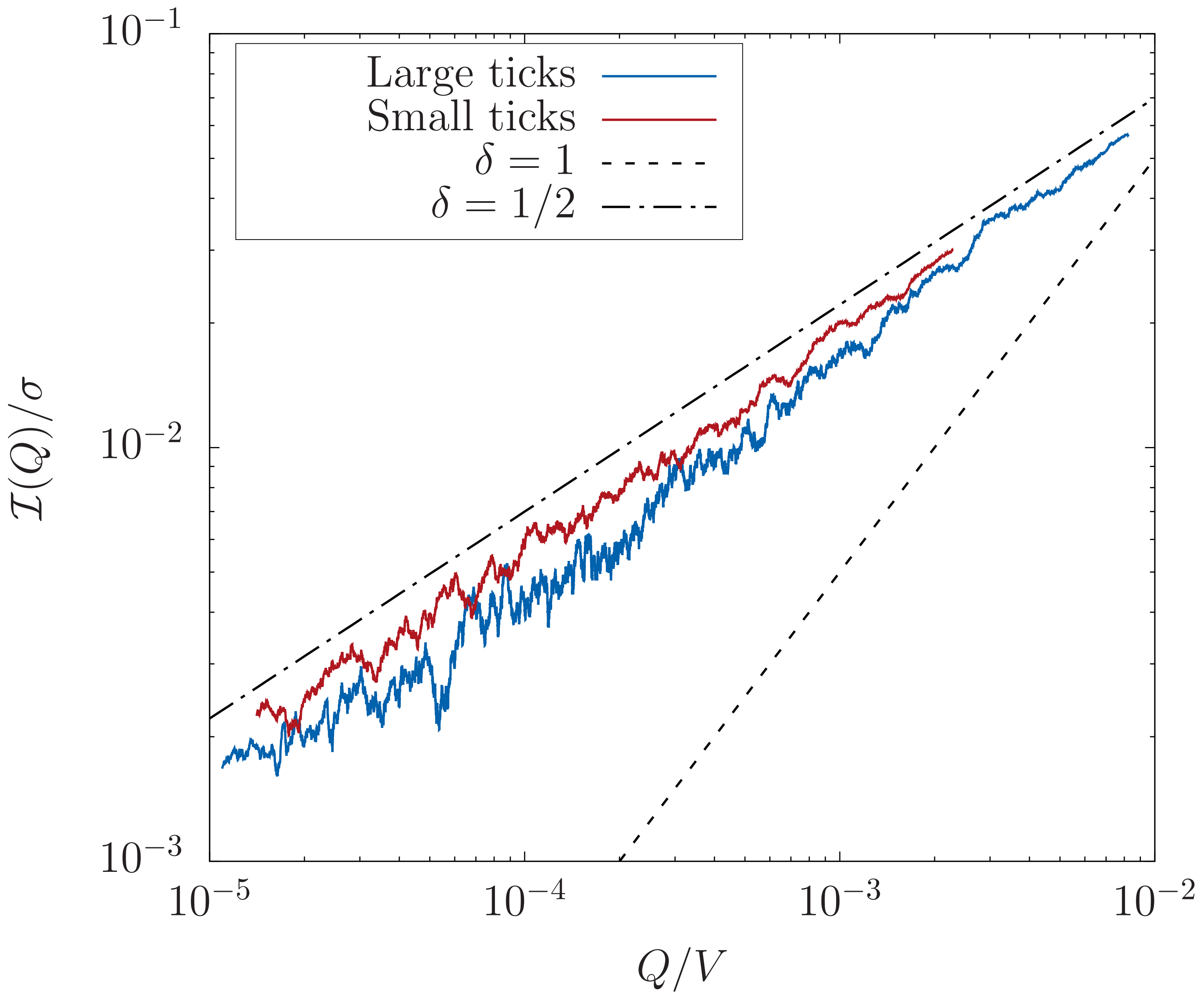
Continuous double auction markets and the limit order book



- **Limit order**: Buy or sell the item at its specified price.
- **Market order**: Buy or sell the item immediately, at the current best price.

Empirical square-root impact

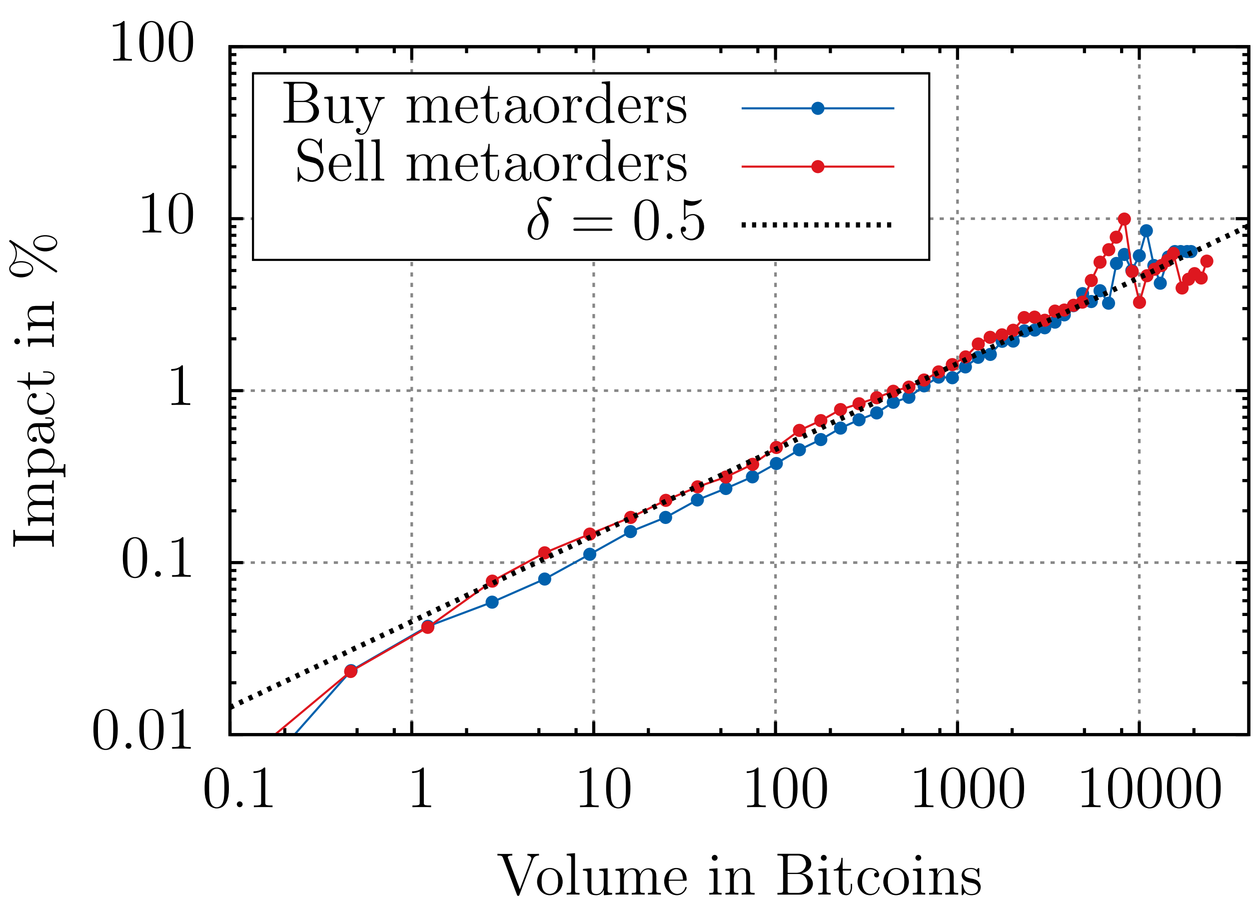


Impact of **Capital Fund Management proprietary trades** on futures markets from June 2007 to December 2010

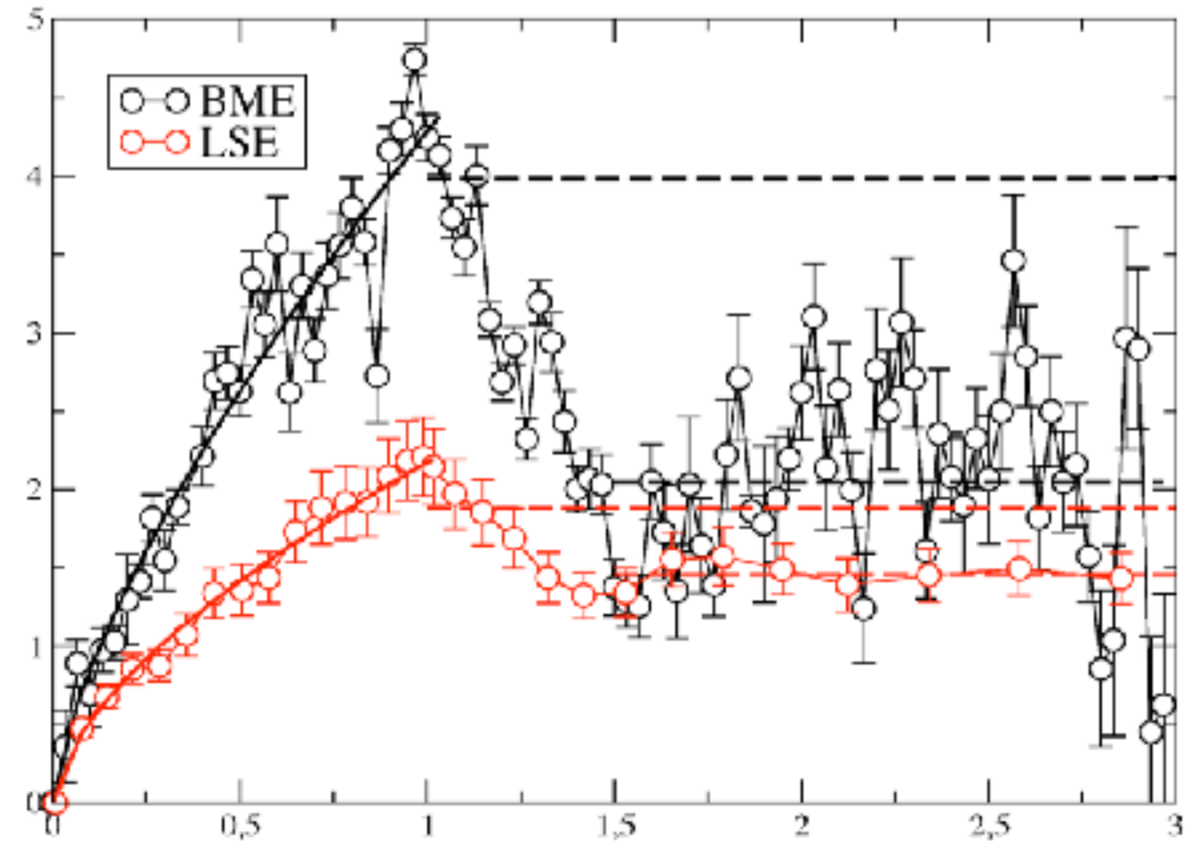
Toth et al. 2011

Empirical square-root impact

Impact of **Bitcoin trades**



Impact decay and permanent impact?



Donier & Bonart 2015

Chapter VIII: Latent order book models

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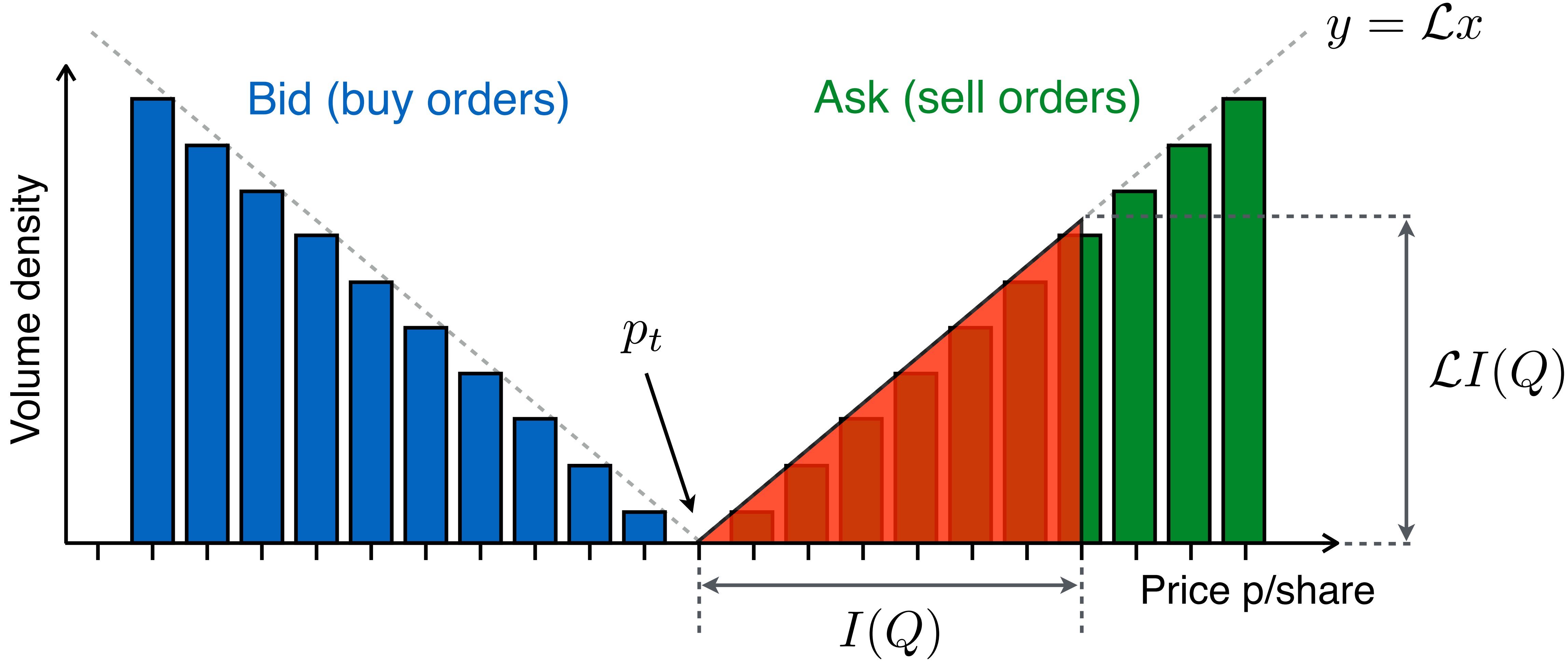
Intuition: For the impact to be square root, the OB must be linear (static vision).



Order of size Q

The area of is: $Q = \frac{1}{2}(I \times \mathcal{L}I)$ $\Longrightarrow$ $I(Q) = \sqrt{\frac{2Q}{\mathcal{L}}}$

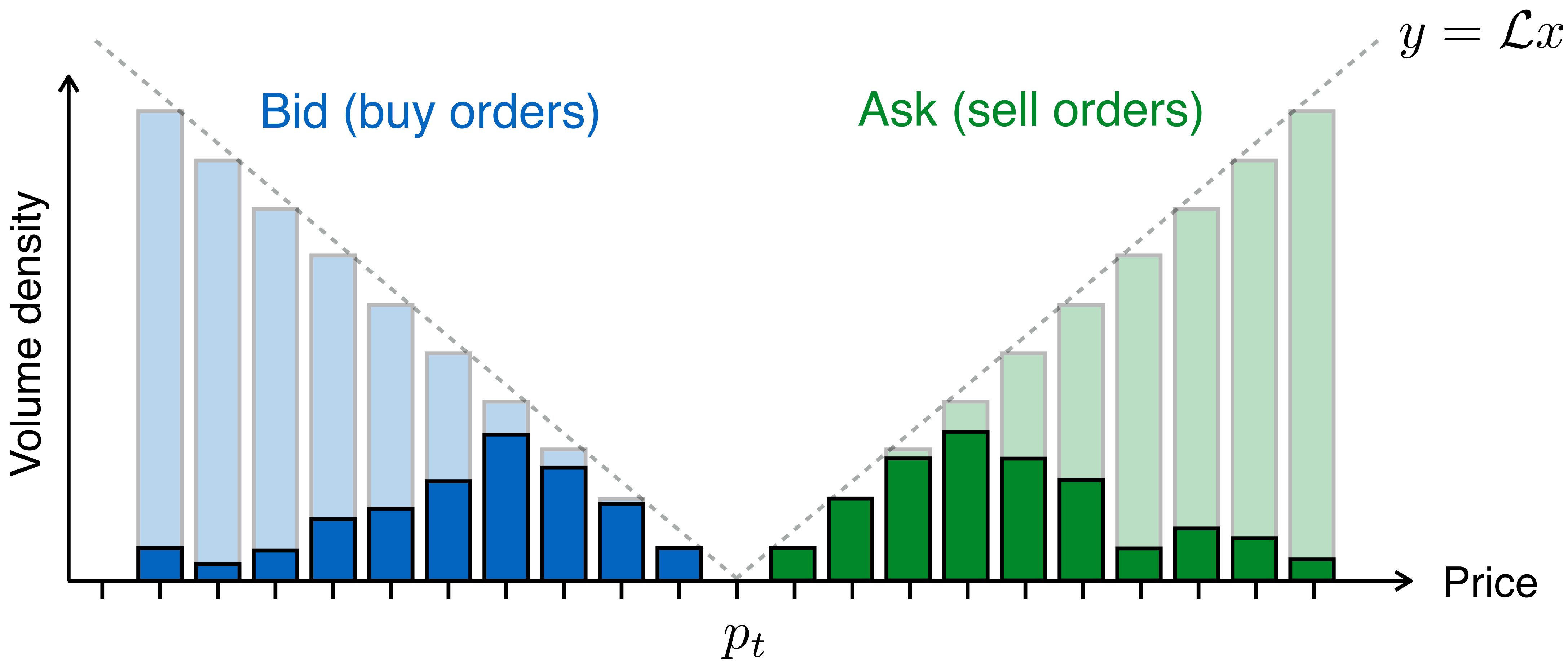
$\mathcal{L}$ is the liquidity of the market.

Square root impact!

Donier & Bonart 2015

The instantly available volume is typically 1% of the daily traded volume

- **Latent order book**

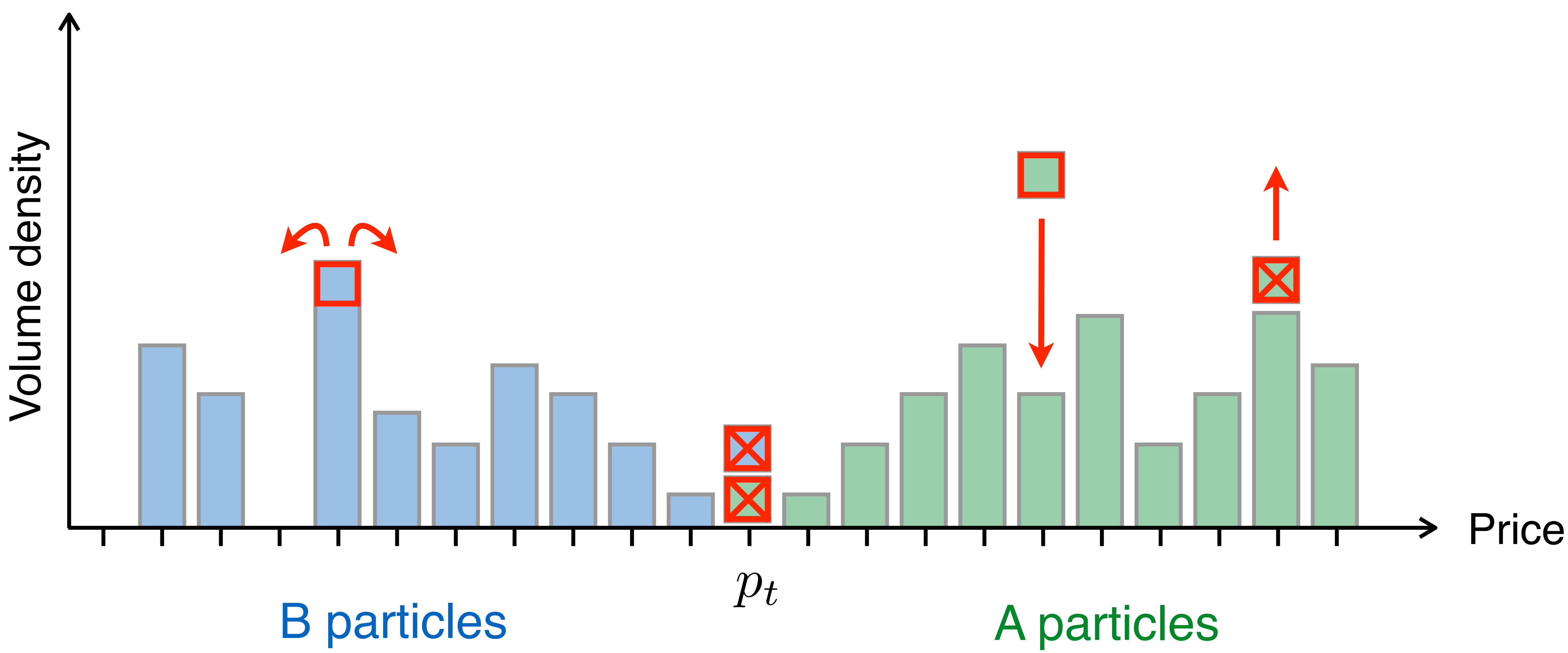


Intentions ( ) reveal themselves ( ) as they get close to the price

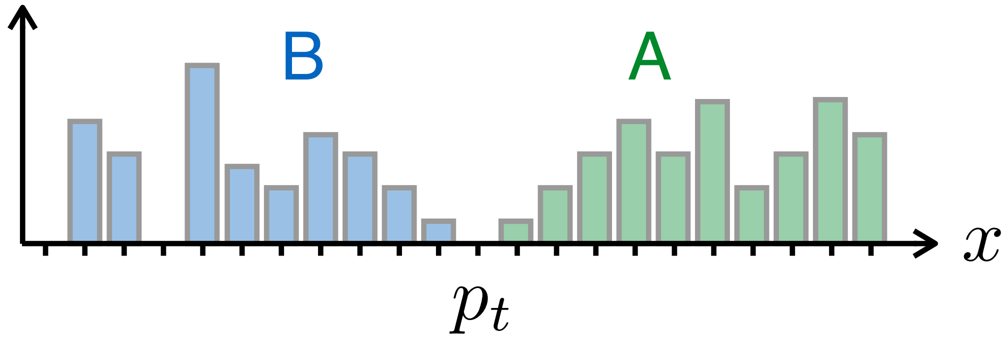
It does make sense: No incentive on giving away private information!

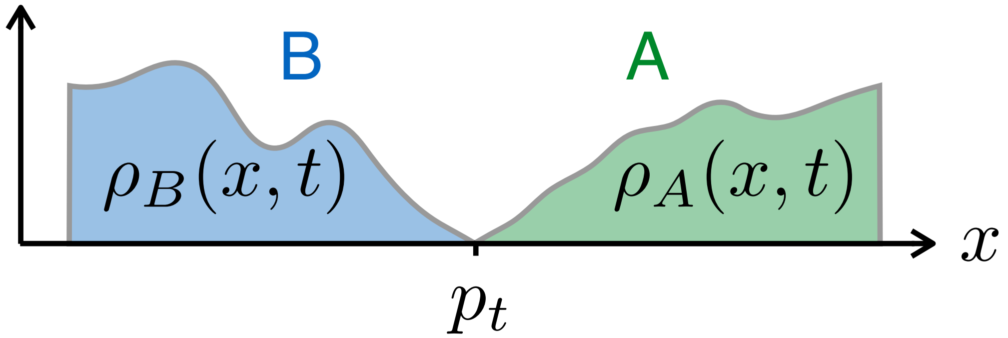
The model

- Drift-diffusion: People can change their minds and revise their intentions (drift V_t, diffusion D)
- Cancellations: Departures (rate ν)
- Depositions: New arrivals (rate λ)
- Reaction: Market-clearing $\mathrm{A} + \mathrm{B} \to \varnothing$ (rate κ)

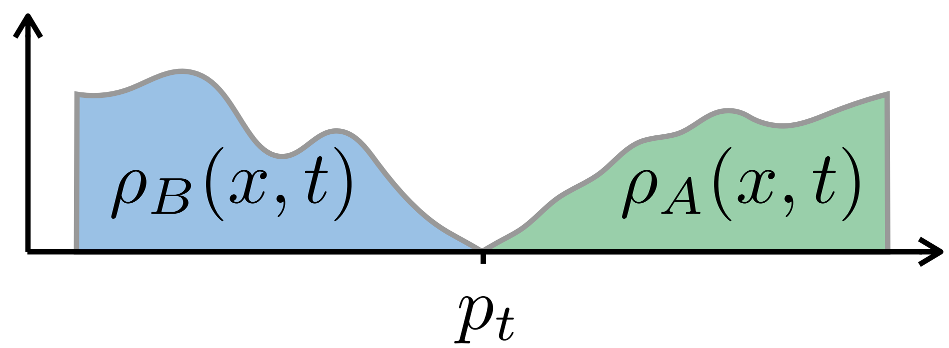


Continuous space approximation





Governing equations



Bid side: $$\partial_t \rho_B = \underbrace{-V_t \partial_x \rho_B + D \partial_{xx} \rho_B}_{\text{Drift}-\text{Diffusion}} \underbrace{- \nu \rho_B}_{\text{Cancel}} + \underbrace{\lambda \Theta(p_t - x)}_{\text{Deposition}} \underbrace{- \kappa \rho_A \rho_B}_{\text{Reaction}}$$

Ask side: $$\partial_t \rho_A = -V_t \partial_x \rho_A + D \partial_{xx} \rho_A - \nu \rho_A + \lambda \Theta(x - p_t) - \kappa \rho_A \rho_B$$

Price equation: $$\rho_B(p_t, t) = \rho_A(p_t, t)$$

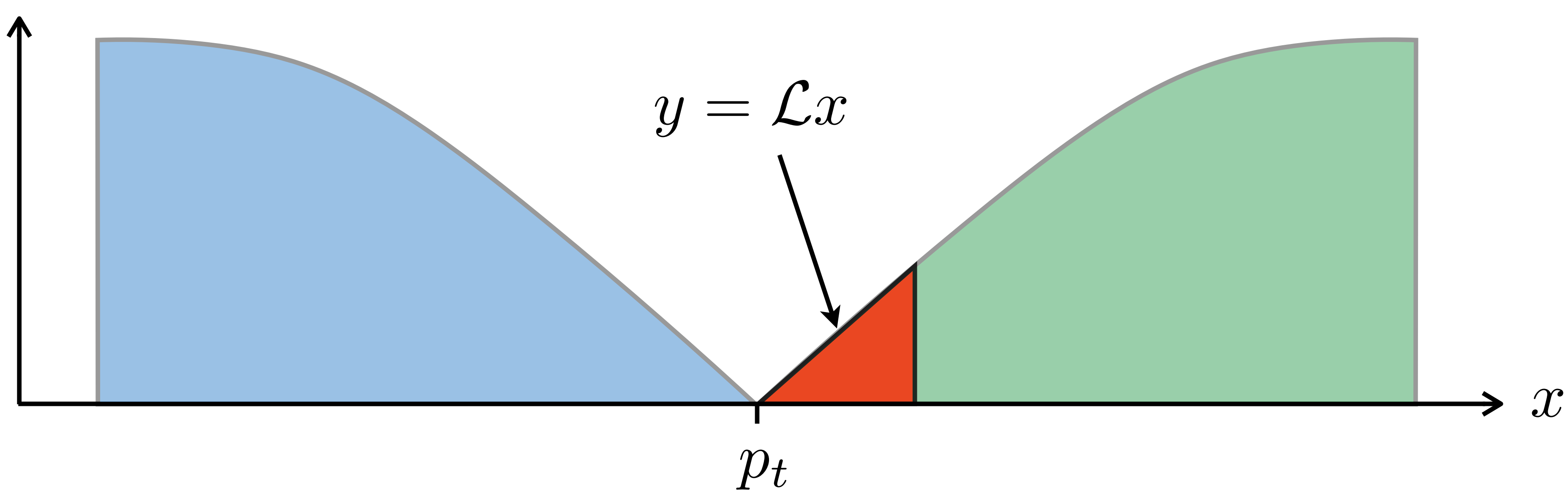
- The drift term can be reabsorbed through: $x \to x - \int_0^t \mathrm{d}t\, V_t$
- The nonlinear reaction term can be disposed of by considering:

$$\phi(x,t) = \rho_B(x,t) - \rho_A(x,t)$$

Governing equations

$$\partial_t \phi \ = D\partial_{xx}\phi - \nu\phi + \lambda\phi \ \mathrm{sgn}(p_t - x)$$
$$\phi(p_t, t) = 0$$

Equilibrium book



- Locally linear order book (LLOB)

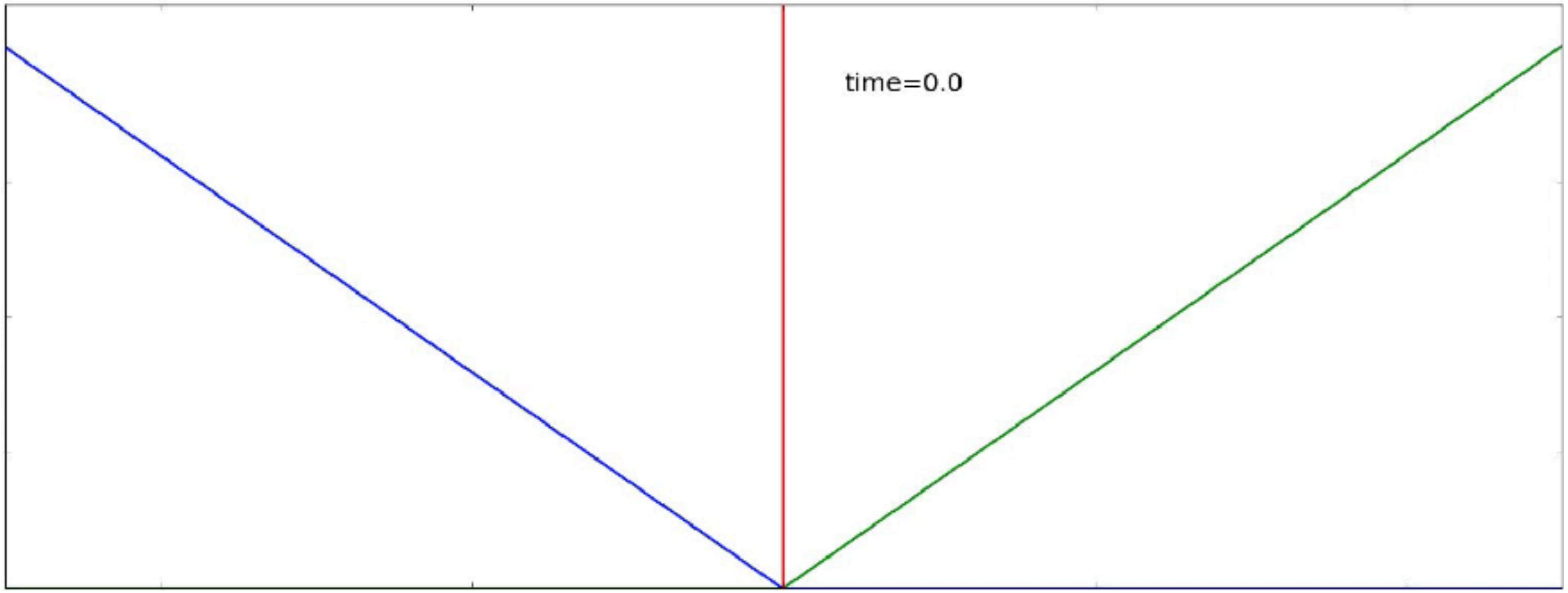
Liquidity $\mathcal{L} = \lambda\sqrt{D/\nu}$

Metaorder $\partial_t \phi = D\partial_{xx}\phi - \nu\phi + \lambda\phi\,\mathrm{sgn}(p_t - x) + \boldsymbol{m_t \delta(x - p_t)}$

$\phi(p_t, t) = 0$

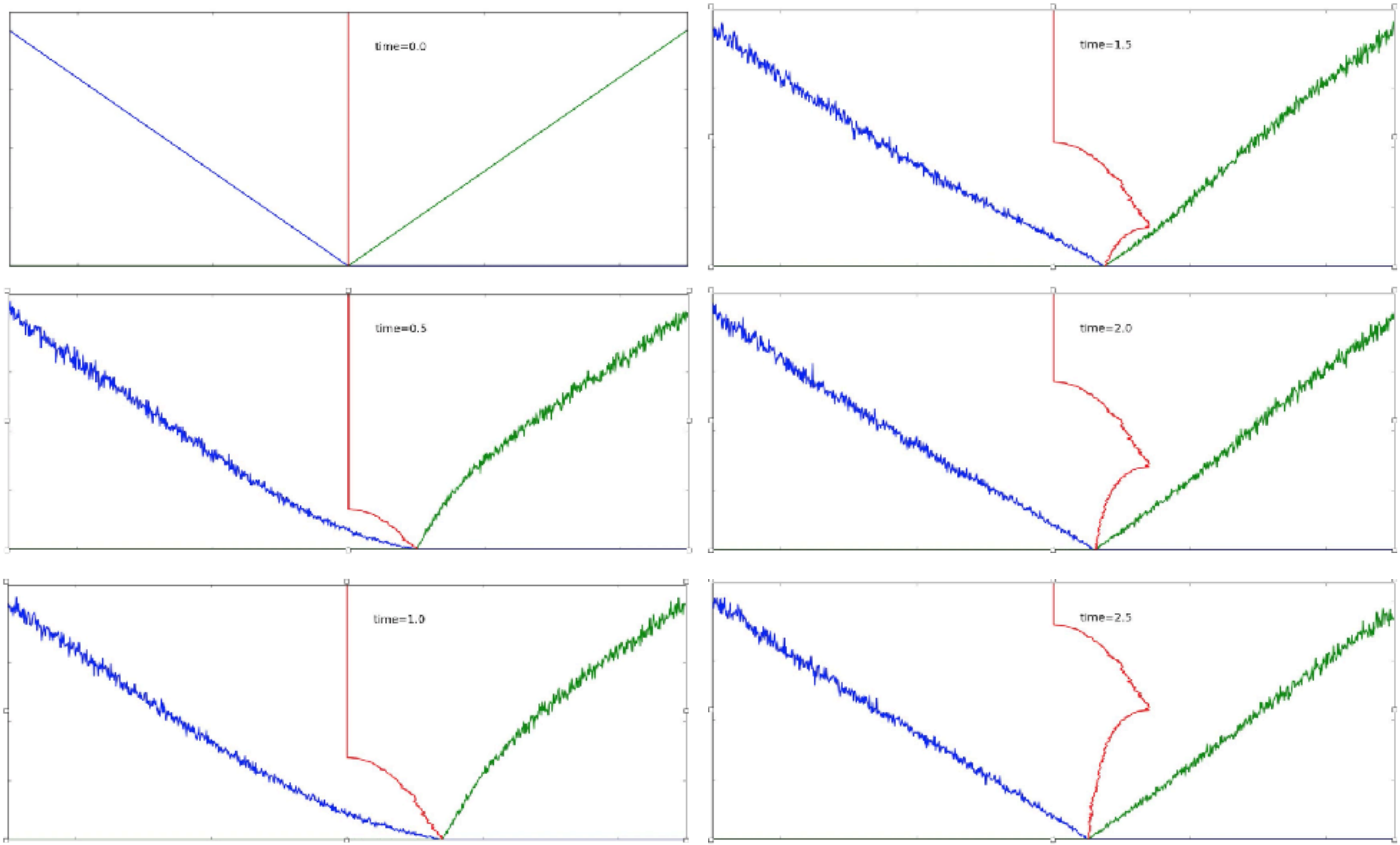
Additional flux of orders at p_t

Price dynamics: Constant execution rate $\boldsymbol{m_t = m_0}$ for $t \in [0, T]$



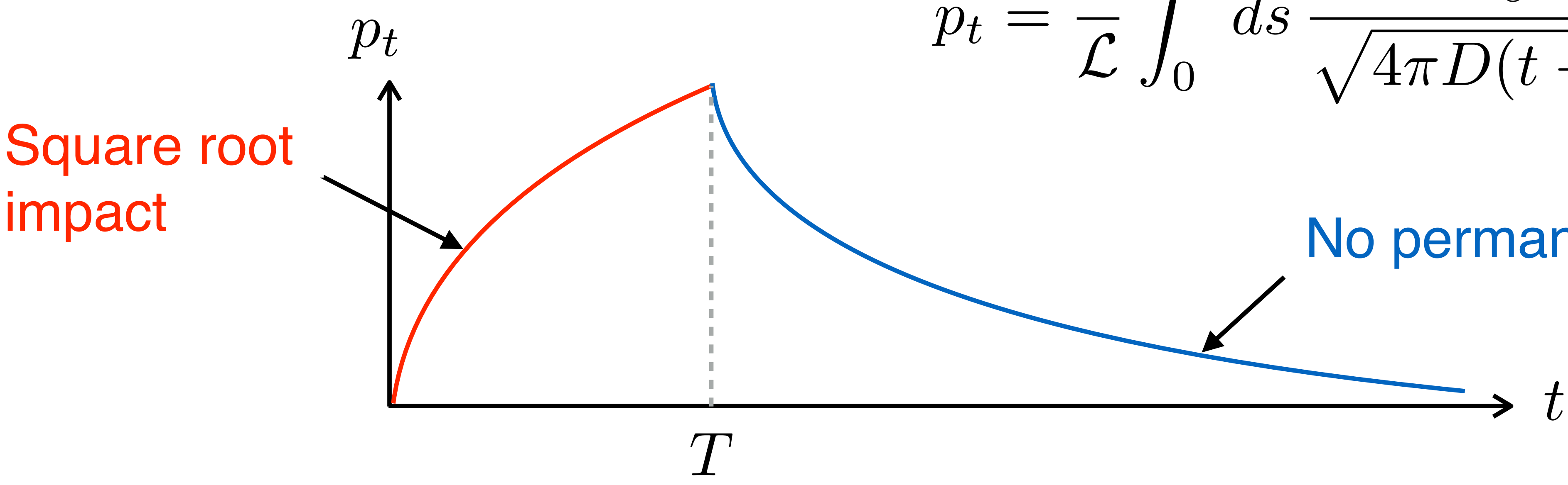
Courtesy of J. Donier

Reaction-diffusion order book model

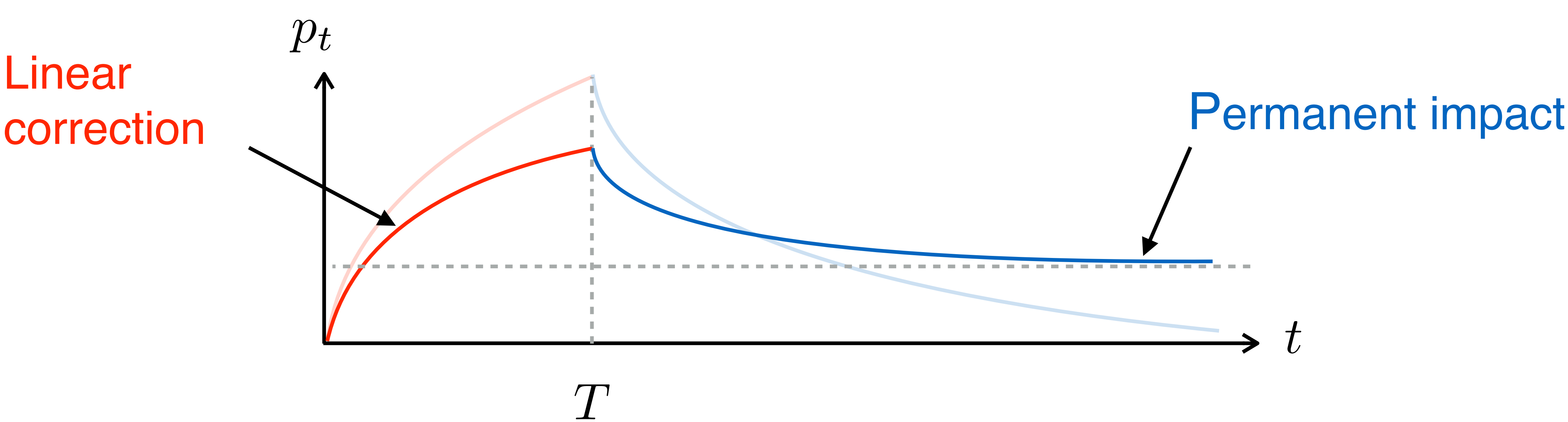


- **Infinite memory** $(\nu, \lambda \to 0)$

$$p_t = \frac{1}{\mathcal{L}} \int_0^t ds \, \frac{m_s}{\sqrt{4\pi D(t-s)}} \, e^{-\frac{(p_t - p_s)^2}{4D(t-s)}}$$



- **Finite memory** $(\nu, \lambda \neq 0)$



Chapter IX: Dimensional analysis in finance

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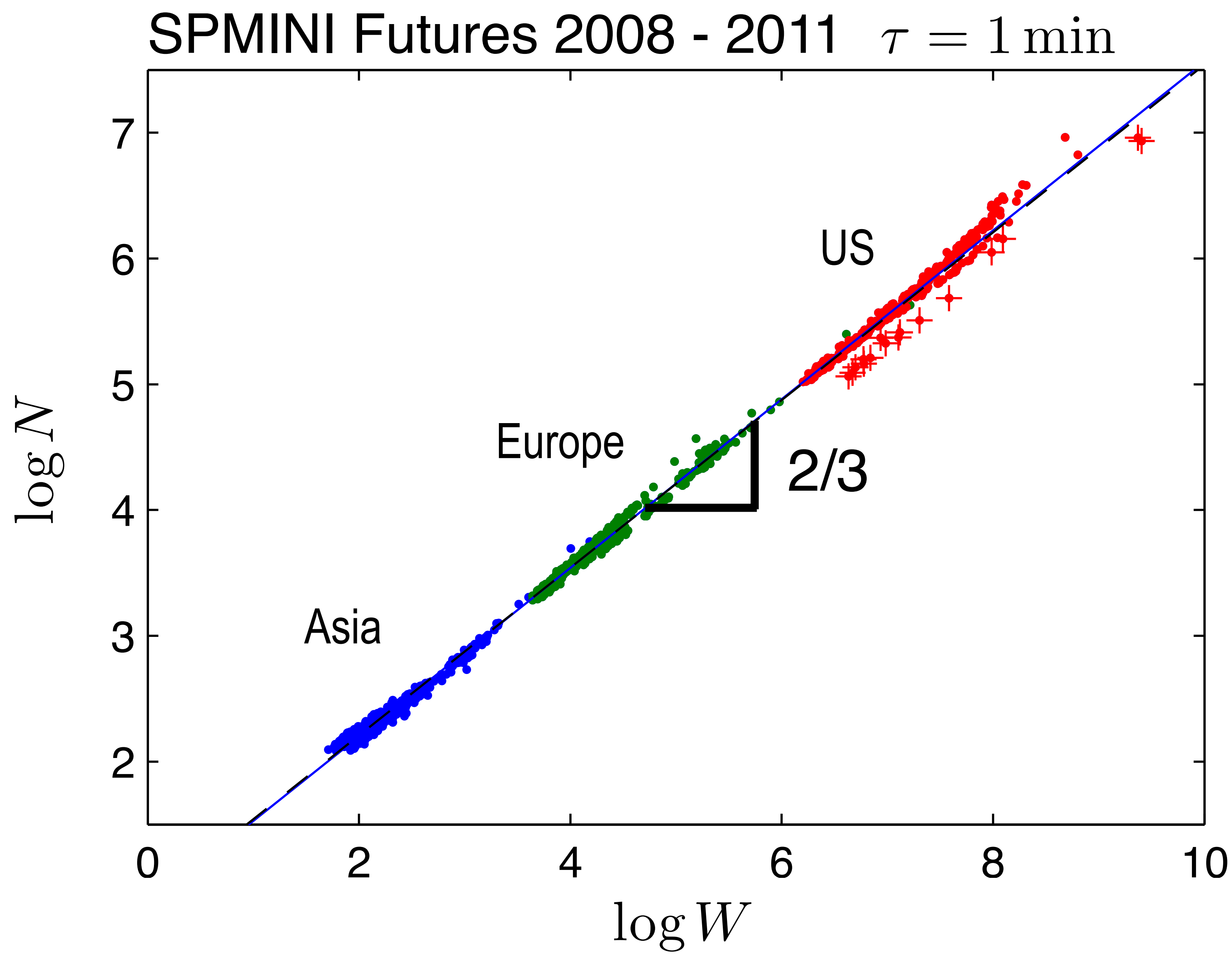
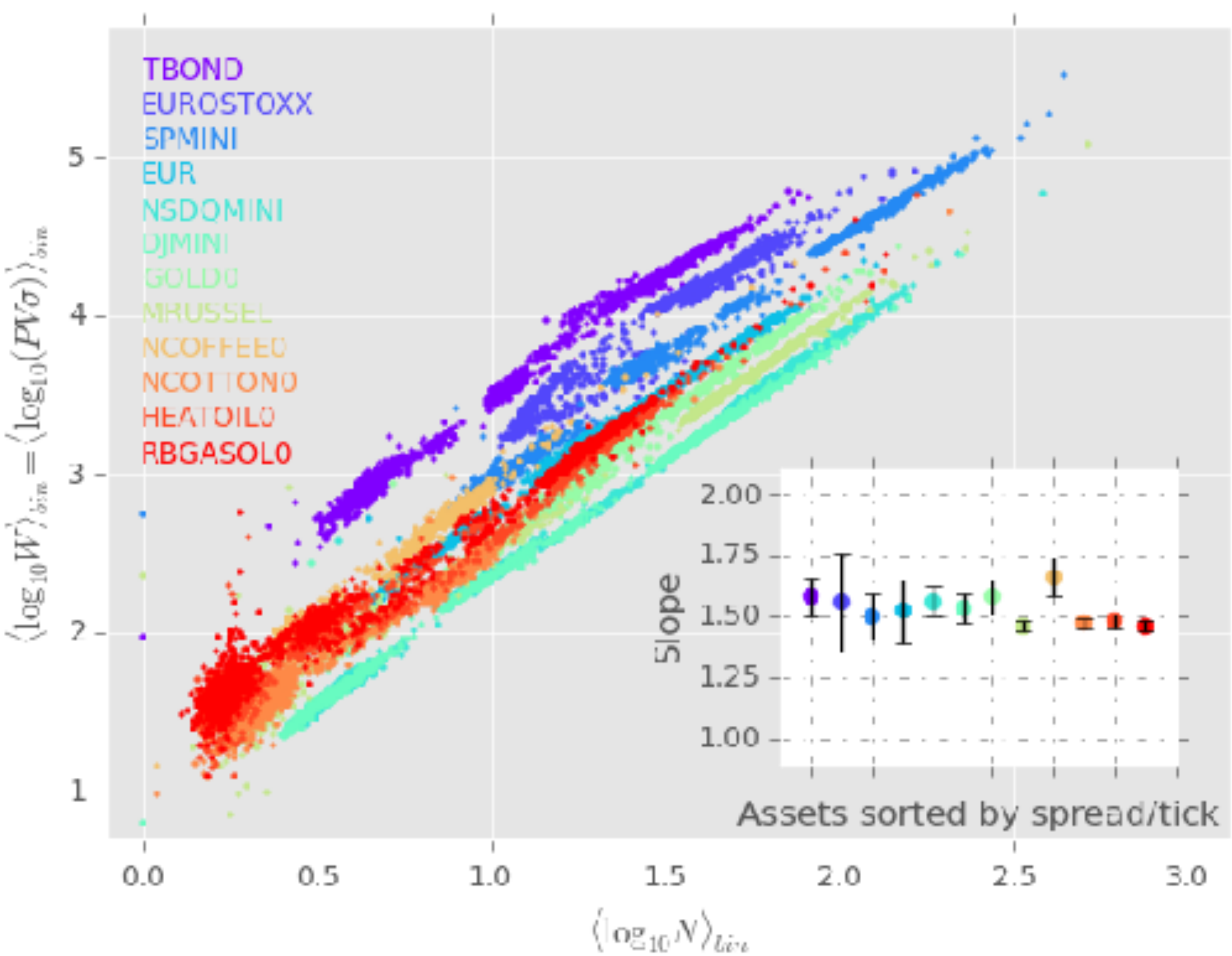


Andersen et al. 2015

Trading invariance hypothesis

Futures

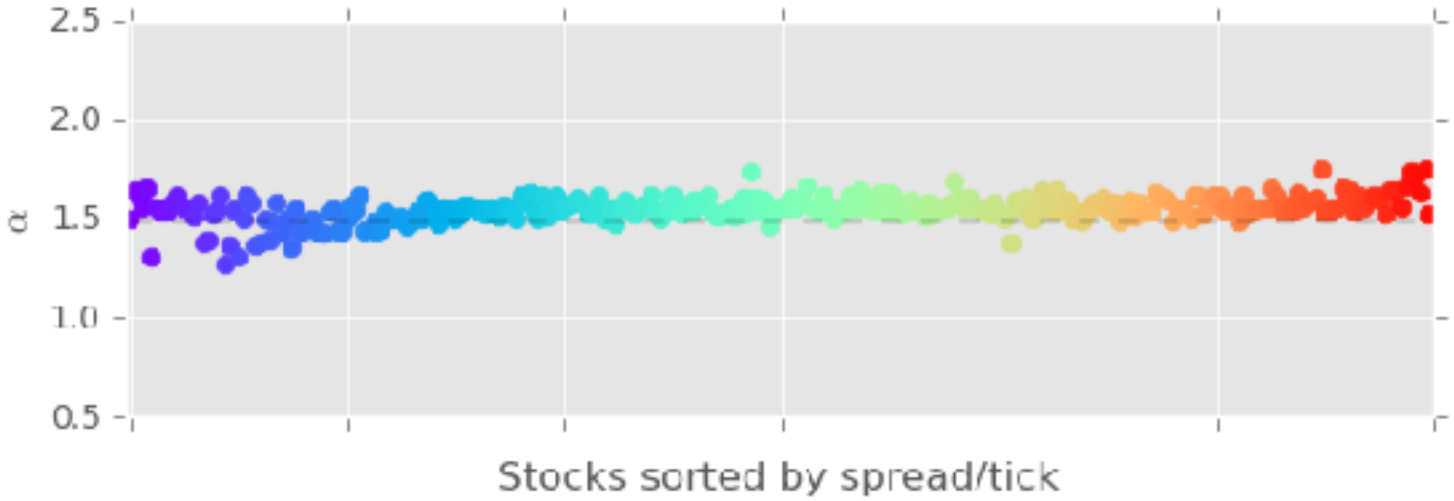
$\tau = 1\,\mathrm{min}$



Name	⟨Spread⟩	⟨N⟩	⟨Q⟩	⟨V⟩	⟨P⟩	⟨σ⟩
TBOND	1.007	18	10.8	191	140960	1.22
EUROSTOXX	1.011	36	23.8	855	28683	1.36
SPMINI	1.011	45	11.2	499	81025	1.46
EUR	1.021	22	4.1	89	163831	1.70
NSDQMINI	1.120	17	2.8	47	58969	2.04
DJMINI	1.157	13	2.5	33	73777	1.98
GOLD0	1.323	21	2.2	45	153092	2.61
MRUSSEL	1.362	14	1.9	27	97573	2.05
NCOFFEE0	2.066	6	2.0	12	59407	1.57
NCOTTON0	3.628	5	2.0	10	40375	2.38
HEATOIL0	4.950	9	1.7	15	122626	6.56
RBGASOL0	6.052	9	1.7	15	116292	7.49

Benzaquen & Bouchaud 2016

Stocks



Cross-sectional determination of the slope $\alpha = 1.54 \pm 0.11$

Benzaquen & Bouchaud 2016

Chapter X: Conclusion

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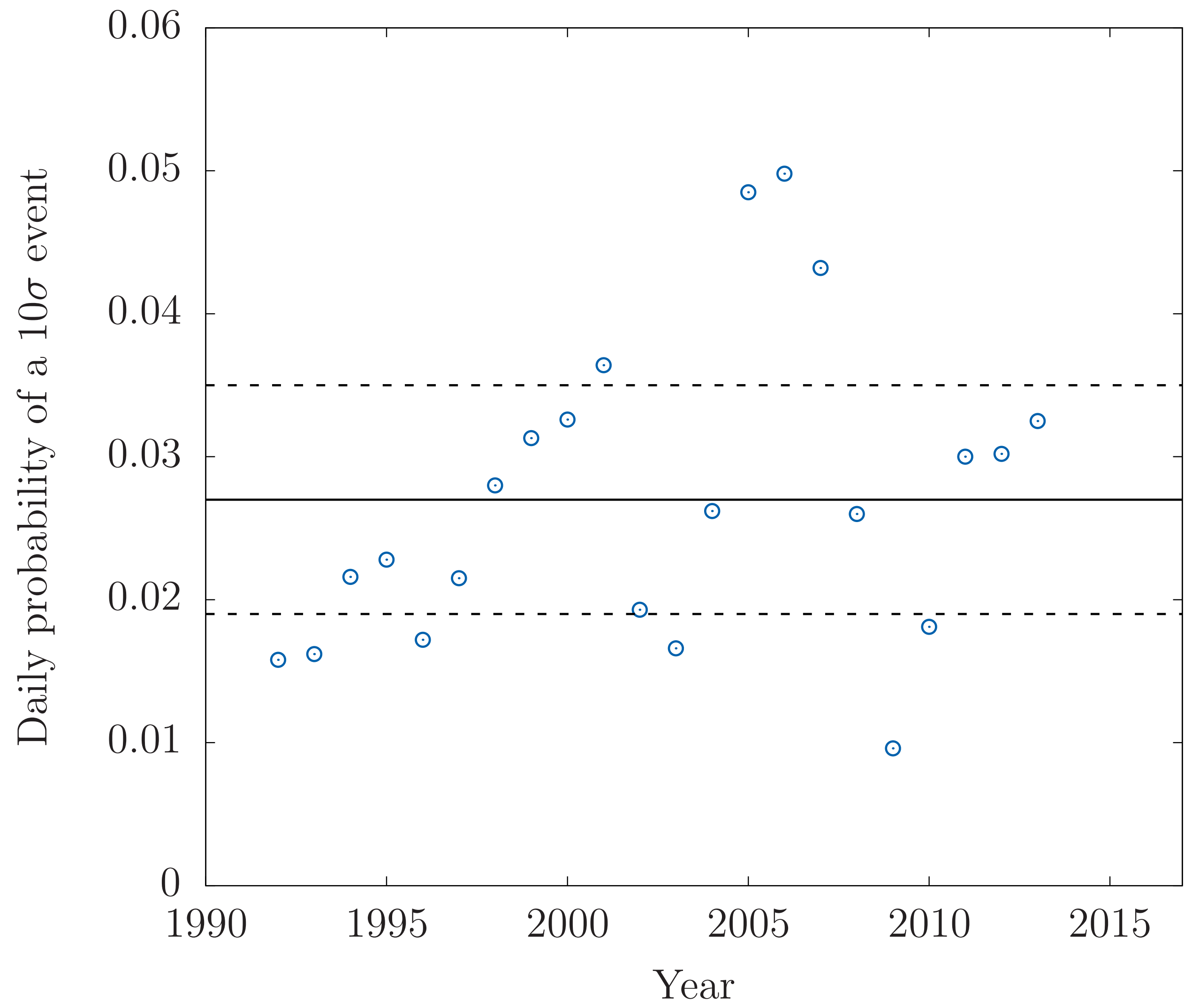
Market stability

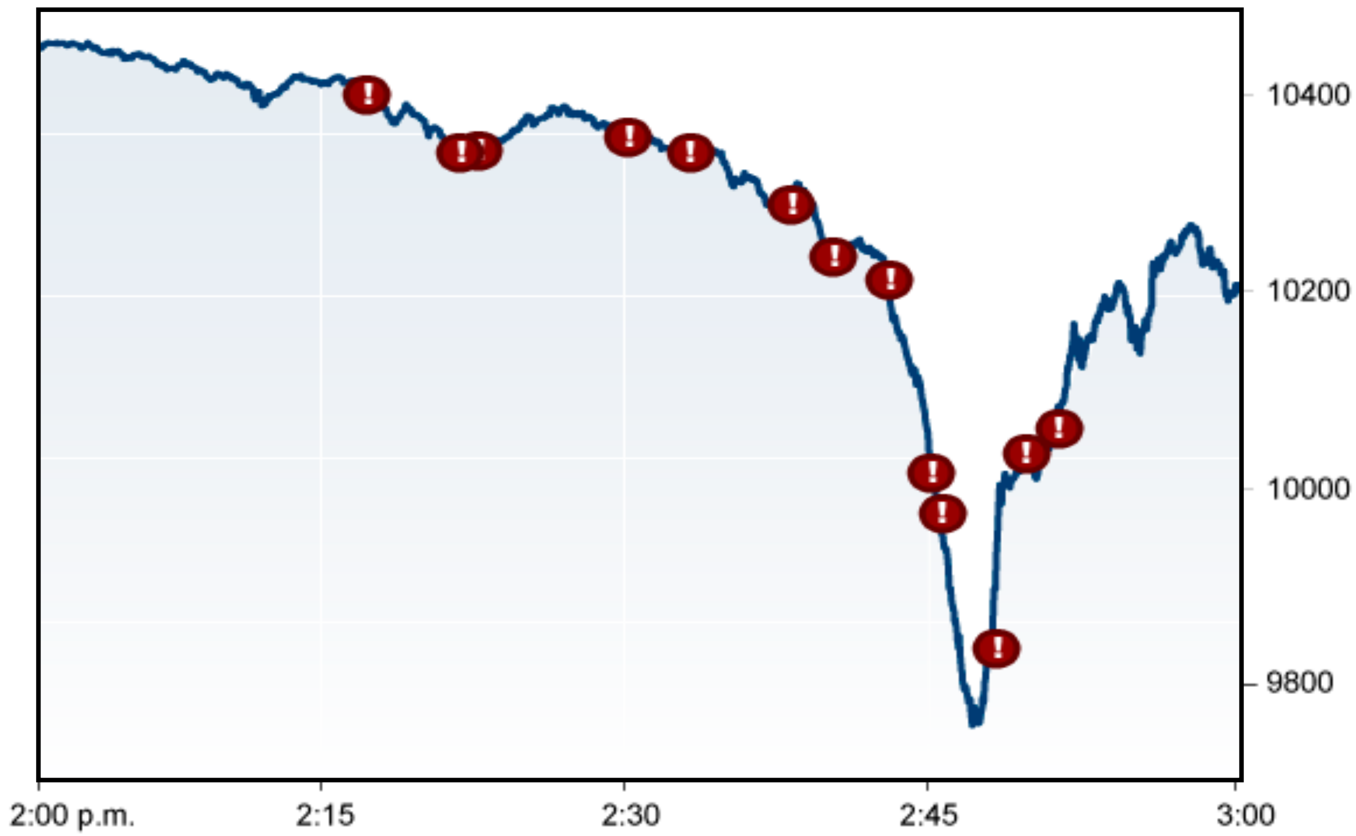
Empirical probability of a 10-sigma price jump on a given day for any of the stocks in the S&P 500, from 1992 to 2013. We estimate the daily volatility for each stock and each year separately. The horizontal line shows the full-sample mean 0.027, and the dashed lines at ±0.008 correspond to one standard deviation (neglecting correlations, which are in fact present and would lead to wider error-bars).



There is no significant change between the pre-HFT era (before about 2000) and the explosion of HFT (after about 2008).

Market stability

"Flash Crash" May 6, 2010



"liquidity is a coward — it's never there when you need it."

When men are in close touch with each other, they no longer decide randomly and independently of each other, they each react to the others. Multiple causes come into play which trouble them and pull them from side to side, but there is one thing that these influences cannot destroy and that is their tendency to behave like Panurges sheep.

Henri Poincaré, Comments on Bachelier's thesis