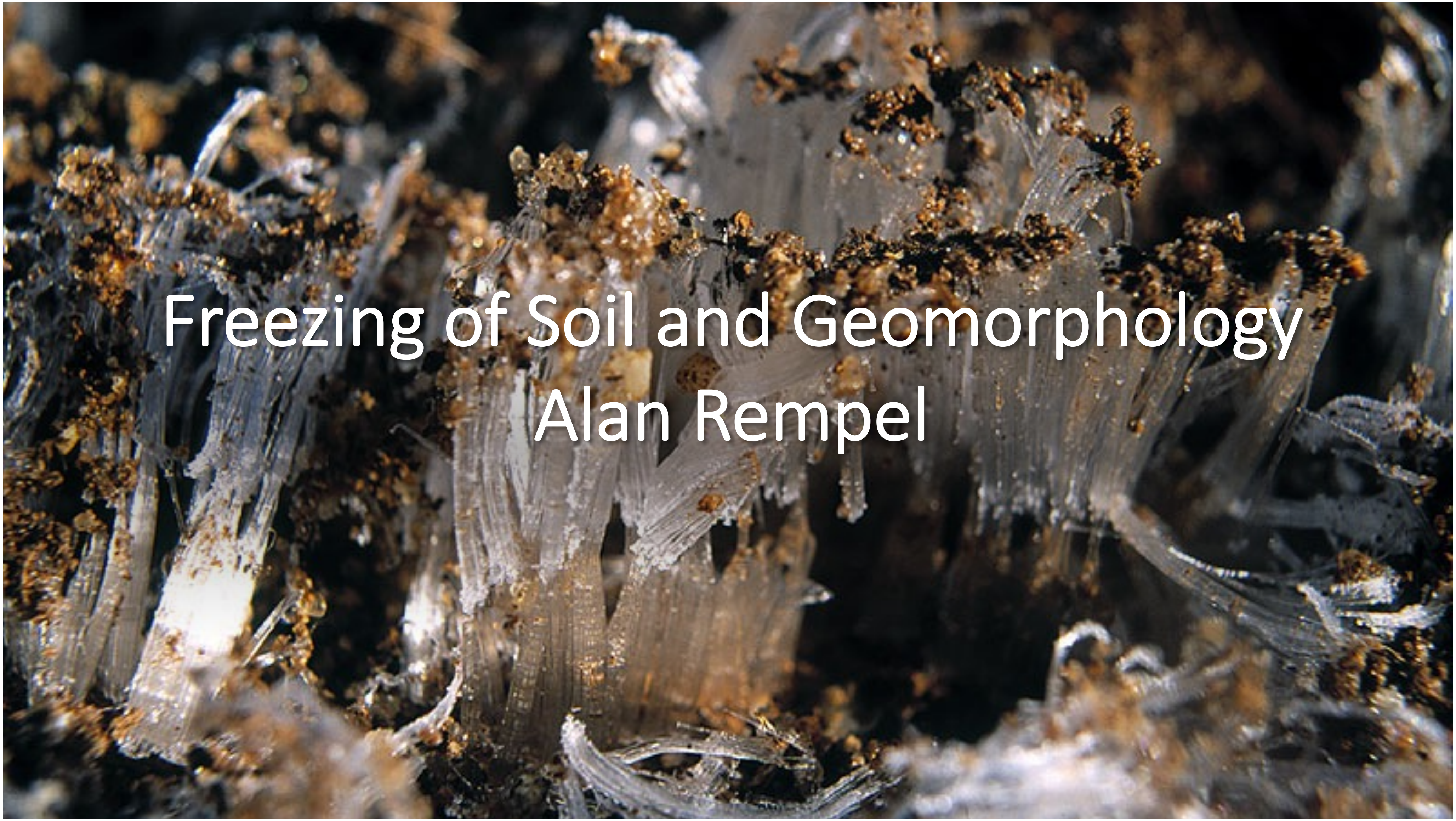
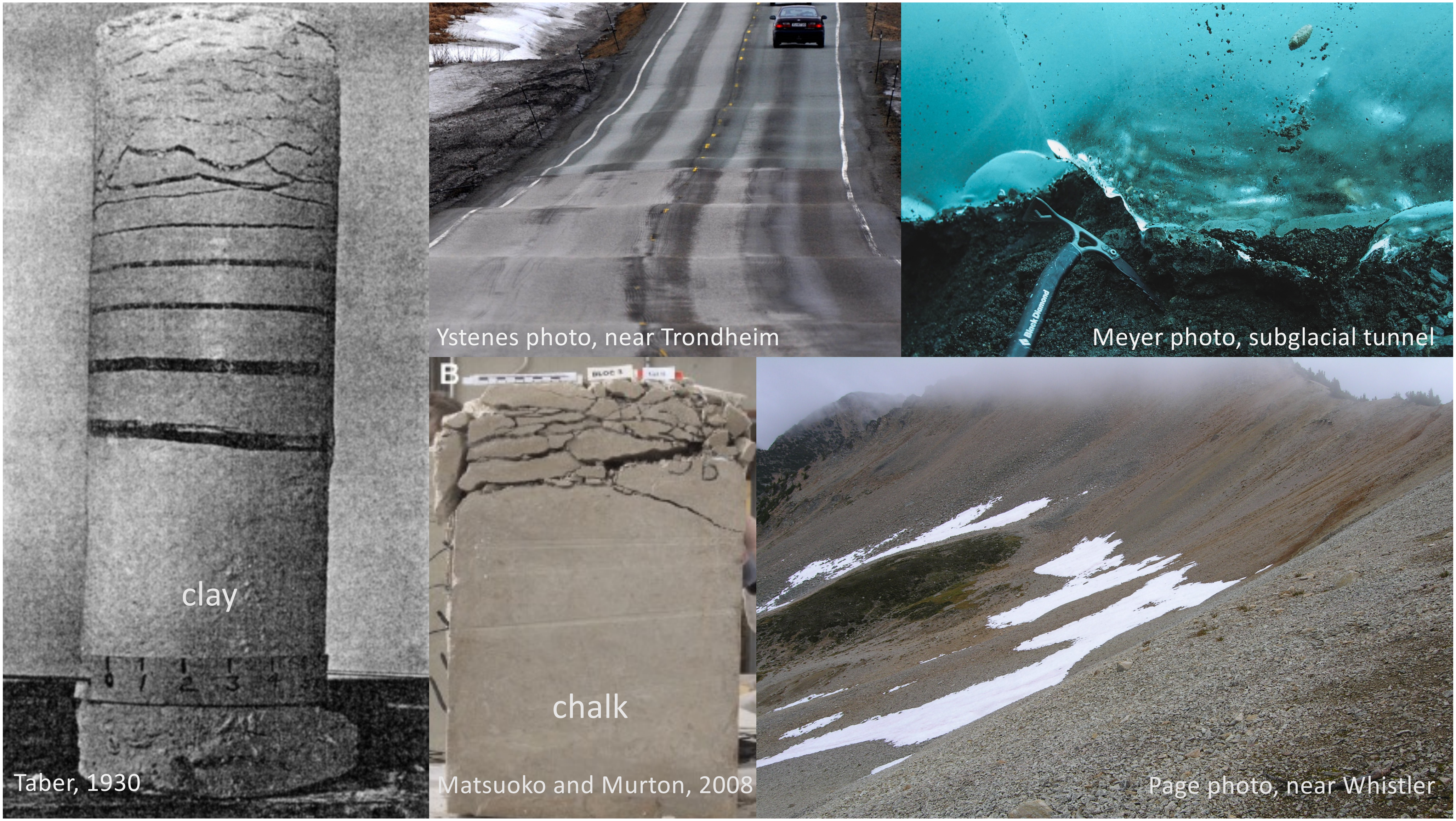
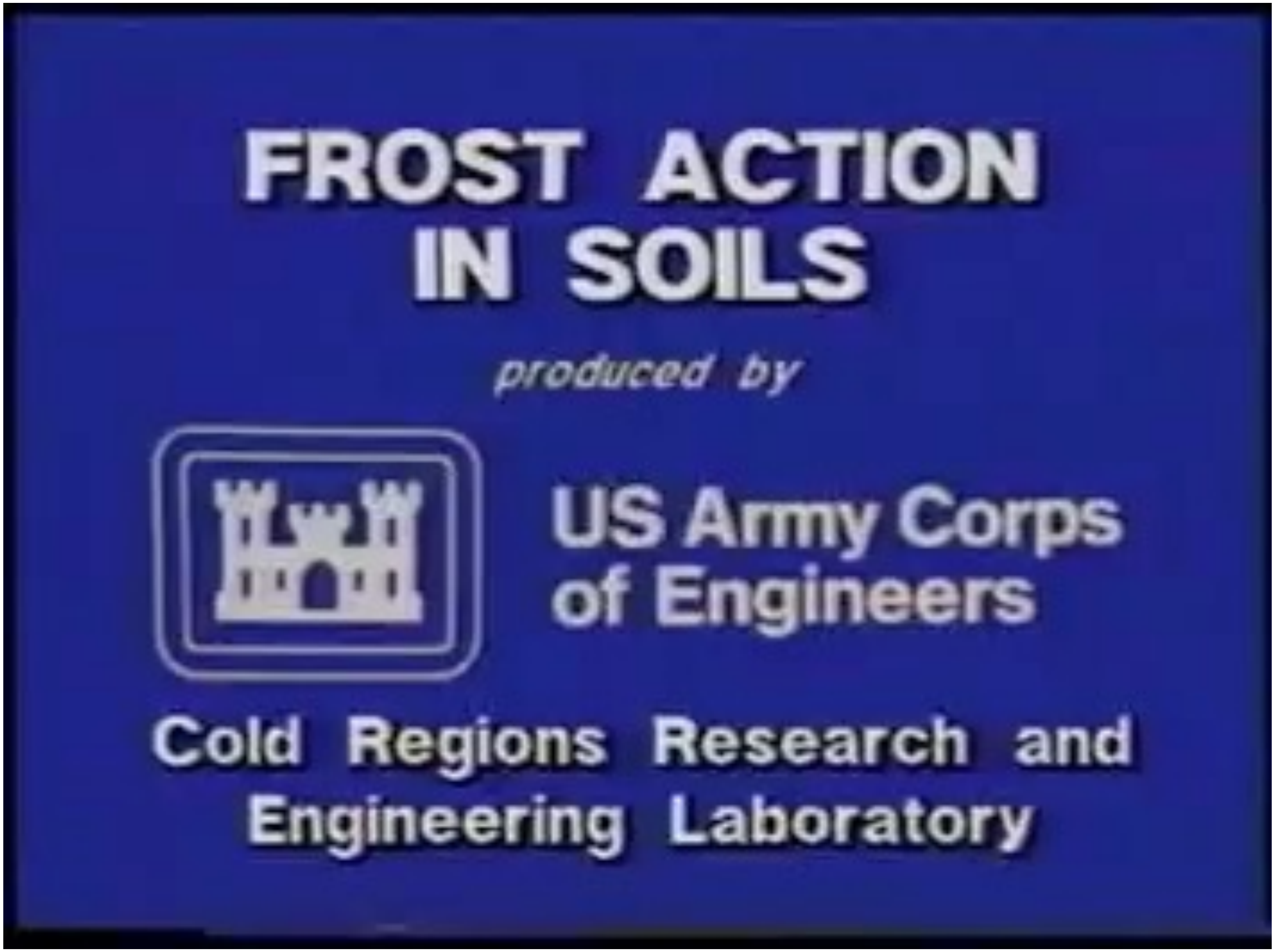


Freezing of Soil and Geomorphology

Alan Rempel

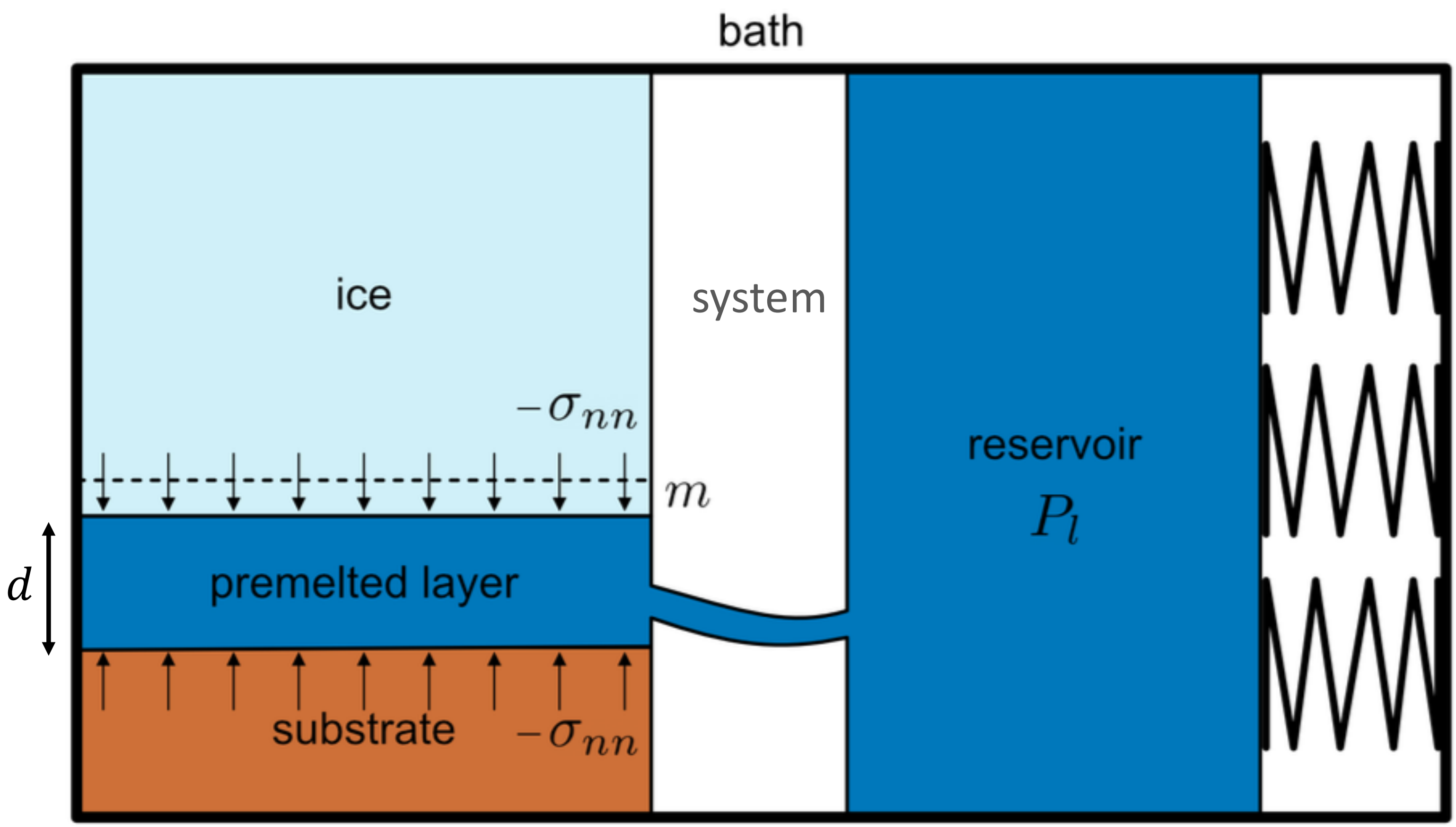

clay
Taber, 1930
Ystenes photo, near Trondheim
Meyer photo, subglacial tunnel
B
chalk
Matsuoko and Murton, 2008
Page photo, near Whistler



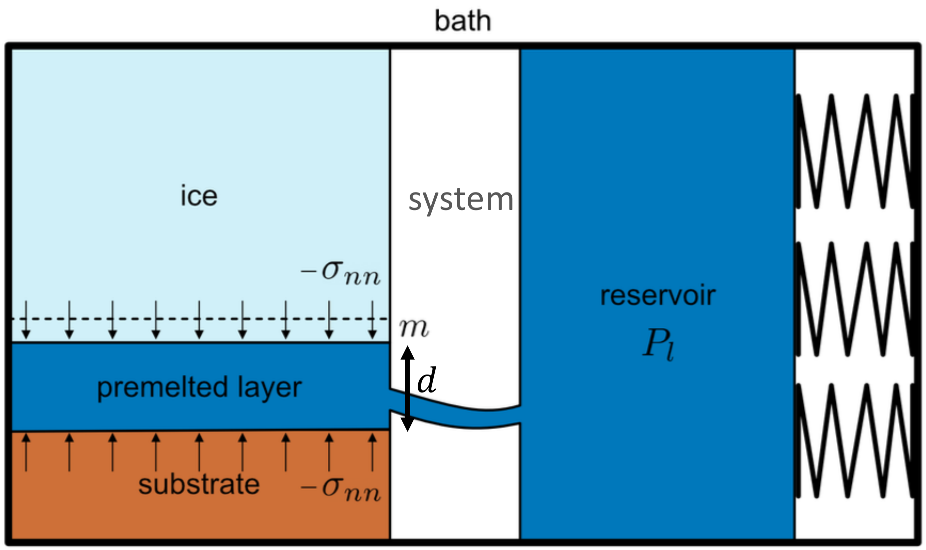
Central issues:

- What controls the pressure ice exerts to cause damage?
- Why does water migrate to supply ice growth?
- What makes soils "frost susceptible"?
- What controls segregated ice growth rates?
- How are ice lenses initiated?

Thermodynamic constraints:
Premelting at equilibrium along a flat ice–liquid interface.



Gerber et al., 2022.



Reversible perturbations to a system immersed in a thermal bath satisfy:

- no change in total entropy

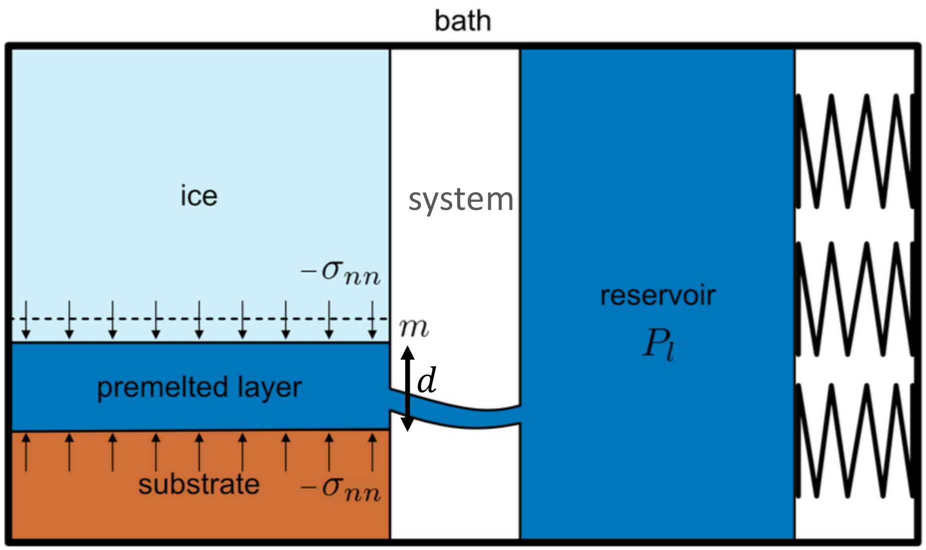
$$\Delta S = 0 = \Delta S_{\text{bath}} + \Delta S_{\text{system}}$$

- no change in total energy

$$\Delta U = 0 = \Delta U_{\text{bath}} + \Delta U_{\text{system}}$$

The bath does no work, so $\Delta U_{\text{bath}} = T\Delta S_{\text{bath}}$

$$\therefore \boxed{\Delta U_{\text{system}} - T\Delta S_{\text{system}} = 0}.$$

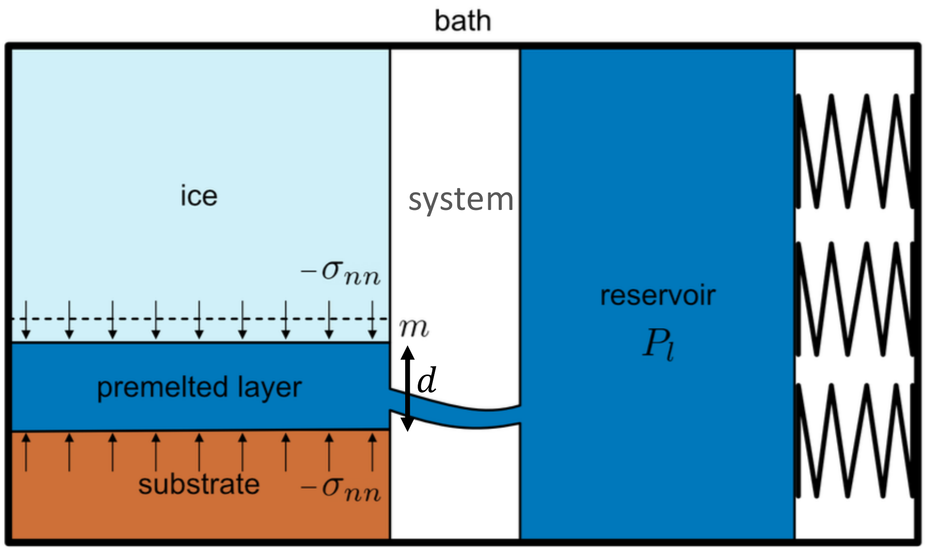


$$\Delta U_{\text{system}} - T\Delta S_{\text{system}} = 0$$

Consider two reversible perturbations:

i) melting a small ice mass

ii) transferring a small mass from the reservoir to the premelted layer



$$\Delta U_{\text{system}} - T\Delta S_{\text{system}} = 0$$

i) Perturb system by melting small ice mass m:

$$\Delta U_{\text{system}} = u_l m - u_i m + P_l v_l m + \sigma_{nn} v_i m$$
$$\Delta S_{\text{system}} = s_l m - s_i m$$

hence

$$-(\sigma_{nn} v_i + P_l v_l) = (u_l - u_i) - T(s_l - s_i).$$

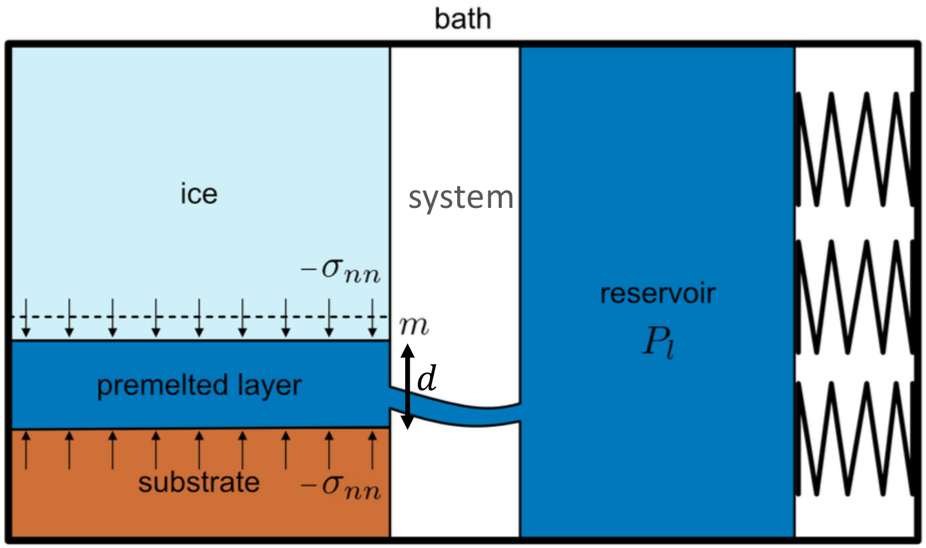
Define reference bulk melting conditions by T_m and $P_m = -\sigma_{nn} = P_l$ to find:

$$(v_i - v_l)P_m = (u_l^o - u_i^o) - T_m(s_l^o - s_i^o).$$

Taking the difference, while neglecting shifts to $u_{l,i}$ and $s_{l,i}$ from $u_{l,i}^o$ and $s_{l,i}^o$ yields the "generalized Clapeyron equation":

$$-\sigma_{nn} - P_l \approx \rho_i L \frac{T_m - T}{T_m} - (P_l - P_m)\frac{\rho_l - \rho_i}{\rho_l},$$

where $\rho_{l,i} = v_{l,i}^{-1}$ and $L = T_m(s_l^o - s_i^o)$.



$$\Delta U_{\text{system}} - T\Delta S_{\text{system}} = 0$$

$$-\sigma_{nn} - P_l \approx \rho_i L \frac{T_m - T}{T_m} - (P_l - P_m)\frac{\rho_l - \rho_i}{\rho_l}$$

ii) Perturb system by removing $m = \rho_l A \Delta d$ from reservoir and increasing d by Δd:

$$\Delta U_{\text{system}} = 0.$$

(No melting takes place so $\Delta S_{\text{system}} = 0$.)

Represent interaction of ice and substrate by a potential* $W_{ils}(d)$:

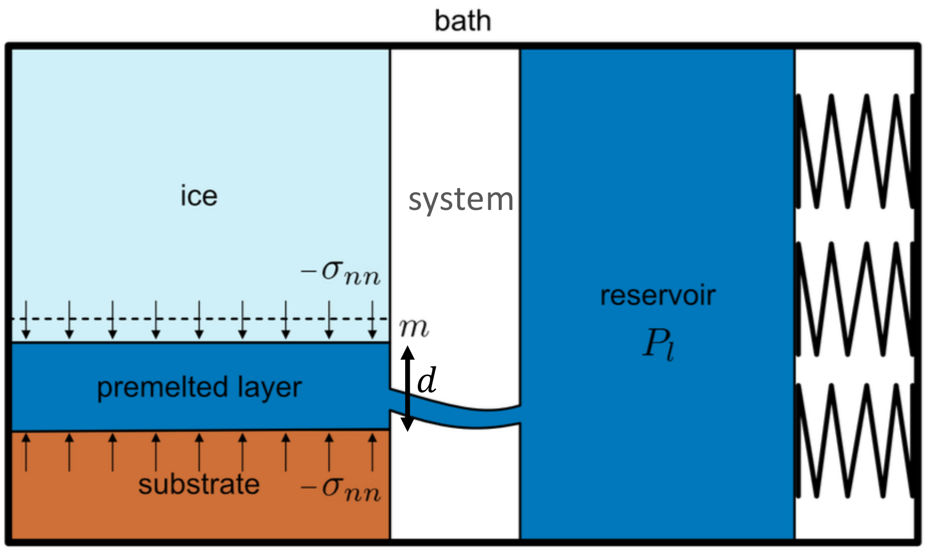
$$\Delta U_{\text{system}} = [W_{ils}(d + \Delta d) - W_{ils}(d)]A$$
$$-(P_l + \sigma_{nn})A\Delta d = 0,$$

hence

$$-\frac{\mathrm{d}W_{ils}}{\mathrm{d}d} = \boxed{\Pi(d) = -\sigma_{nn} - P_l}.$$

*Various sources can contribute to the interaction potential (e.g. van der Waals forces, screened Coulomb interactions, etc.);

At large layer thicknesses ($d \sim O(\mu\mathrm{m})$) the potential vanishes and $-\sigma_{nn} = P_l$.



$$\Delta U_{\text{system}} - T\Delta S_{\text{system}} = 0$$

$$-\sigma_{nn} - P_l \approx \rho_i L \frac{T_m - T}{T_m} - (P_l - P_m)\frac{\rho_l - \rho_i}{\rho_l}$$

$$\Pi(d) = -\sigma_{nn} - P_l$$

Style et al., 2023 discuss 2nd order effects

- What controls the pressure ice exerts to cause damage?

Approximating $\rho_l \approx \rho_i$, as depicted (i.e. flat):

$$\Pi(d) \approx \rho_i L \frac{T_m - T}{T_m} \approx (1.1\ \text{MPa/°C})(T_m - T)$$

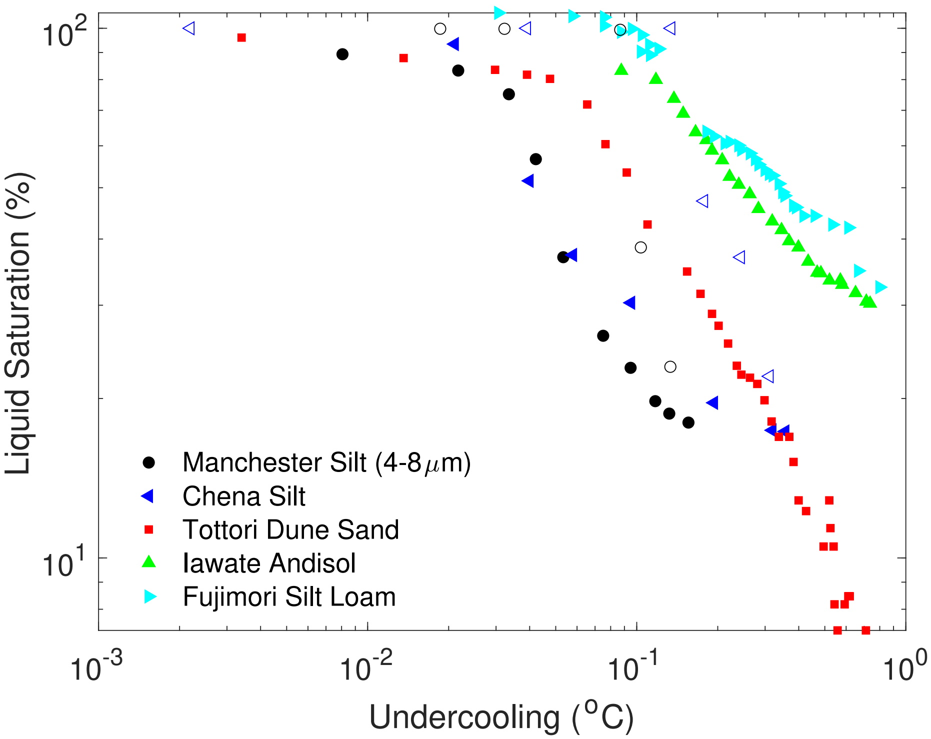
- Why does water migrate to supply ice growth?

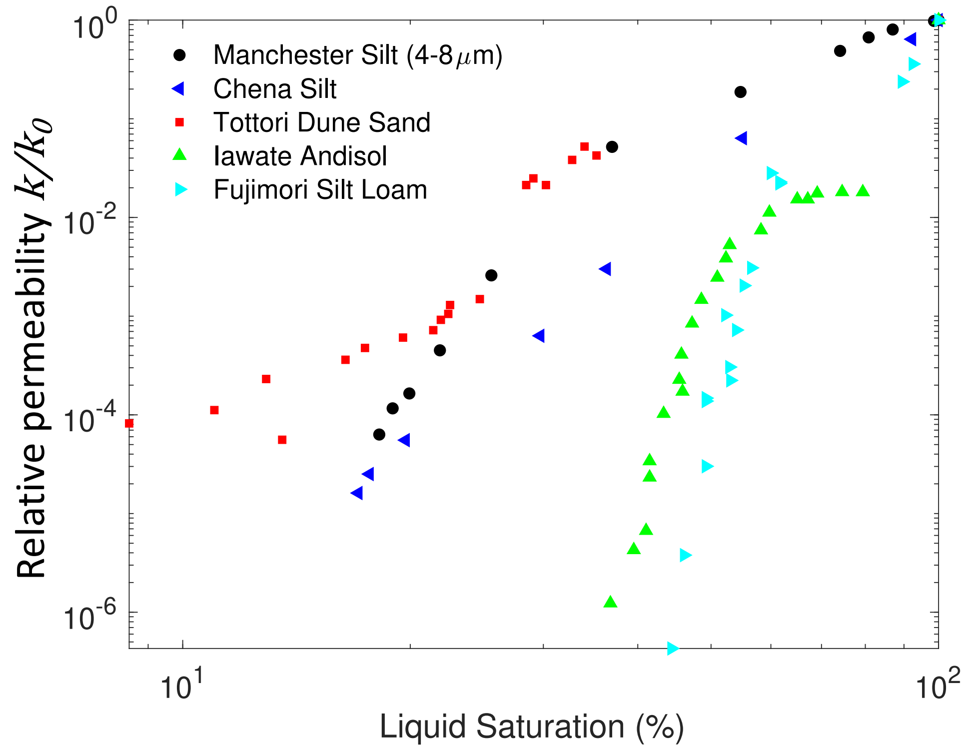
Darcy's law $q = -\frac{k}{\mu}\nabla(P_l + \rho_l g z_{el})$, which for $-\sigma_{nn} = \rho_B g(z_s - z_{el})$:

$$q = -\frac{k}{\mu}\left[\frac{\rho_l L}{T_m}\nabla T + \left(1 - \frac{\rho_B}{\rho_i}\right)\rho_l g \nabla z_{el}\right]$$

- What makes soils "frost susceptible"?

Frost susceptibility depends on soil constitutive behavior (e.g. controls on S_l and k).

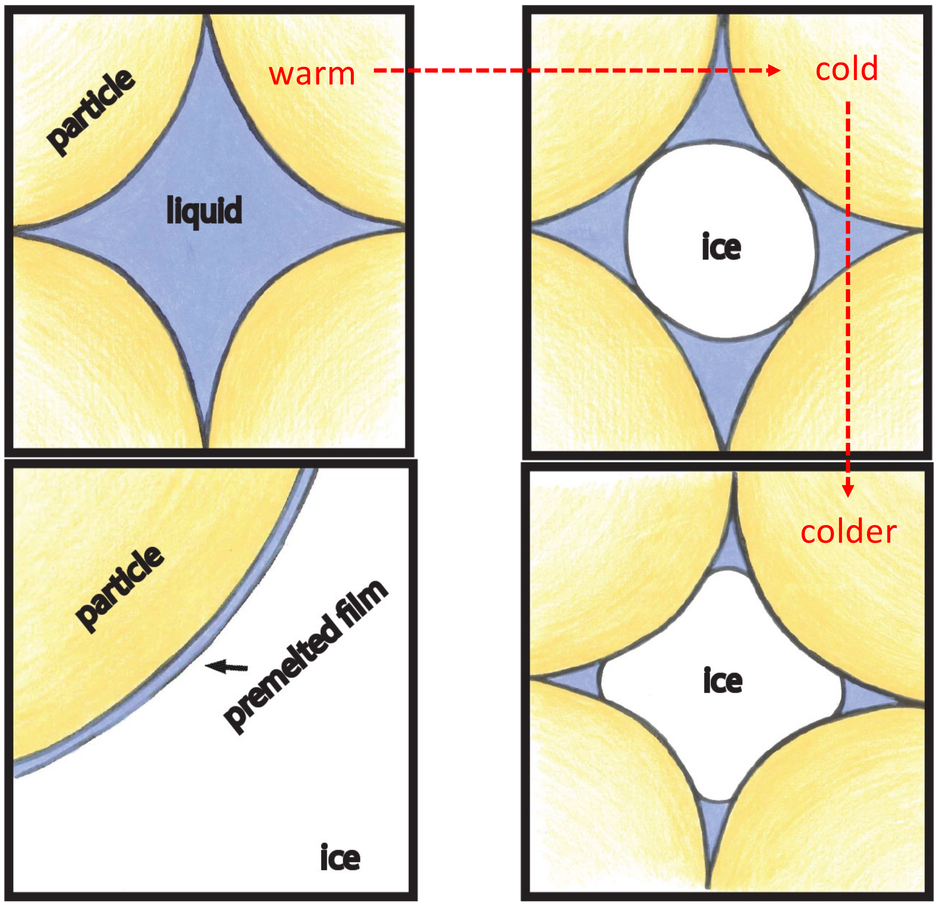




Various probes (e.g. TDR, NMR, calorimetry) can be used to measure how the liquid saturation S_l varies with temperature; measurements of permeability k are more difficult and scarce.

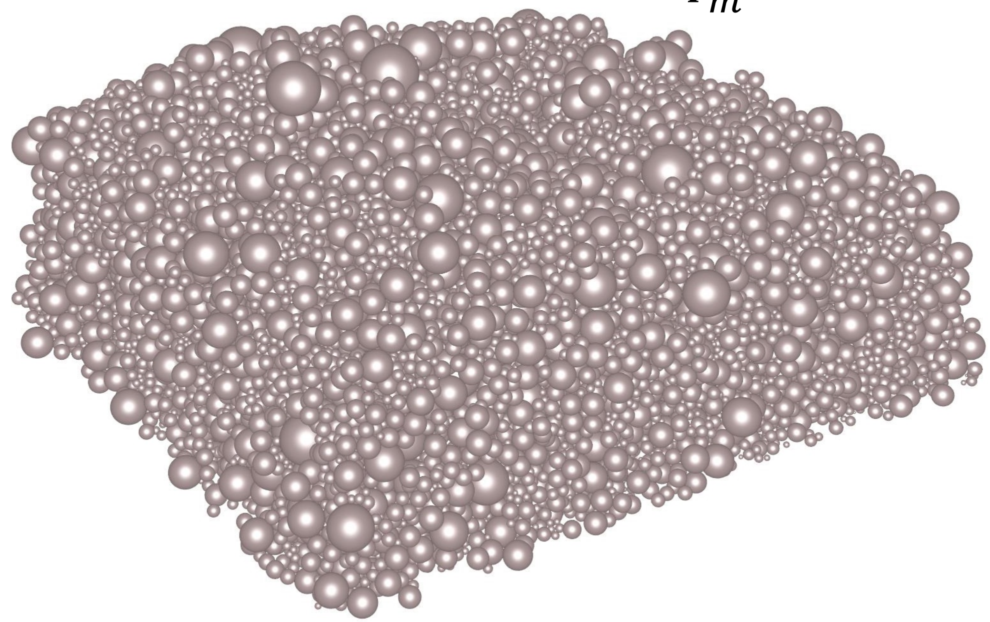
- What makes soils "frost susceptible"?

Frost susceptibility depends on soil constitutive behavior (e.g. controls on S_l and k).



With interfacial curvature $\mathcal{K} = \nabla \cdot \hat{n}$, and surface energy γ_{il} :

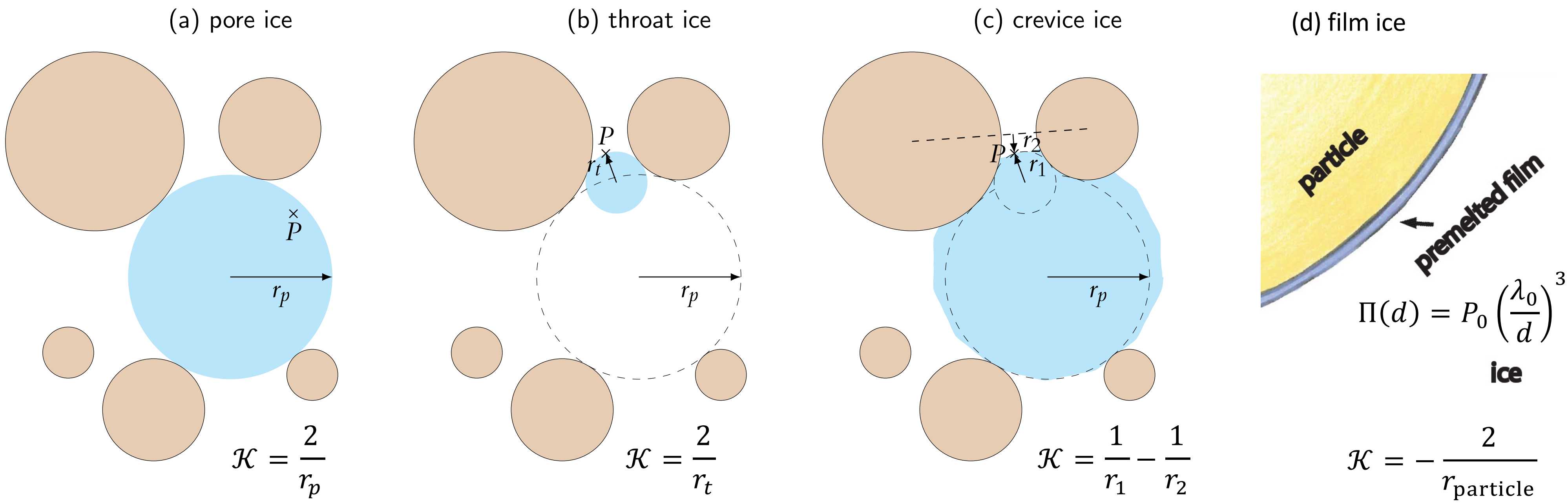
$$\Pi(d) + \gamma_{il}\mathcal{K} \approx \rho_i L \frac{T_m - T}{T_m}$$



Idealized predictive treatments can be instructive.

Begin with a synthetic soil constructed from a specified distribution of spherical particles.

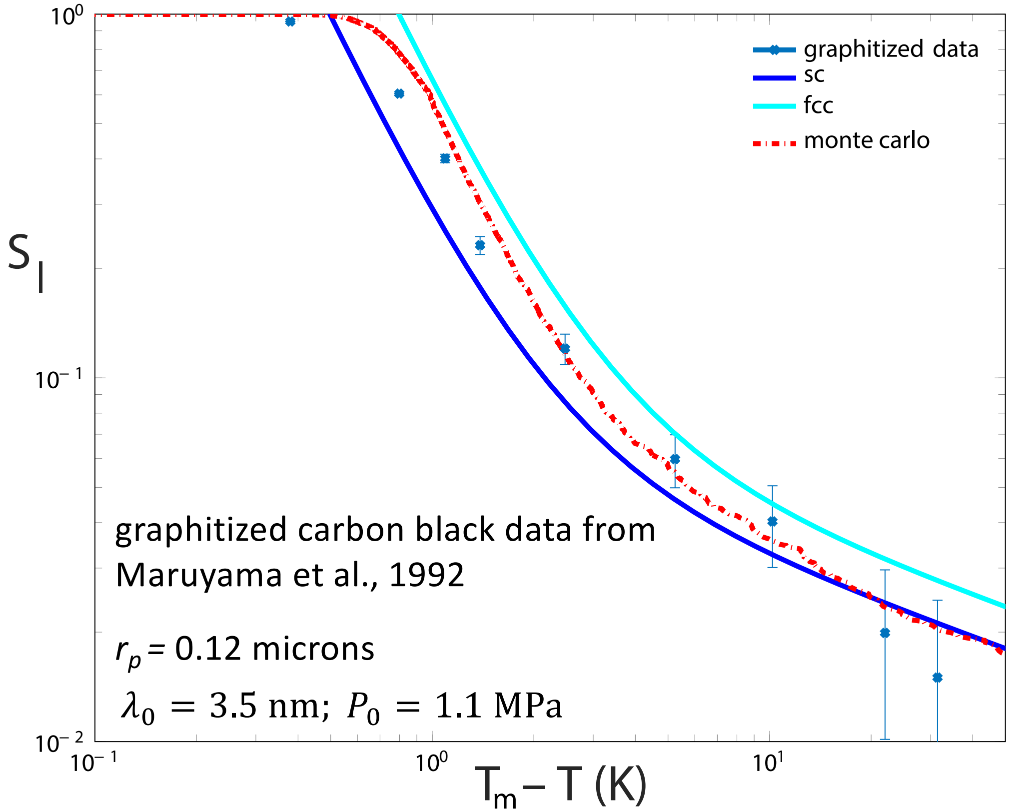
Strategy: determine equilibrium at a large number of points satisfying $\Pi(d) + \gamma_{il}\mathcal{K} \approx \rho_i L \frac{T_m - T}{T_m}$

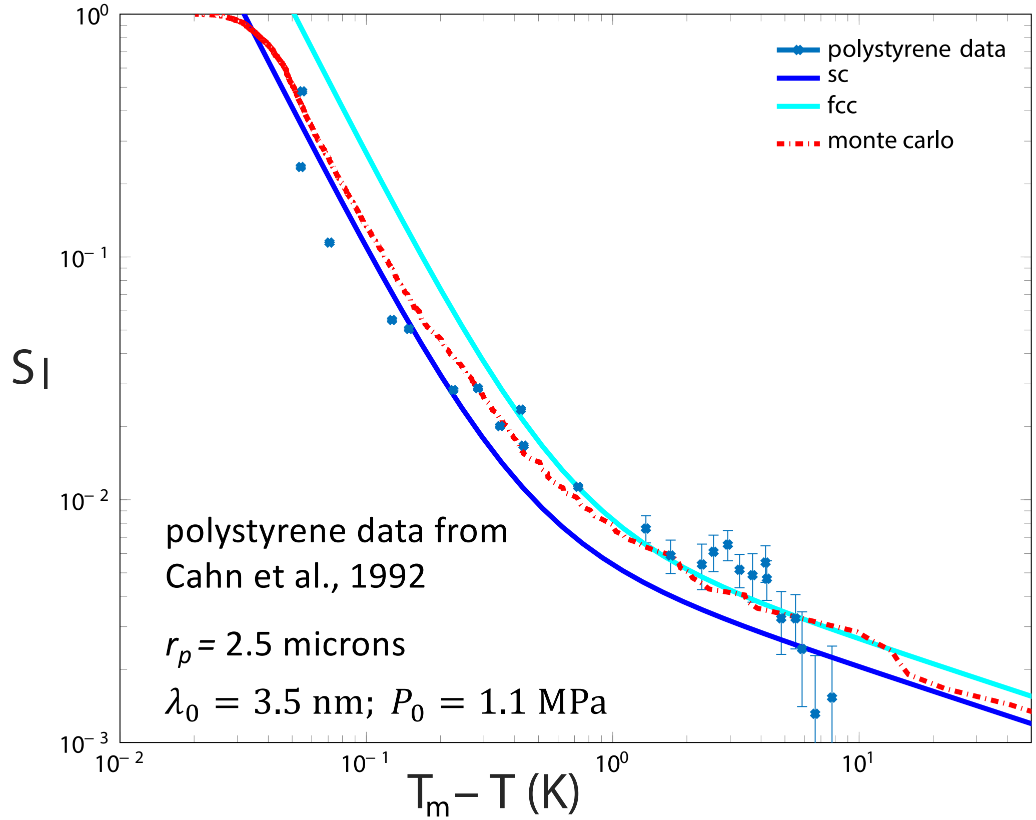


Appollonius' (of Perga) problem – finding tangent spheres is a long-solved problem.

Fraction of points in ice and liquid phases defines freezing constitutive behavior as function of T.

The drop in liquid fraction with undercooling is dominated by curvature at high S_l.
Premelted films are volumetrically significant once only a few percent liquid remains.
Experimental data can be matched at high S_l with no adjustable parameters.

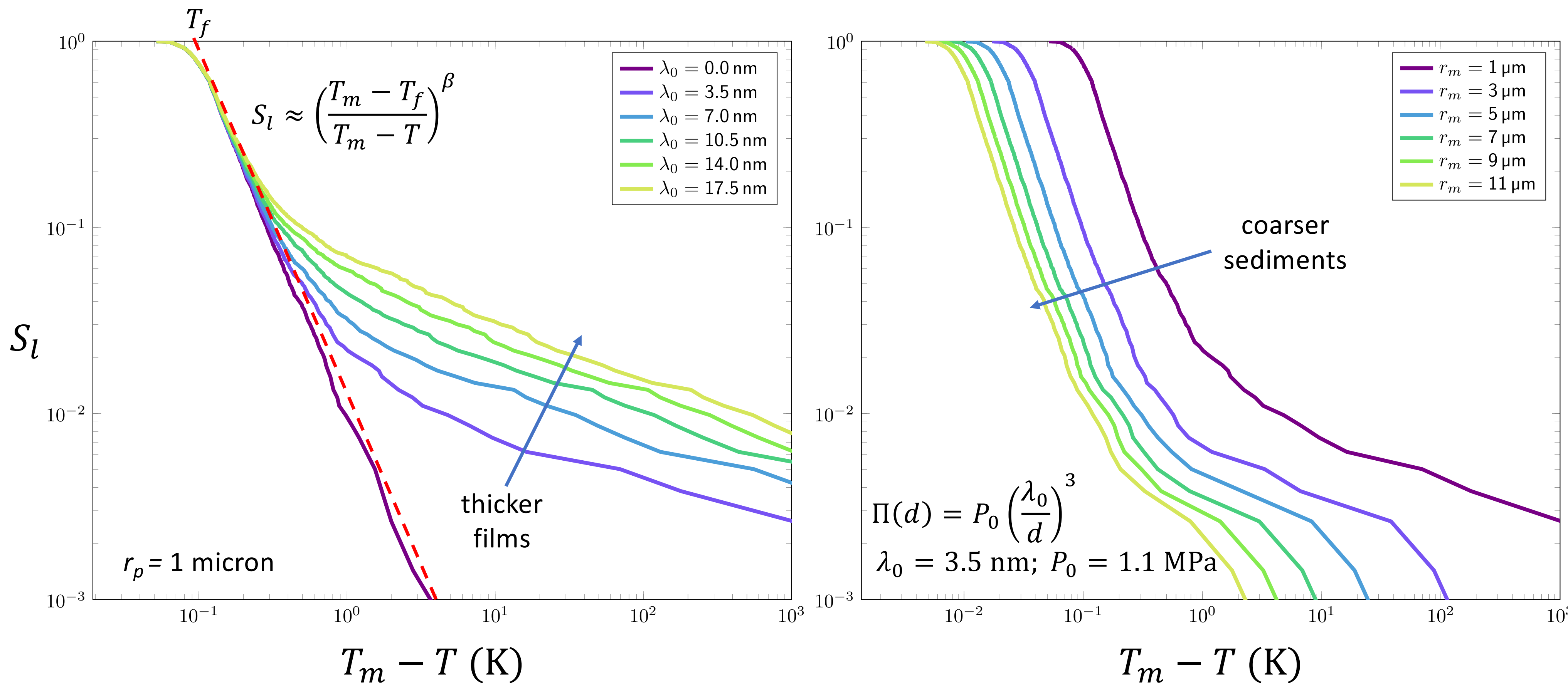




Chen et al., 2020

The drop in liquid fraction with undercooling follows a power law at high S_l.

Once premelted films are volumetrically significant the power-law approximation is poor.



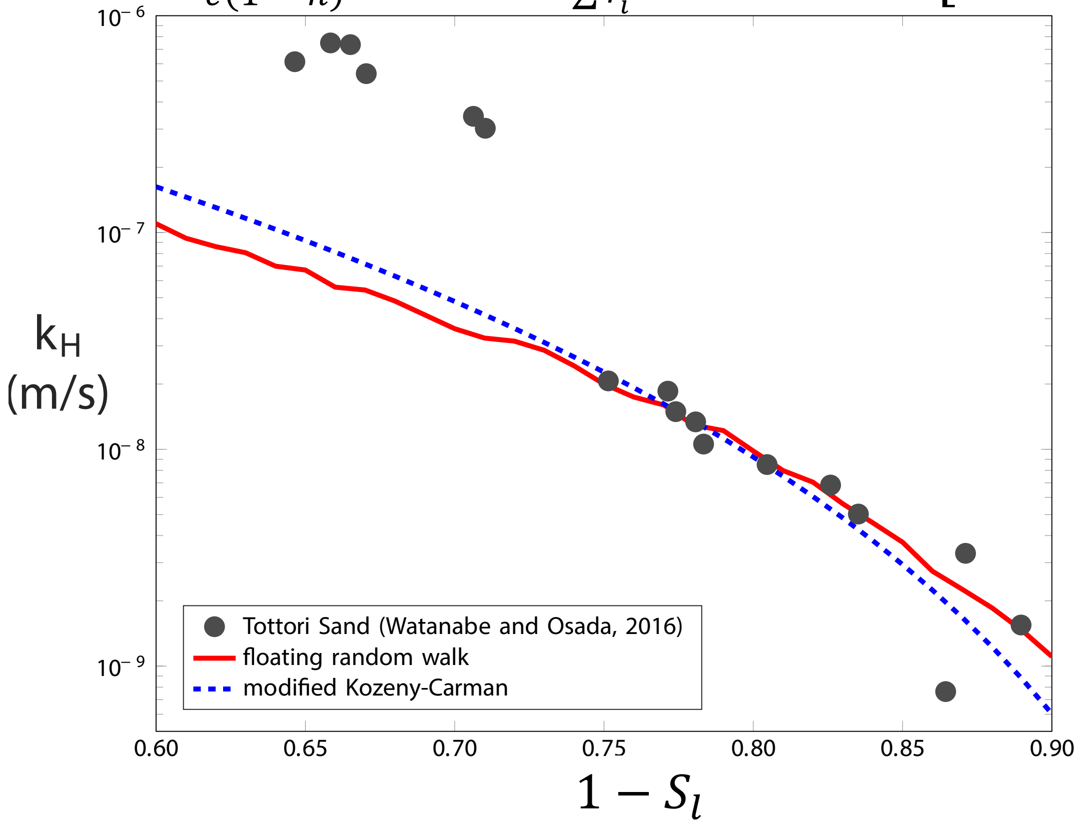
Chen et al., 2020

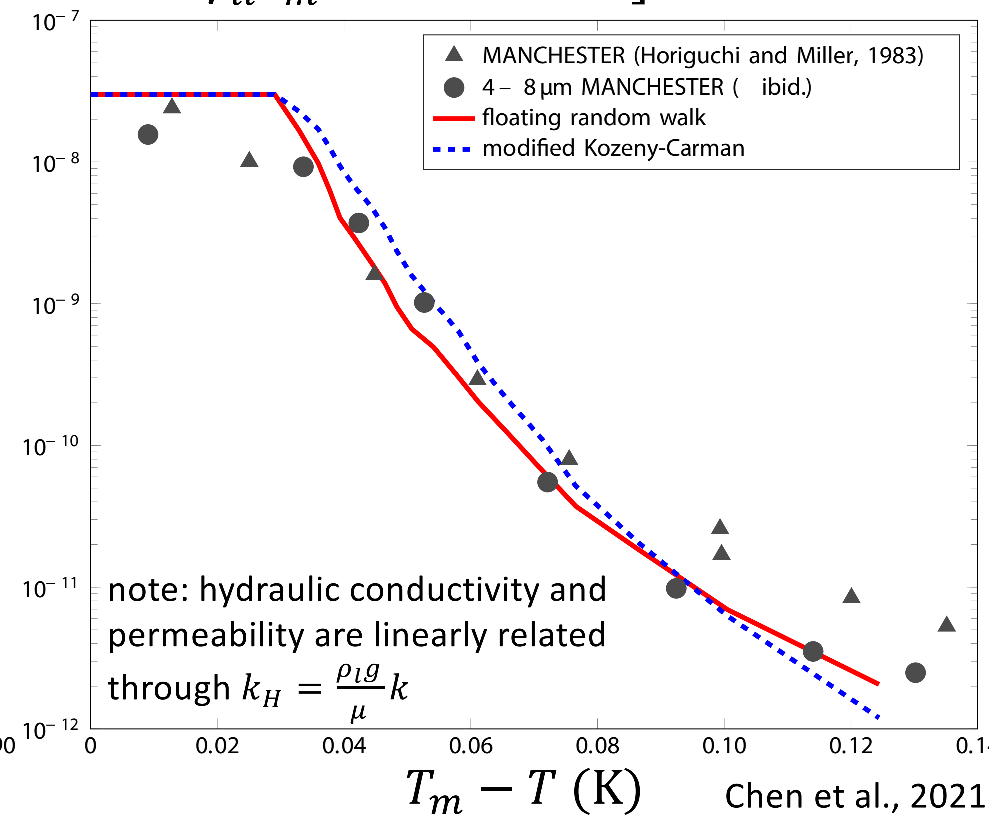
Permeability drops as ice clogs pores. Treat curved ice surfaces as additional particles obstructing a random walk, or with a modified Kozeny–Carman formulation:

K-C ice-free permeability

$$k_0 \approx \frac{n^3}{c(1-n)^2} r_{\text{eff}}^2 \; ; \; r_{\text{eff}} = \frac{\sum r_i^3}{\sum r_i^2}$$

$$\frac{k}{k_0} \approx \frac{S_l^3 (1-n)^2}{\left[1 - n + \frac{r_{\text{eff}} \rho_i L (T_m - T)}{2 \gamma_{il} T_m} n (1 - S_l)\right]^2}$$





Chen et al., 2021

- What makes soils "frost susceptible"?

The "disjoining pressure" must be sufficient to produce deformation:

$$\Pi(d) \approx \rho_i L \frac{T_m - T}{T_m} \approx (1.1\ \text{MPa}/°\text{C})(T_m - T)$$

Water must flow to supply continued ice growth:

$$q = -\frac{k}{\mu}\left[\frac{\rho_l L}{T_m}\nabla T + \left(1 - \frac{\rho_B}{\rho_i}\right)\rho_l g \nabla z_{el}\right]$$

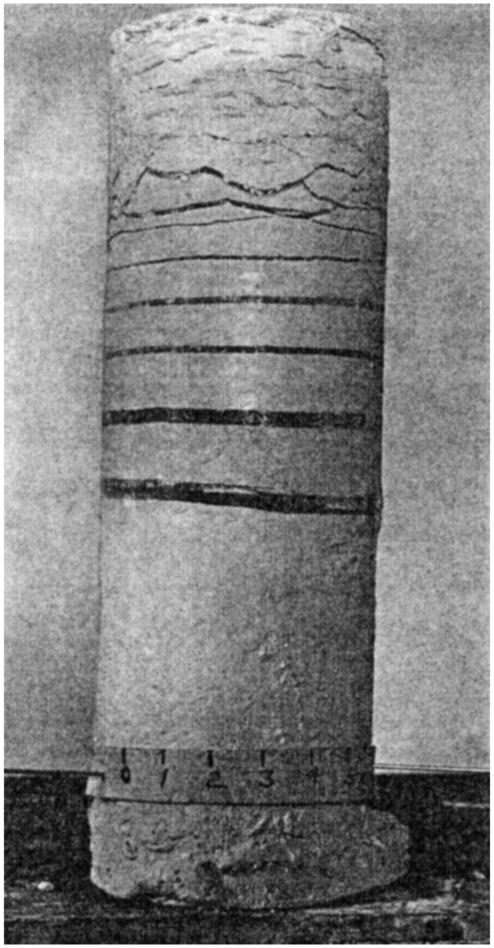
Require water to be present at moderately large undercoolings $T_m - T$.

Approximating the liquid saturation as a power law:

$$S_l \approx \left(\frac{T_m - T_f}{T_m - T}\right)^\beta$$

And the relative permeability as a rapidly decreasing function of S_l:

$$k \approx k_0 S_l^\alpha$$



Need fine-grained soils to make T_f sufficiently cold, but not so fine that k_0 prevents transport.

- What controls segregated ice growth rates?
- How are ice lenses initiated?

Taber 1929/1930 - refrigerated experiments

Beskow 1935 - rails and roads

Everett 1961 - capillary rise (persistent, flawed idea)

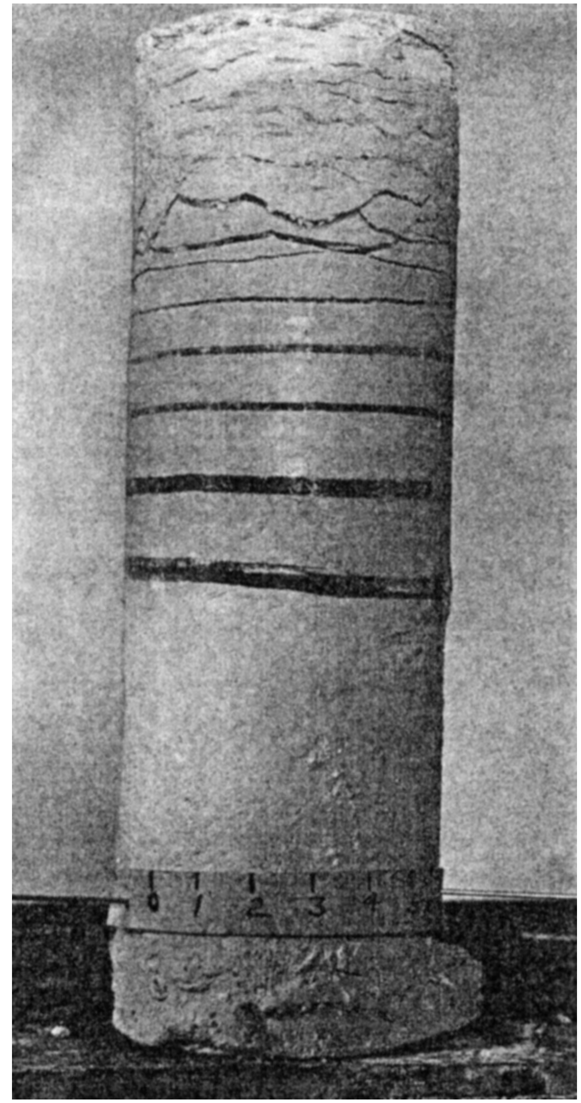
Gilpin 1980 - surface interactions

O'Neill & Miller 1985 - rigid-ice

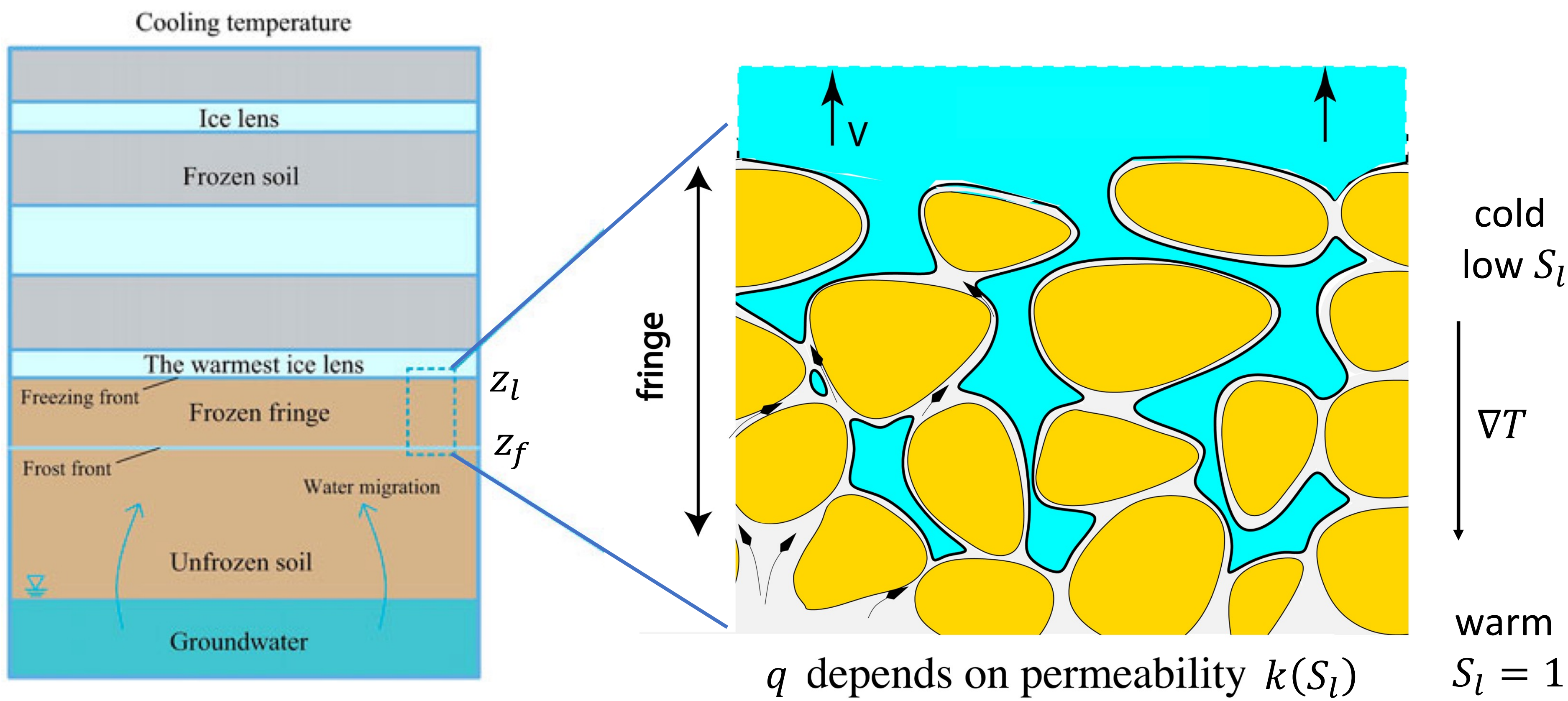
Fowler & Krantz 1994 - asymptotics

Peppin et al. 2006 - morphological instability

Style et al. 2011 - **geometrical supercooling**



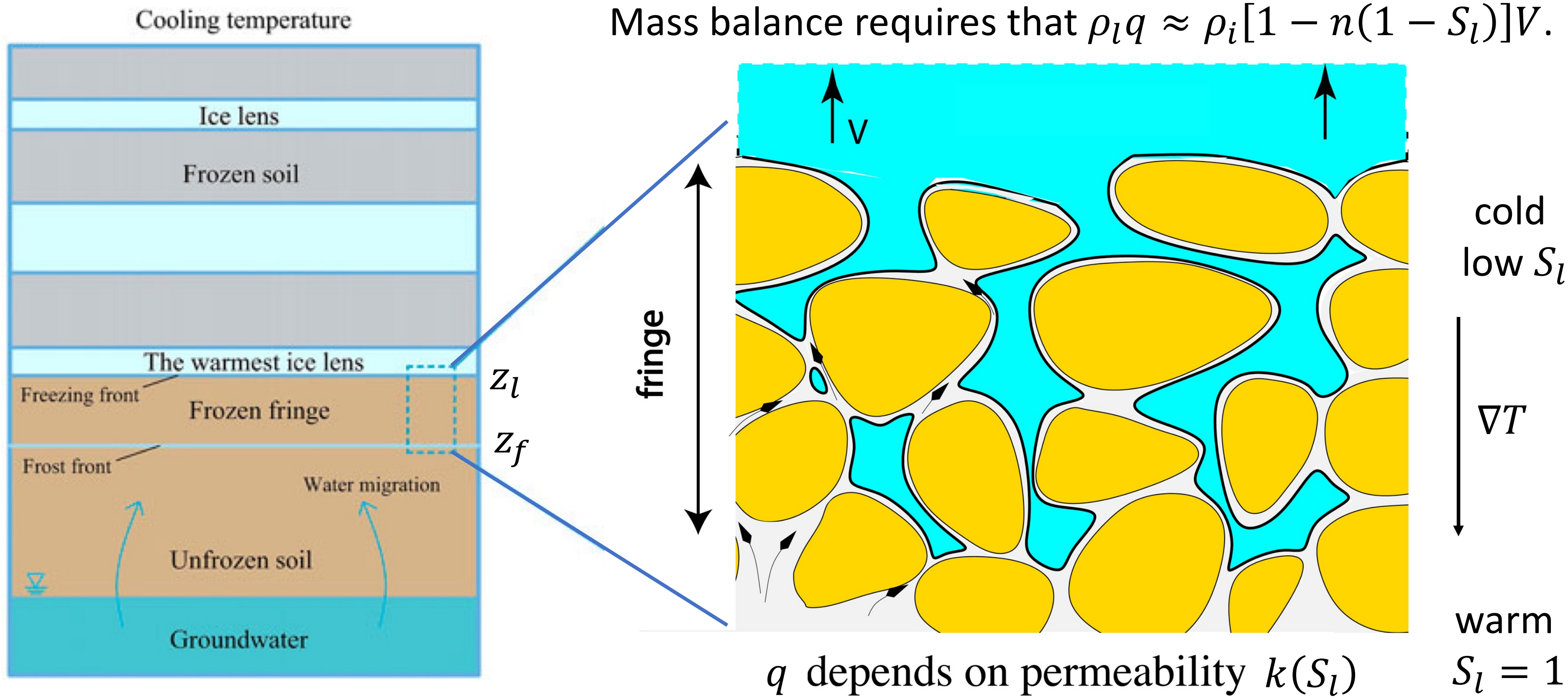
"Something there is that doesn't love a wall,
that sends the frozen-ground-swell under it,
and spills the upper boulders in the sun;"
Robert Frost 1915



In the "rigid ice" model of O'Neill and Miller, a fringe of ice connected to the 'active' warmest lens regelates upwards at the lens growth rate V, supplied by Darcy transport q.

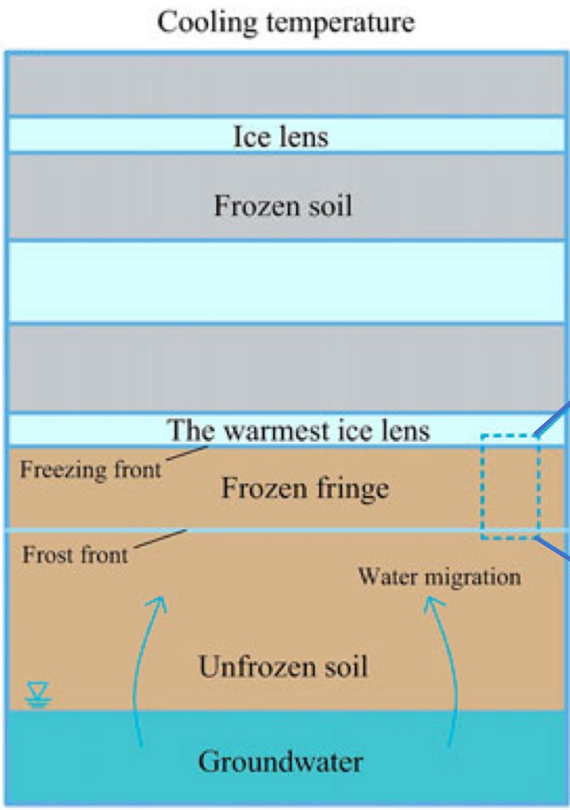
Deformation between or below lenses is assumed negligible; S_l and k are independent of V.

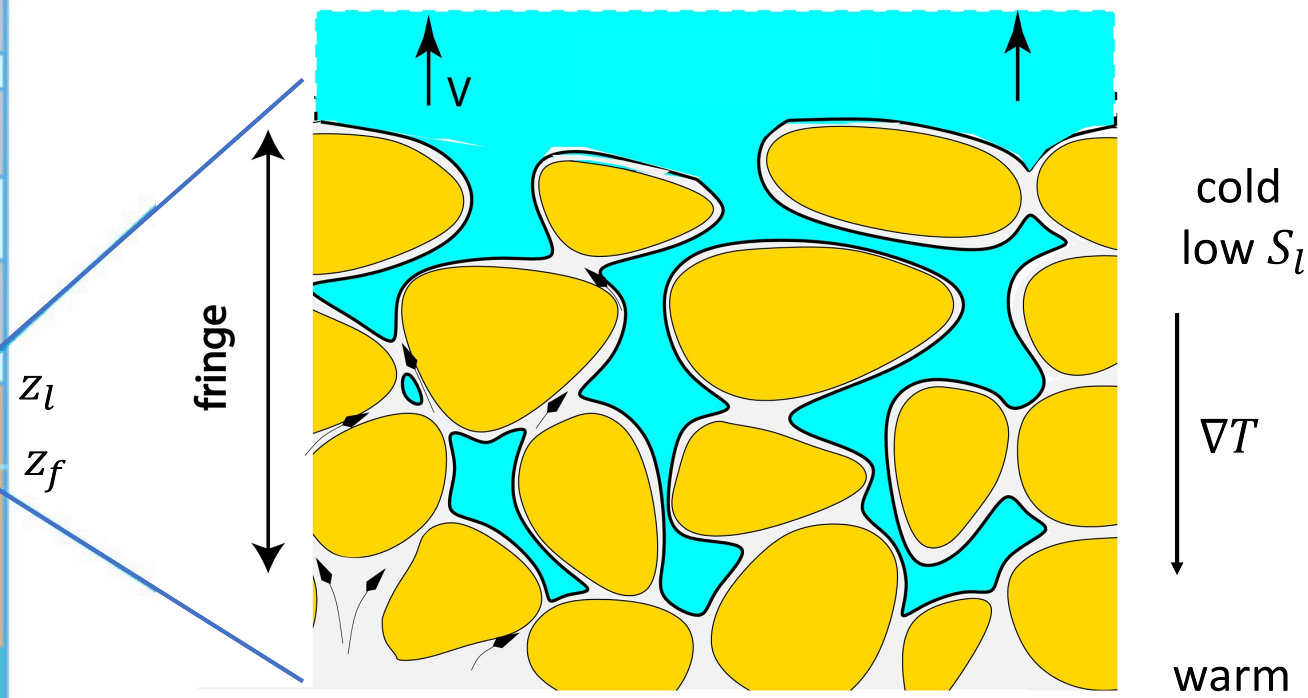
Mass balance requires that $\rho_l q \approx \rho_i[1 - n(1 - S_l)]V$.



The force balance on the growing lens involves gravity ↓, the hydrodynamic pressure P_l on the ice surface ↓, and the net disjoining pressure $\Pi(d)$ between the ice and particles of the fringe ↑.

Mass balance requires that $\rho_l q \approx \rho_i[1 - n(1 - S_l)]V$.



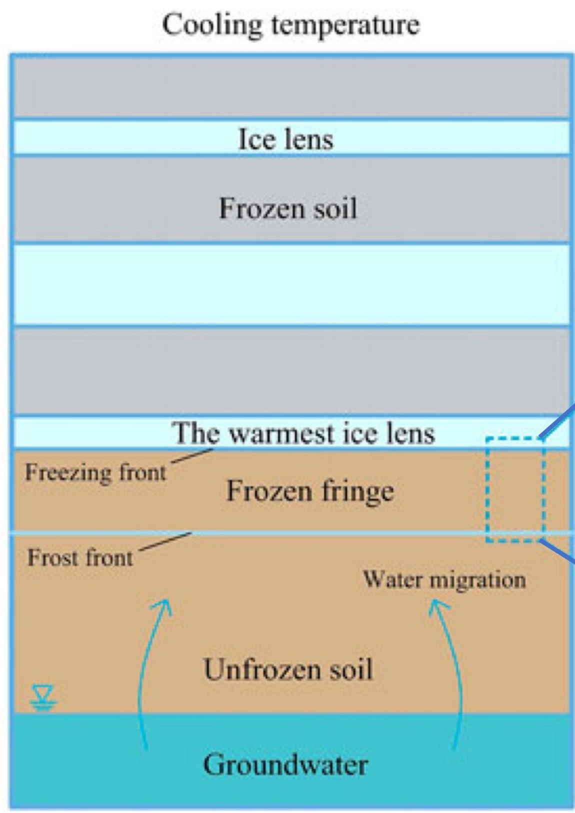


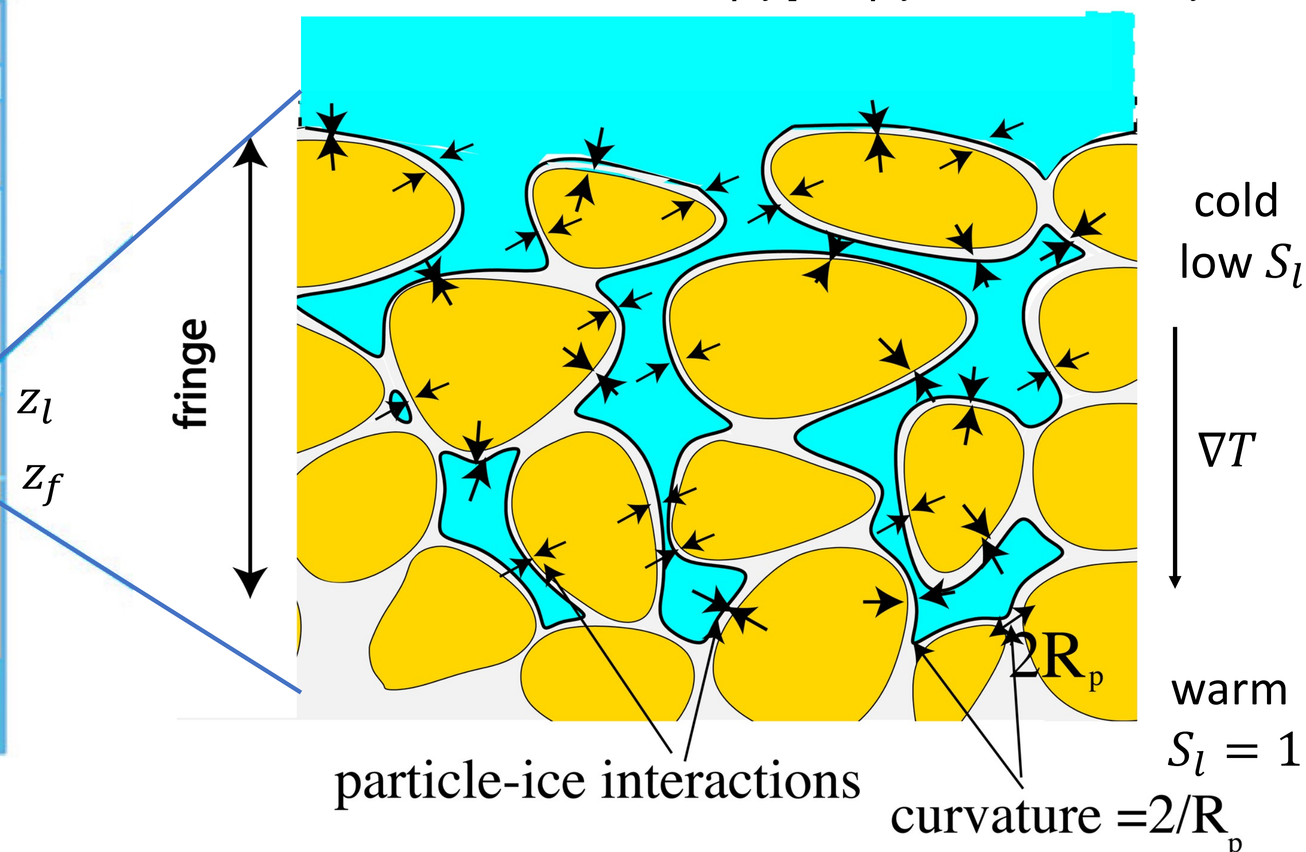
q depends on permeability $k(S_l)$

The force balance on the growing lens involves gravity ↓, the hydrodynamic pressure P_l on the ice surface ↓, and the net disjoining pressure $\Pi(d)$ between the ice and particles of the fringe ↑.

The net hydrodynamic force per unit area can be expressed as: $P_l(z_f) + \mu V \int_{z_f}^{z_l} \frac{[1-n(1-S_l)]^2}{k} \mathrm{d}z$

Mass balance requires that $\rho_l q \approx \rho_i[1 - n(1 - S_l)]V$.

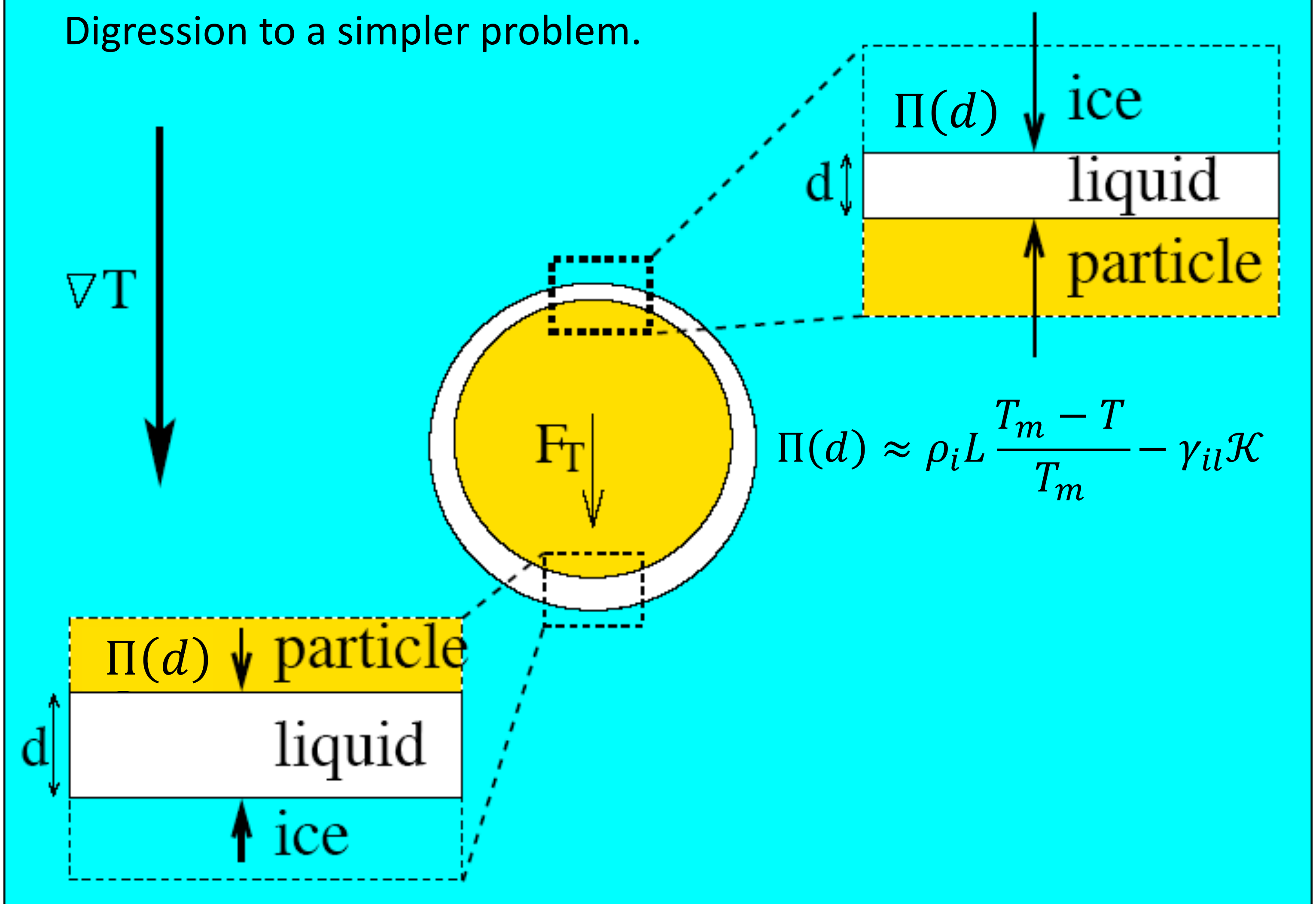




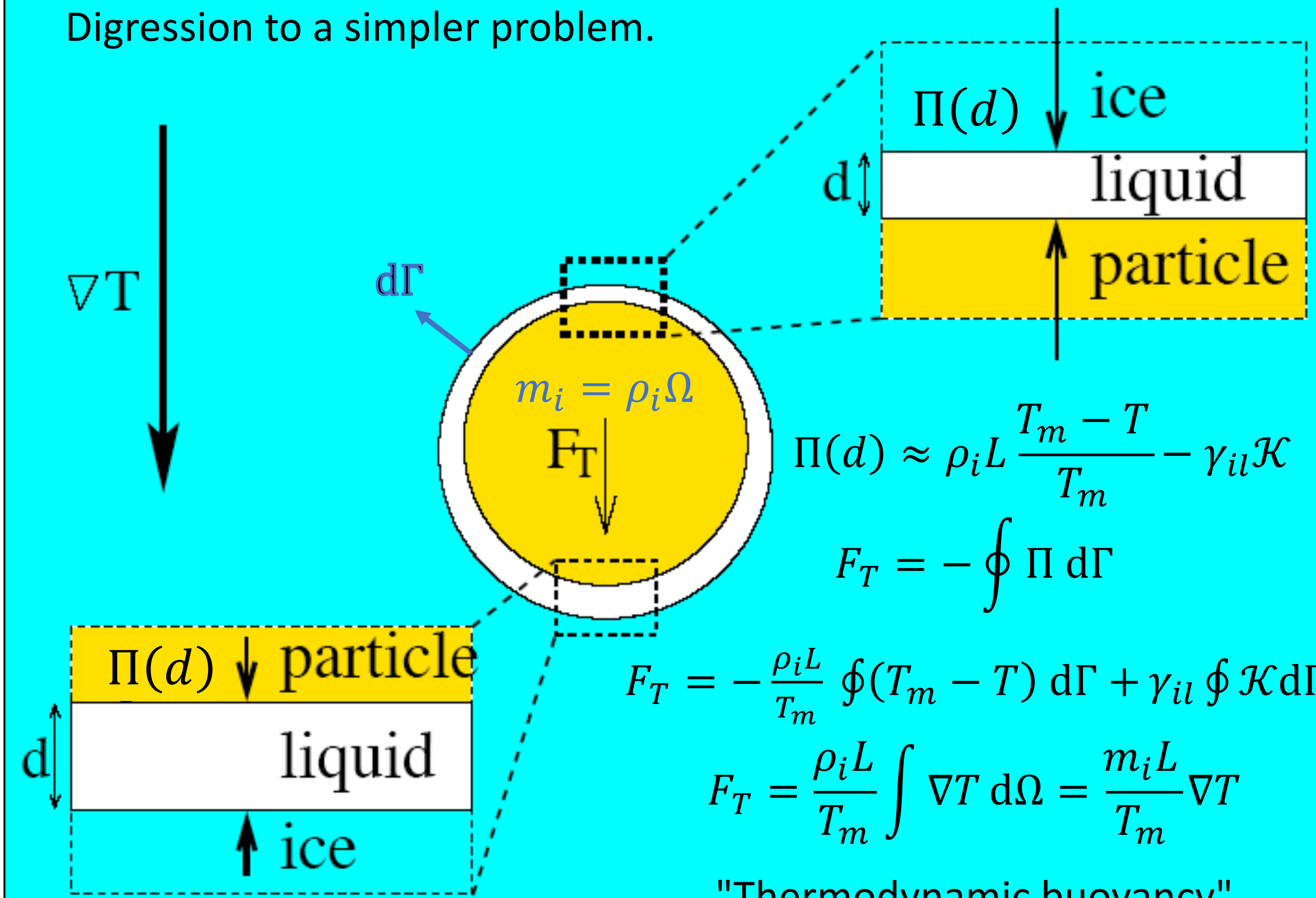
The force balance on the growing lens involves gravity ↓, the hydrodynamic pressure P_l on the ice surface ↓, and the net disjoining pressure $\Pi(d)$ between the ice and particles of the fringe ↑.

The strength of ice–particle interactions varies according to $\Pi(d) \approx \rho_i L \frac{T_m - T}{T_m} - \gamma_{il}\mathcal{K}$.

Digression to a simpler problem.



Digression to a simpler problem.



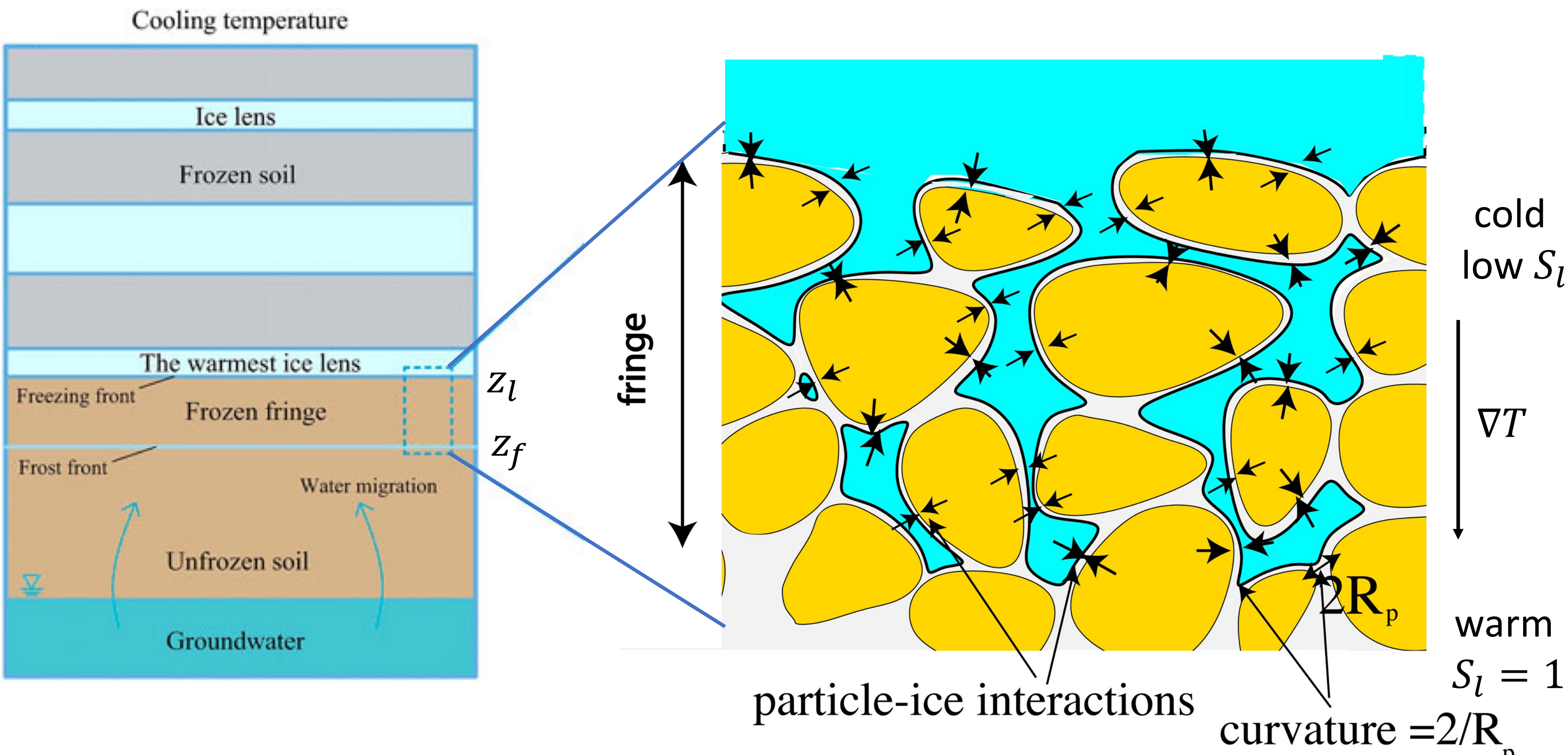
$$\Pi(d) \approx \rho_i L \frac{T_m - T}{T_m} - \gamma_{il}\mathcal{K}$$

$$F_T = -\oint \Pi \, d\Gamma$$

$$F_T = -\frac{\rho_i L}{T_m} \oint (T_m - T)\, d\Gamma + \gamma_{il} \oint \mathcal{K} d\Gamma$$

$$F_T = \frac{\rho_i L}{T_m} \int \nabla T \, d\Omega = \frac{m_i L}{T_m} \nabla T$$

"Thermodynamic buoyancy"

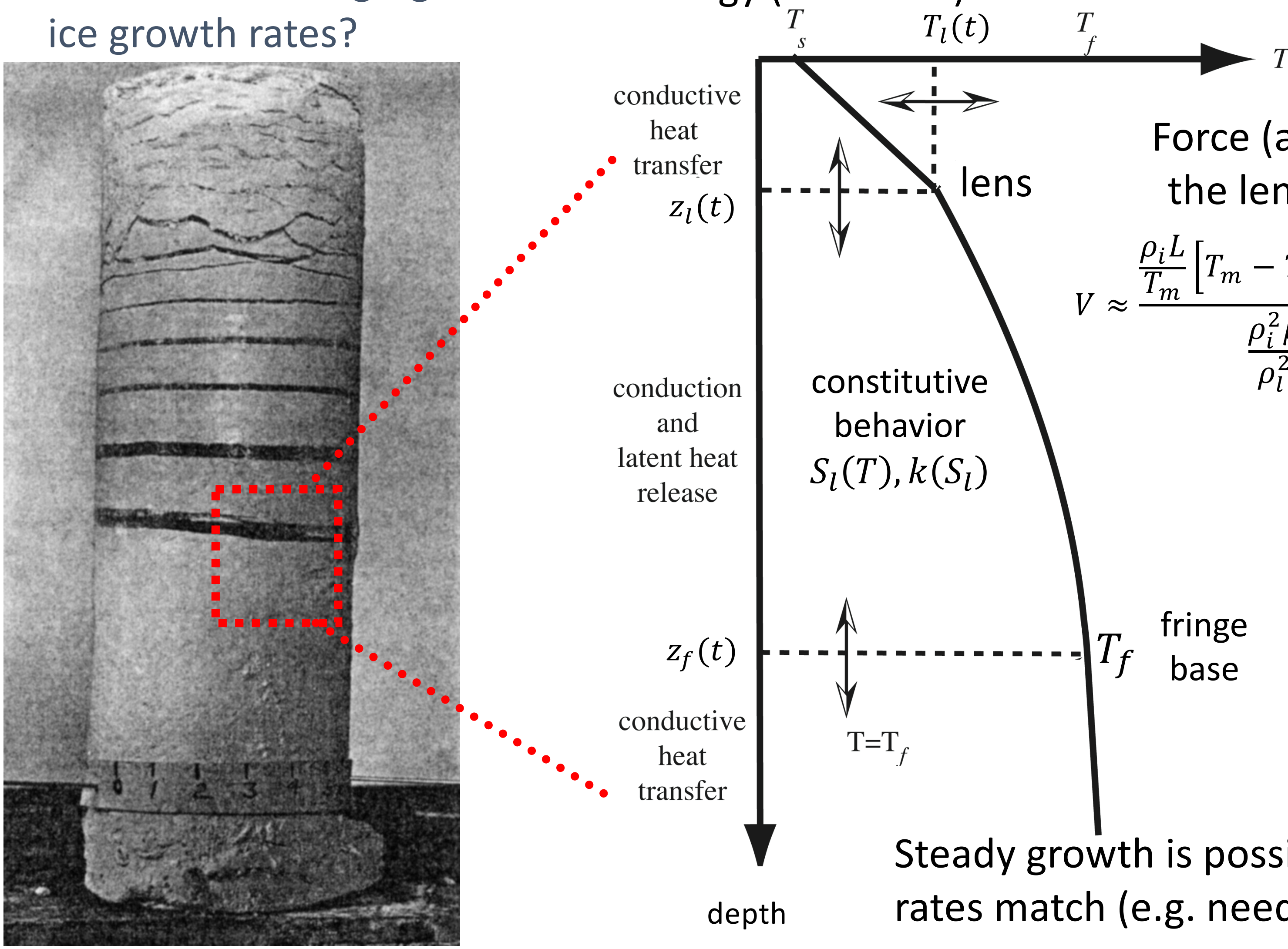


The force balance on the growing lens involves gravity ↓, the hydrodynamic pressure P_l on the ice surface ↓, and the net disjoining pressure $\Pi(d)$ between the ice and particles of the fringe ↑.

With $\oint \mathcal{K} \mathrm{d}\Gamma = 0$, we have $\int \Pi \, \mathrm{d}\Gamma = \frac{\rho_i L}{T_m} \int_{z_f}^{z_l} [1 - n(1 - S_l)] \, \nabla \mathrm{T} \, \mathrm{d}z = \frac{\rho_i L}{T_m} \int_{T_f}^{T_l} [1 - n(1 - S_l)] \, \mathrm{d}T$

- What controls segregated ice growth rates?

Energy (and mass) balance dictates thermal field.



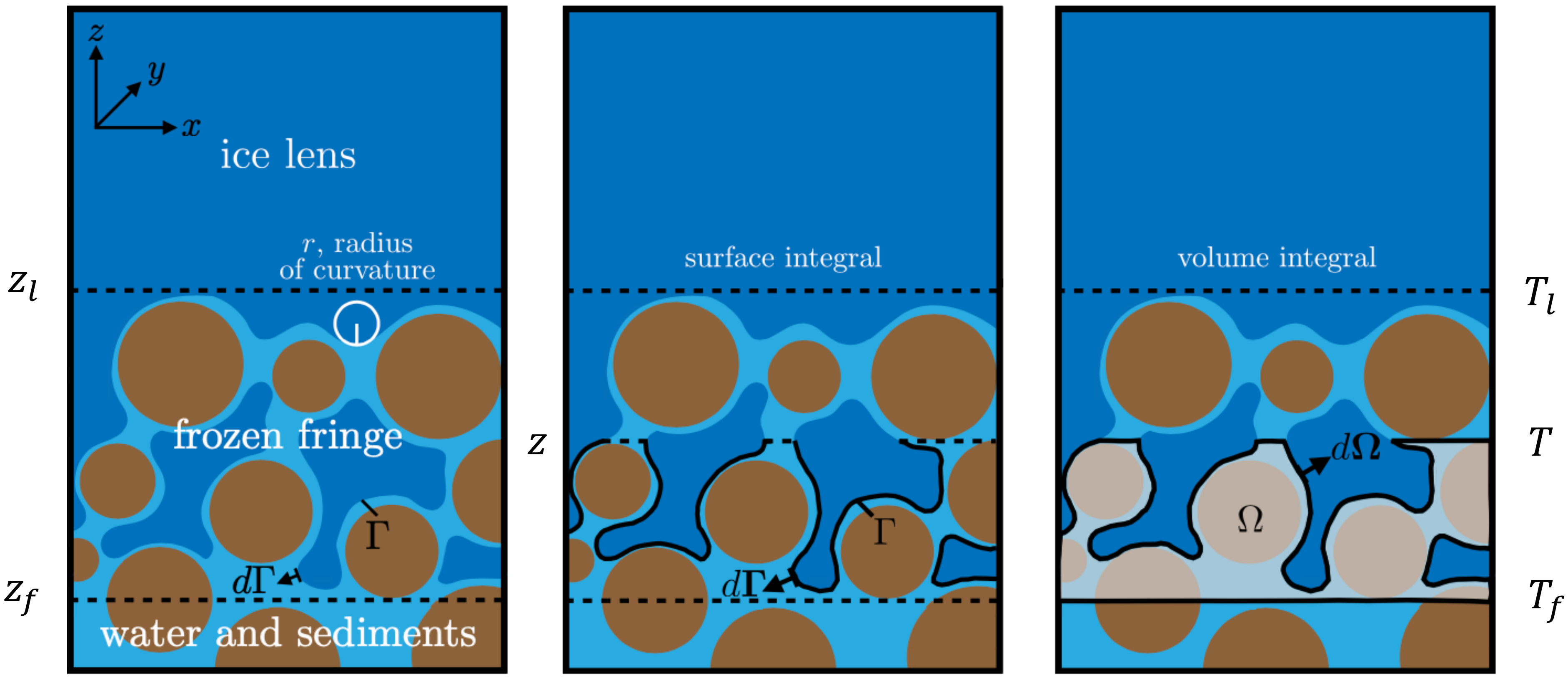
Force (and mass) balance on the lens/fringe ice surface:

$$V \approx \frac{\frac{\rho_i L}{T_m}\left[T_m - T_l + n\int_{T_f}^{T_l}(1-S_l)\,\mathrm{d}T\right] - \sigma_{\mathrm{eff}}(z_f)}{\frac{\rho_i^2\mu}{\rho_l^2}\int_{z_f}^{z_l}\frac{[1-n(1-S_l)]^2}{k}\,\mathrm{d}z}$$

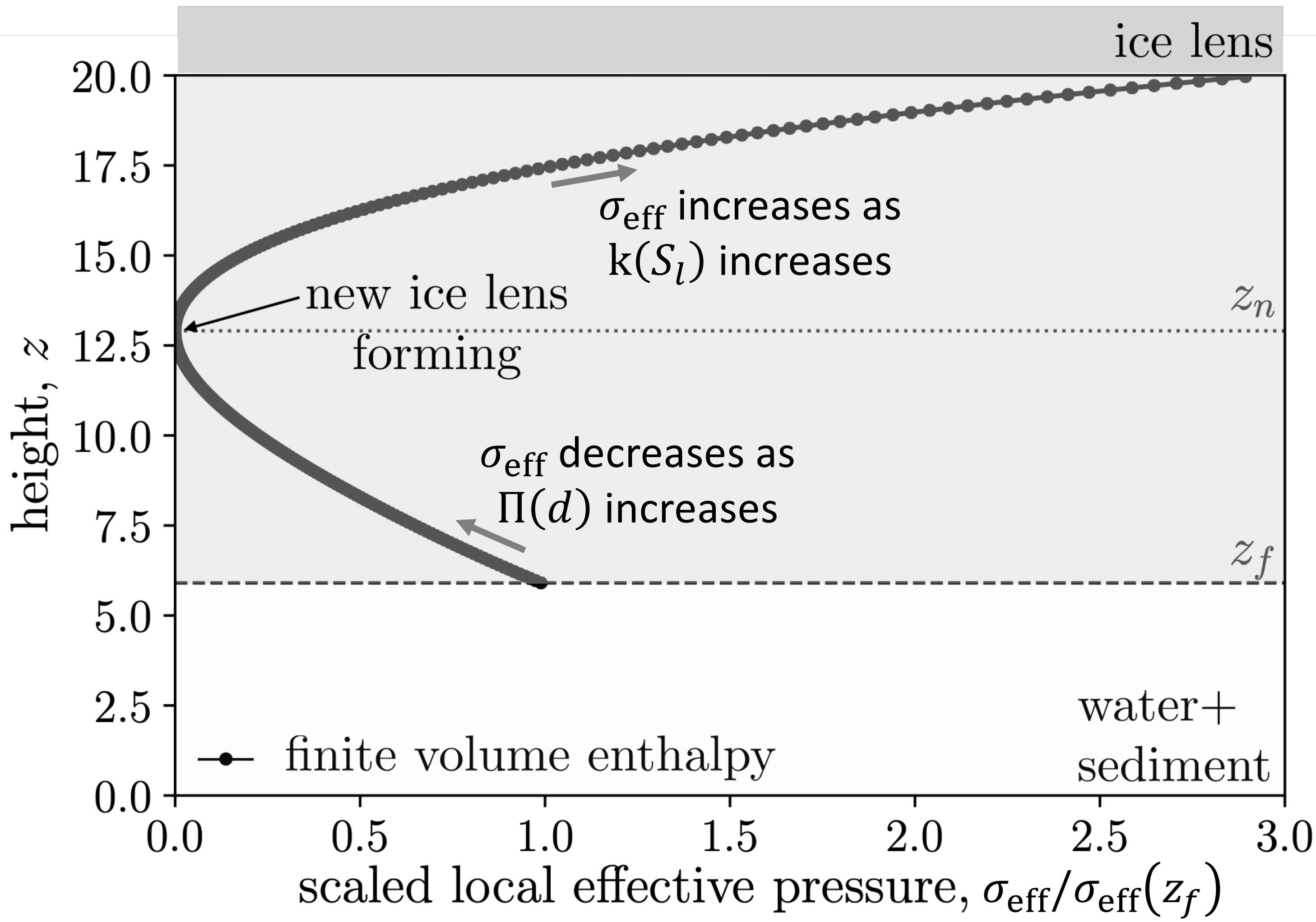
Steady growth is possible if freezing and heave rates match (e.g. needle ice, pingos?).

Steady growth is possible if freezing and heave rates match (e.g. needle ice, pingos?).

The fringe thickens if lens growth lags isotherm advance.



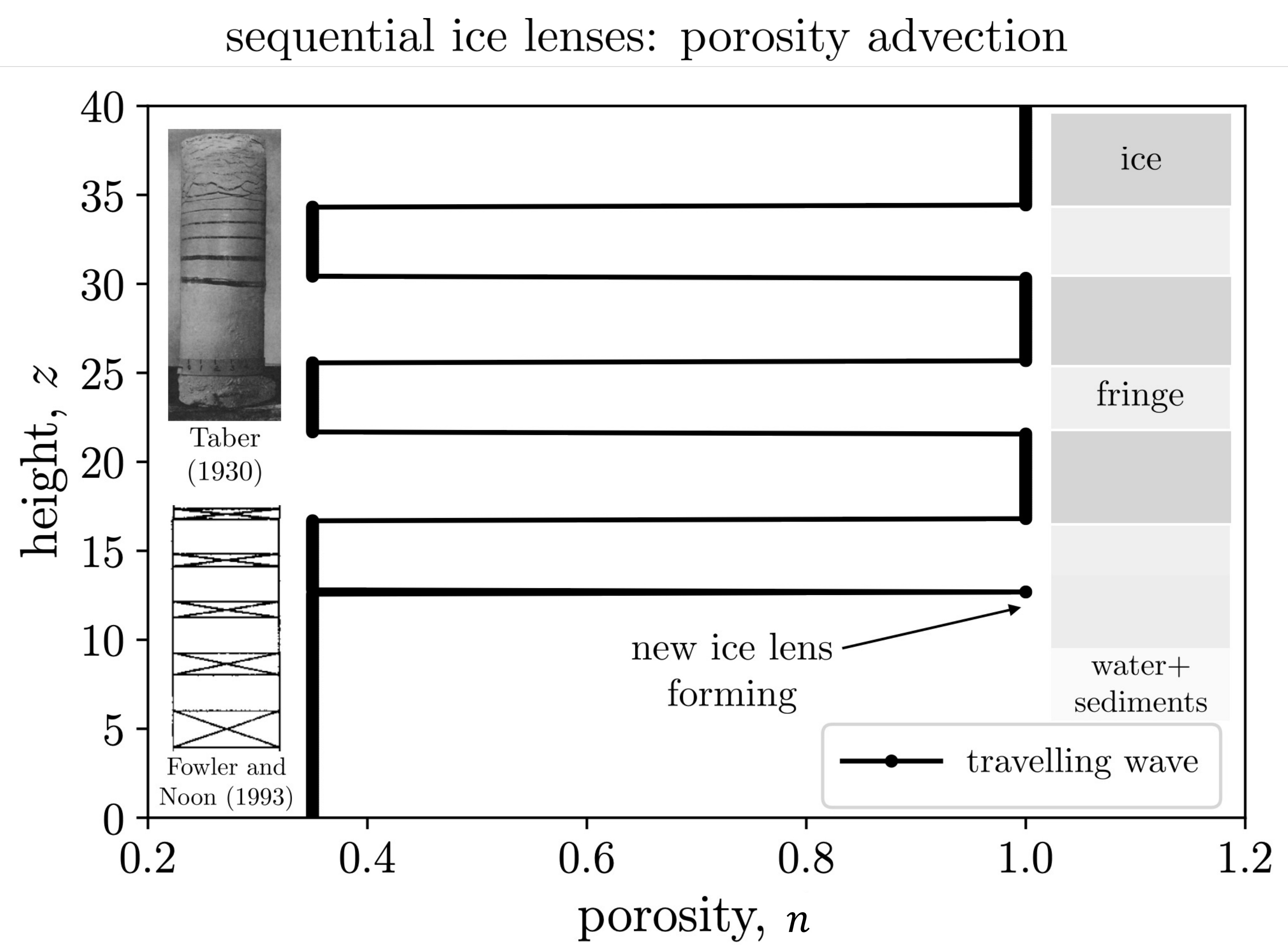
For $z_l > z > z_f$, the effective stress supported by particle contacts σ_{eff} balances any mismatch between gravity, hydrodynamic pressure and the net $\Pi(d)$ transmitted through the fringe ice.



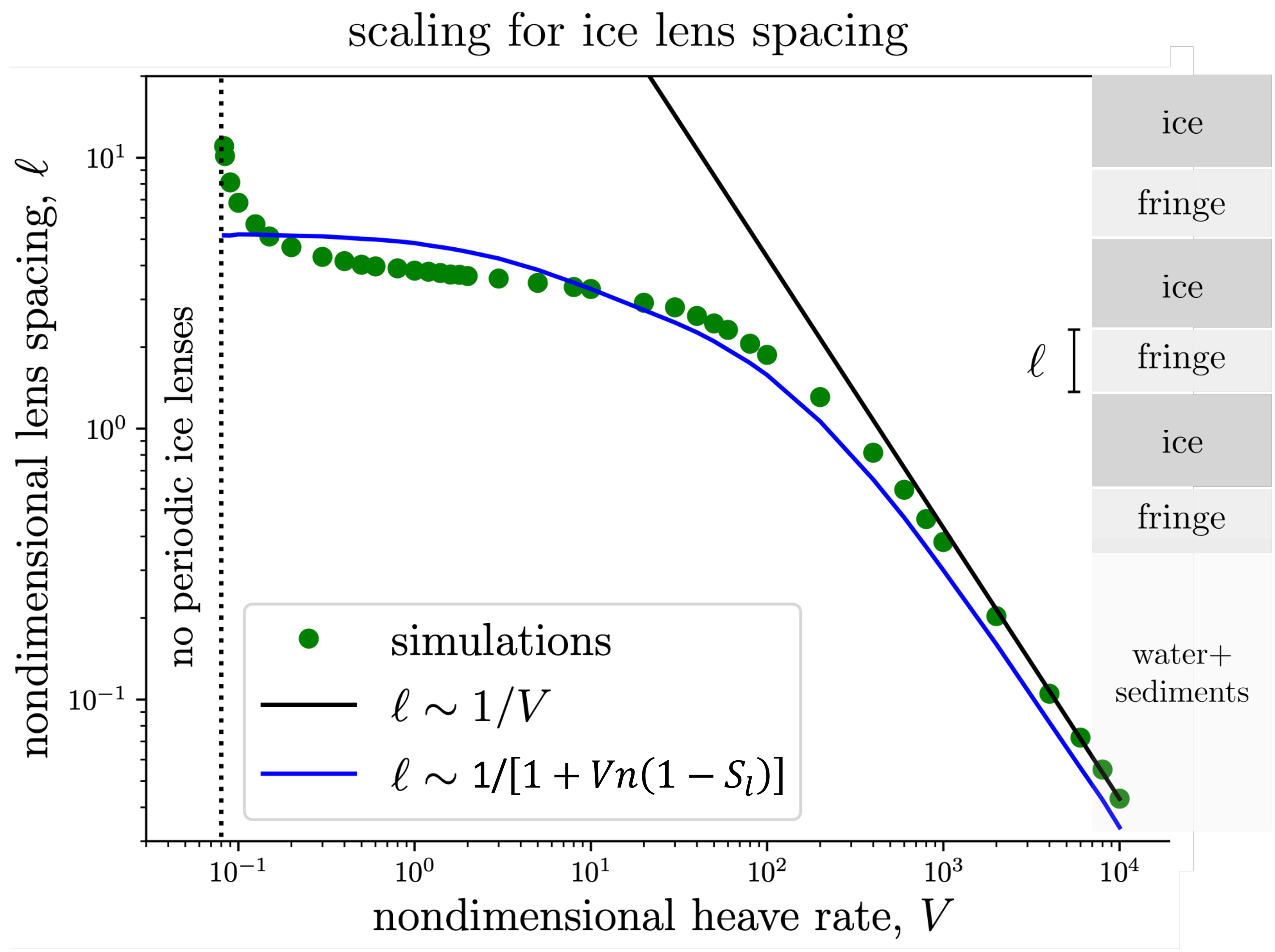
- How are ice lenses initiated?

i) sediment contacts can be unloaded by stress transfer through the connected ice of the fringe

Meyer et al., 2023

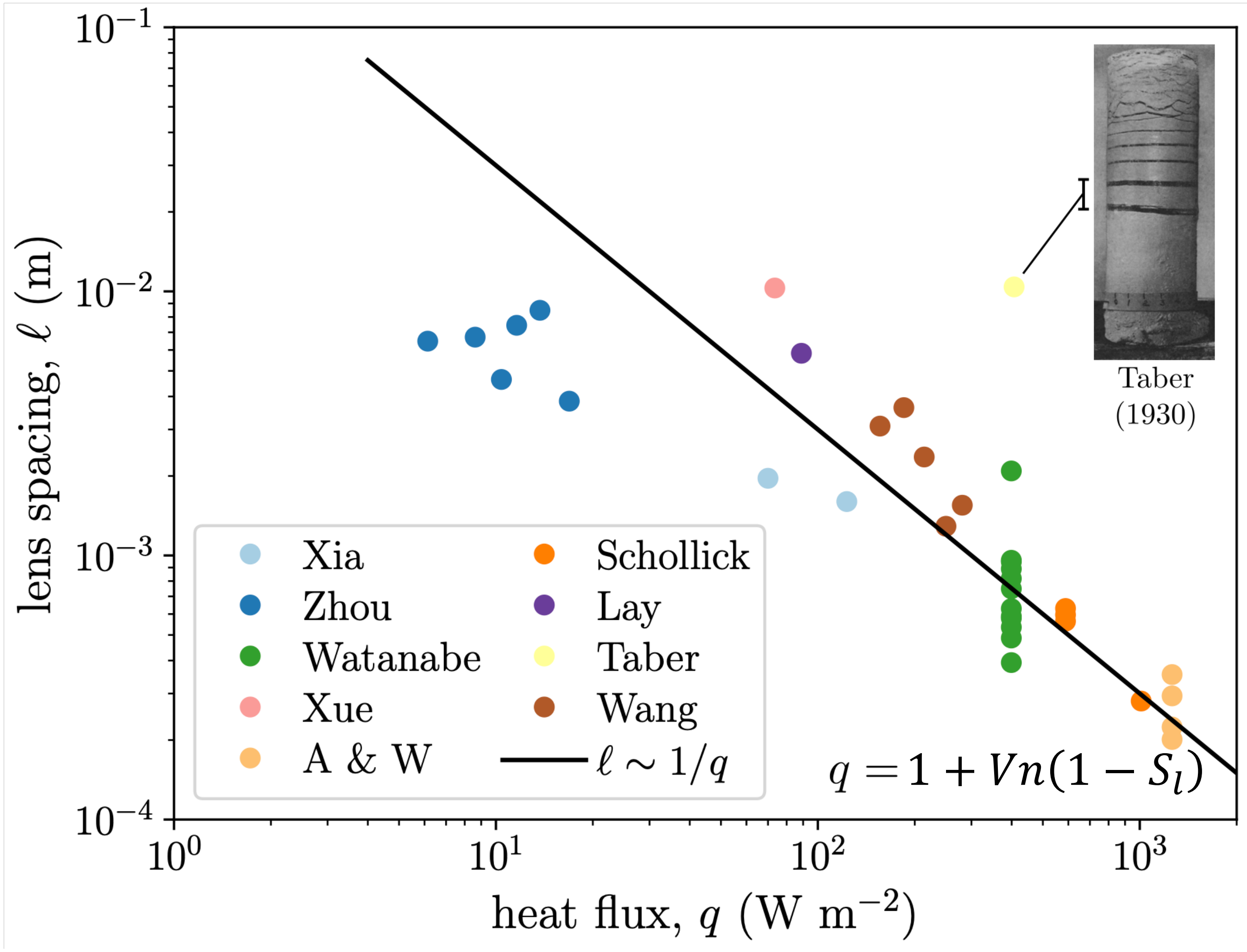


sequential ice lenses: porosity advection
height, z
porosity, n
0
5
10
15
20
25
30
35
40
0.2
0.4
0.6
0.8
1.0
1.2
Taber (1930)
Fowler and Noon (1993)
ice
fringe
water+ sediments
new ice lens forming
travelling wave

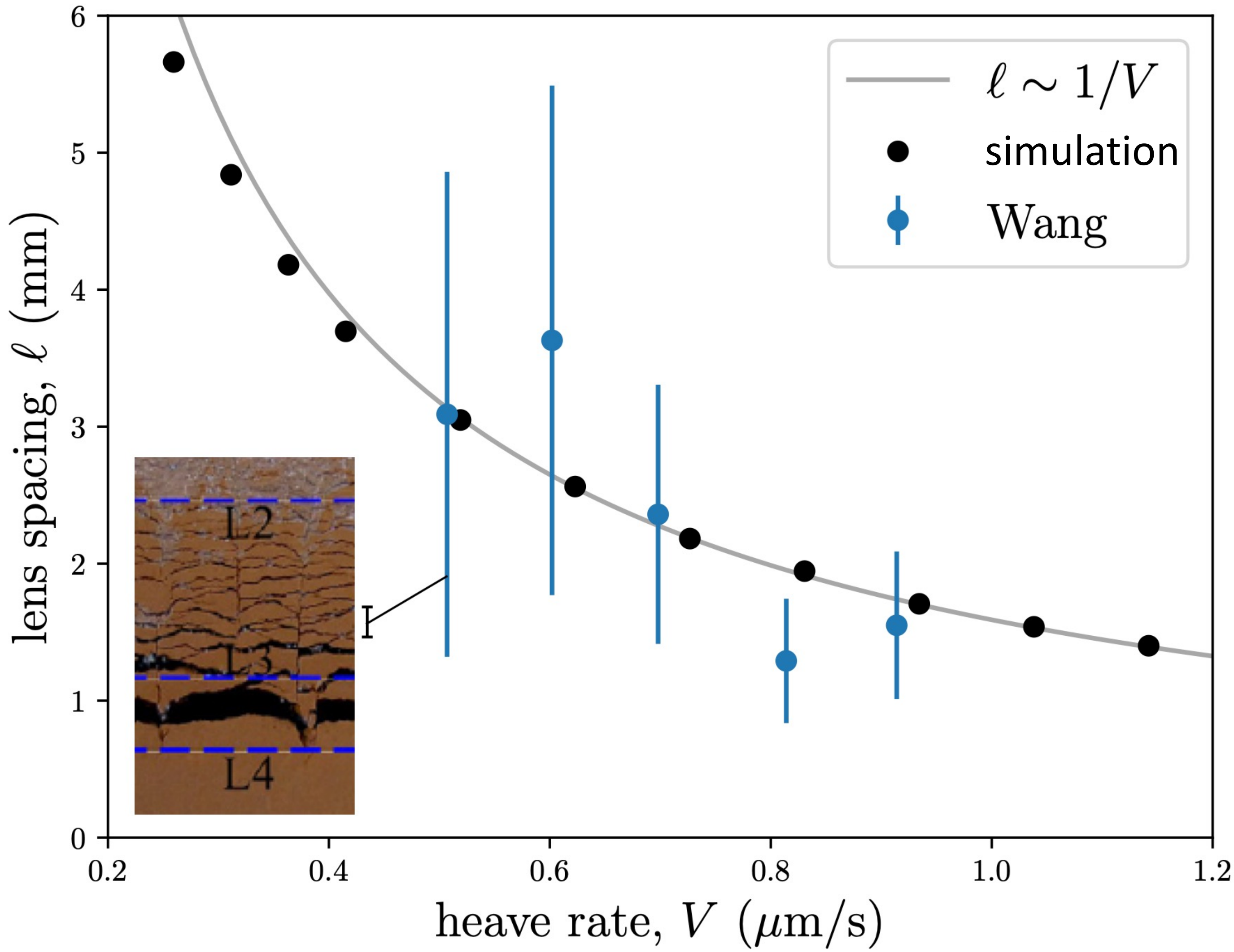


scaling for ice lens spacing
nondimensional lens spacing, ℓ
nondimensional heave rate, V
no periodic ice lenses
simulations
$\ell \sim 1/V$
$\ell \sim 1/[1 + Vn(1 - S_l)]$
ice
fringe
ice
ℓ
fringe
ice
fringe
water+
sediments
10^{1}
10^{0}
10^{-1}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}

scaling for ice lens spacing: flux data



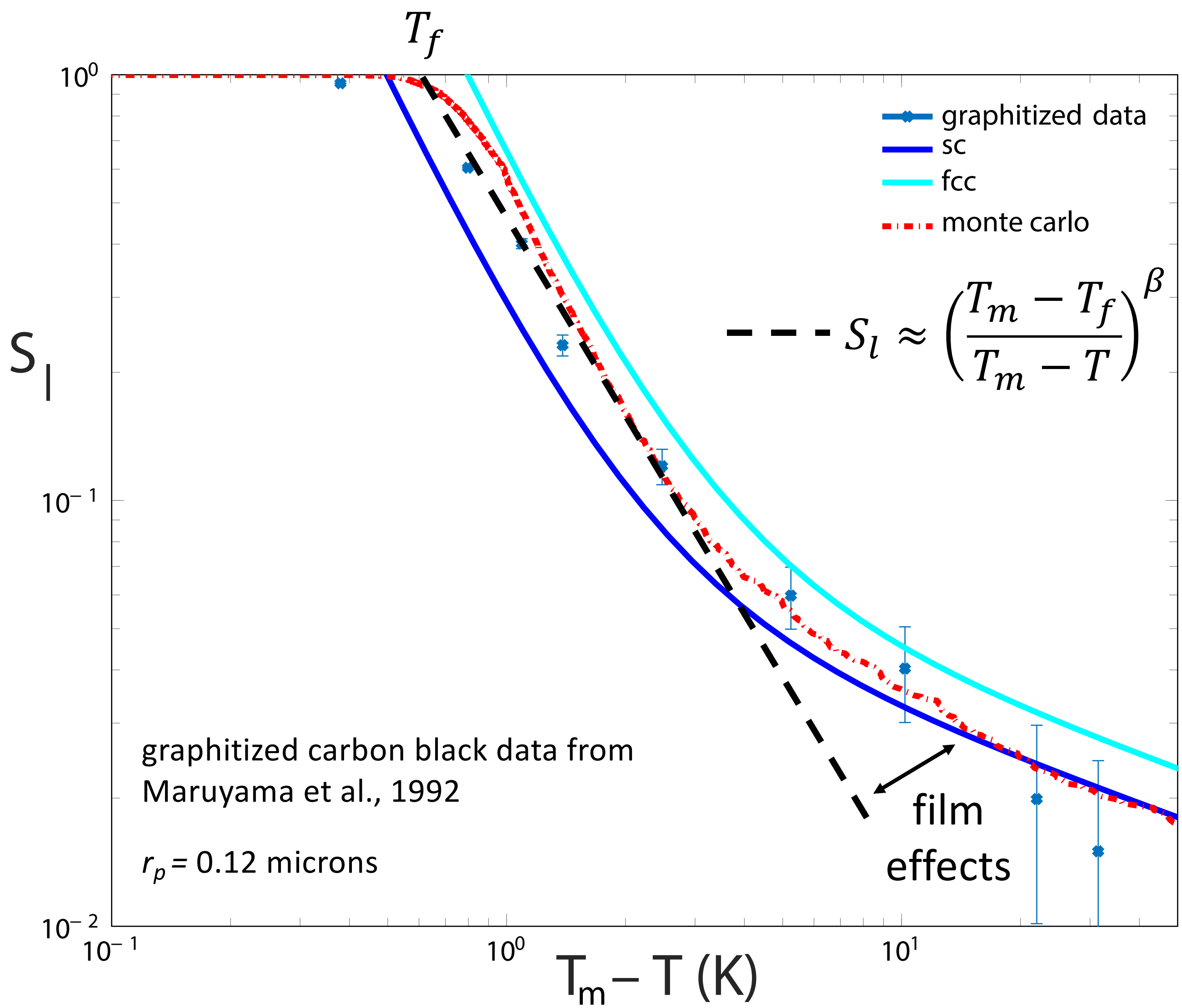
scaling for ice lens spacing: Wang data



- How are ice lenses initiated?

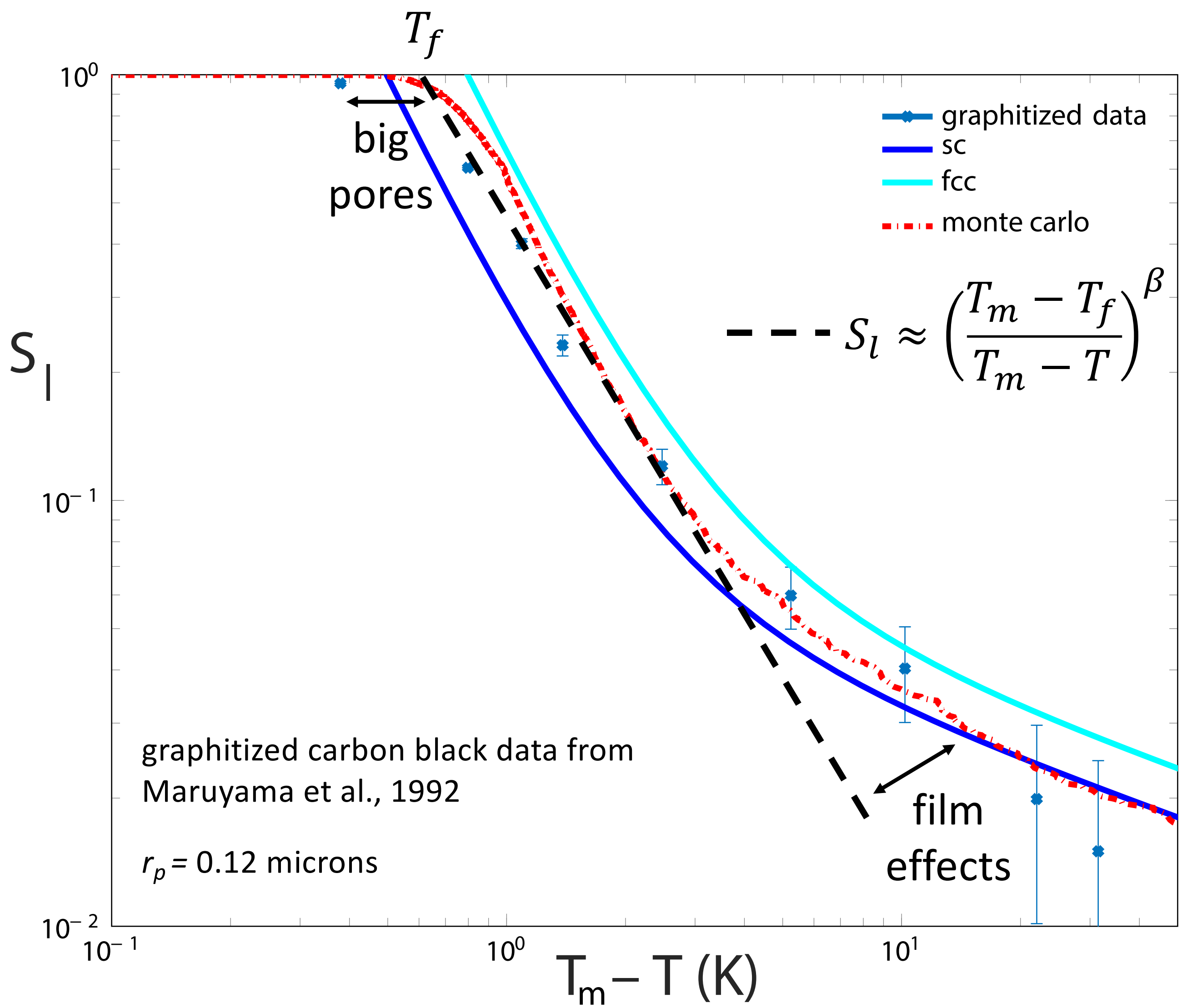
i) sediment contacts can be unloaded by stress transfer through the connected ice of the fringe

Data supporting the idealized 1D treatment isn't overwhelming.



Idealized 1D treatments only allow deformation during lens initiation.

Ice might infiltrate anomalously large pores that can themselves be enlarged through a mechanism akin to crack growth.



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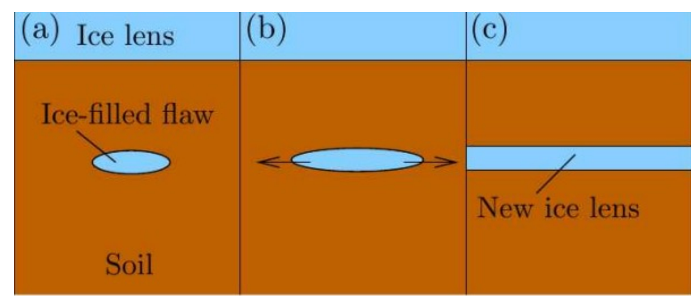
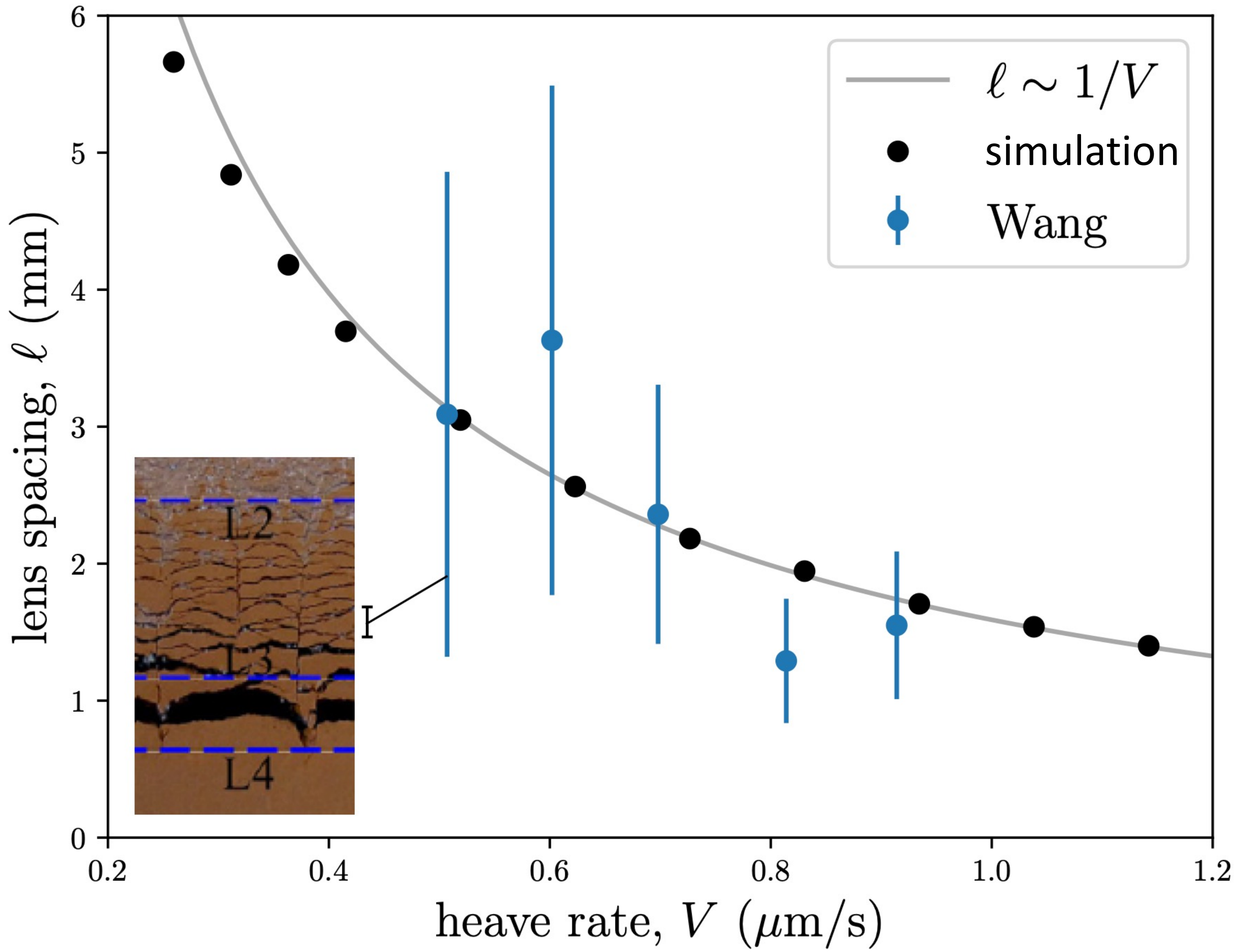


"geometrical supercooling", Style et al., 2011

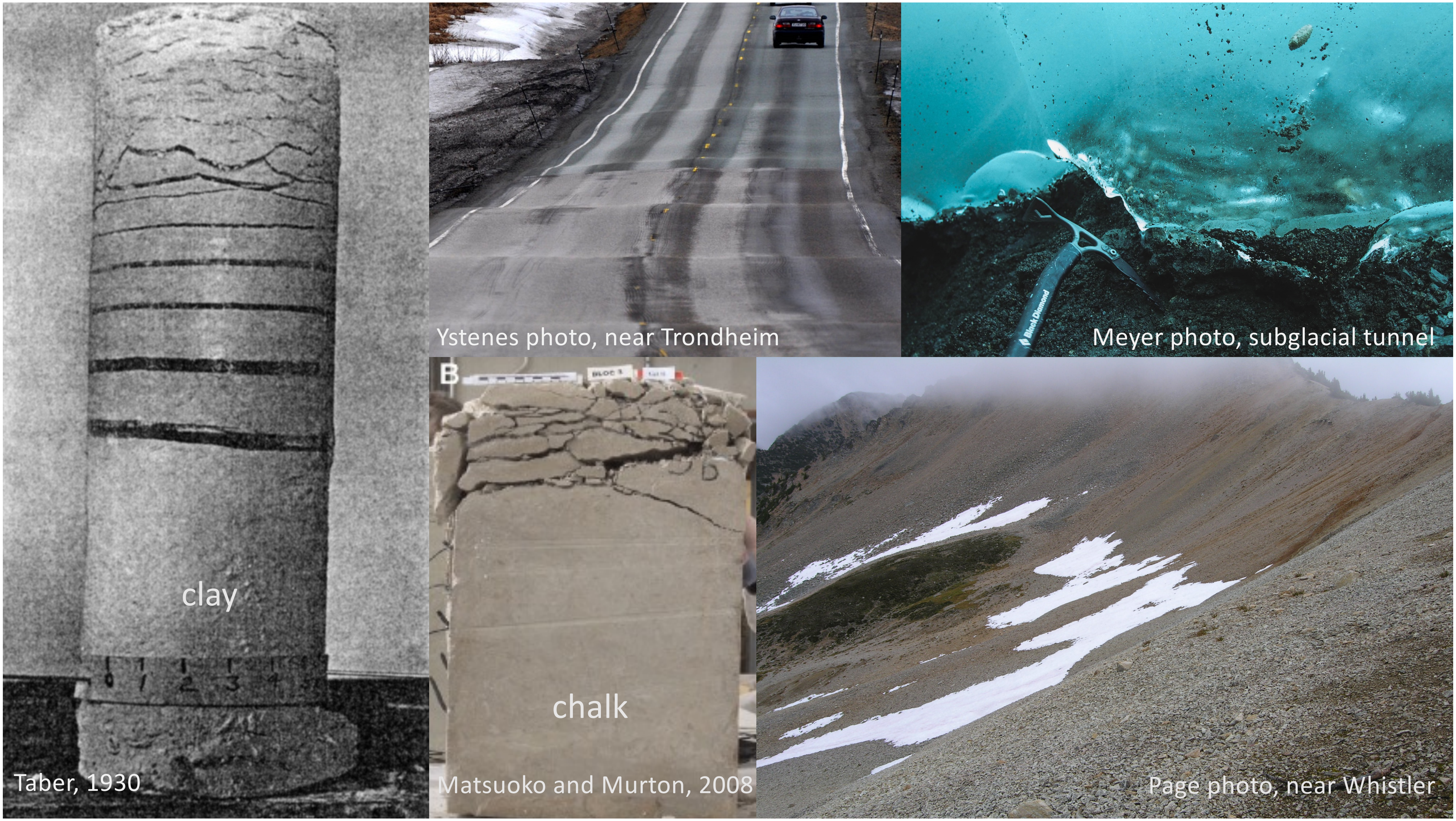
scaling for ice lens spacing: Wang data



- How are ice lenses initiated?

i) sediment contacts can be unloaded by stress transfer through the connected ice of the fringe

ii) ice growth that initiates in large pores can propagate cracks that evolve into ice lenses


clay
Taber, 1930
Ystenes photo, near Trondheim
Meyer photo, subglacial tunnel
B
chalk
Matsuoko and Murton, 2008
Page photo, near Whistler

Sources (ordered by slide number of first appearance)

2 Taber, S. (1930). The mechanics of frost heaving. *The Journal of Geology*, *38*(4), 303-317.
Matsuoka, N., & Murton, J. (2008). Frost weathering: recent advances and future directions. *Permafrost and Periglacial Processes*, *19*(2), 195-210.
4 Gerber, D., Wilen, L. A., Poydenot, F., Dufresne, E. R., & Style, R. W. (2022). Stress accumulation by confined ice in a temperature gradient. *Proceedings of the National Academy of Sciences*, *119*(31), e2200748119.
9 Style, R. W., Gerber, D., Rempel, A. W., & Dufresne, E. R. (2023). The generalized Clapeyron equation and its application to confined ice growth. *Journal of Glaciology*, in press.
10 Horiguchi, K. and Miller, R.D. (1983) Hydraulic conductivity functions of frozen materials. Proceedings of 4th International Permafrost Conference. pp. 504–508.
Watanabe, K., & Osada, Y. (2016). Comparison of hydraulic conductivity in frozen saturated and unfrozen unsaturated soils. *Vadose Zone Journal*, *15*(5).
11 Chen, J., Mei, S., Irizarry, J. T., & Rempel, A. W. (2020). A Monte Carlo approach to approximating the effects of pore geometry on the phase behavior of soil freezing. *Journal of Advances in Modeling Earth Systems*, *12*(10), e2020MS002117.
13 Maruyama, M., Bienfait, M., Dash, J. G., & Coddens, G. (1992). Interfacial melting of ice in graphite and talc powders. *Journal of crystal growth*, *118*(1-2), 33-40.
Cahn, J. W., Dash, J. G., & Fu, H. (1992). Theory of ice premelting in monosized powders. *Journal of crystal growth*, *123*(1-2), 101-108.
15 Chen, J., Mei, S., & Rempel, A. W. (2021). Estimating permeability of partially frozen soil using floating random walks. *Water Resources Research*, *57*(12), e2021WR030598.
17 Taber, S. (1929). Frost heaving. *The Journal of Geology*, *37*(5), 428-461.
Black, P. B., & Hardenberg, M. J. (1991). *Historical perspectives in frost heave research: the early works of S. Taber and G. Beskow*. Cold Regions Research & Engineering Lab Hanover NH.
Everett, D. H. (1961). The thermodynamics of frost damage to porous solids. *Transactions of the Faraday society*, *57*, 1541-1551.
Gilpin, R. (1980). A model for the prediction of ice lensing and frost heave in soils. *Water Resources Research*, *16*(5), 918-930.
O'Neill, K., & Miller, R. D. (1985). Exploration of a rigid ice model of frost heave. *Water Resources Research*, *21*(3), 281-296.
Fowler, A. C., & Krantz, W. B. (1994). A generalized secondary frost heave model. *Siam Journal on Applied Mathematics*, *54*(6), 1650-1675.
Peppin, S. S. L., Elliott, J. A. W., & Worster, M. G. (2006). Solidification of colloidal suspensions. *Journal of Fluid Mechanics*, *554*, 147-166.
Style, R. W., Peppin, S. S., Cocks, A. C., & Wettlaufer, J. S. (2011). Ice-lens formation and geometrical supercooling in soils and other colloidal materials. *Physical Review E*, *84*(4), 041402.
Frost, R. (1915) Mending Wall. *North of Boston*.
18 Rempel, A. W., Wettlaufer, J. S., & Worster, M. G. (2004). Premelting dynamics in a continuum model of frost heave. *Journal of Fluid Mechanics*, *498*, 227-244.
22 Rempel, A. W., Wettlaufer, J. S., & Worster, M. G. (2001). Interfacial premelting and the thermomolecular force: thermodynamic buoyancy. *Physical Review Letters*, *87*(8), 088501.
25 Rempel, A. W. (2007). Formation of ice lenses and frost heave. *Journal of Geophysical Research: Earth Surface*, *112*(F2).
26 Meyer, C. R., Schoof, C., & Rempel, A. W. (2023). A thermomechanical model for frost heave and subglacial frozen fringe. *J. Fluid Mech.* in press.
30 Xia, D., Arenson, L. U., Biggar, K. W., & Sego, D. C. (2005). Freezing process in Devon silt-using time-lapse photography. In *58th Canadian geotechnical conference, Saskatoon, Sask*.
Zhou, J., Wei, C., Wei, H., & Tan, L. (2014). Experimental and theoretical characterization of frost heave and ice lenses. *Cold Regions Science and Technology*, *104*, 76-87.
Watanabe, K. (2002). Relationship between growth rate and supercooling in the formation of ice lenses in a glass powder. *Journal of crystal growth*, *237*, 2194-2198.
Xue, K., Wen, Z., Zhu, Z., Wang, D., Luo, F., & Zhang, M. (2021). An experimental study of the relationship between the matric potential, unfrozen water, and segregated ice of saturated freezing soil. *Bulletin of Engineering Geology and the Environment*, *80*, 2535-2544.
Anderson, A. M., & Worster, M. G. (2012). Periodic ice banding in freezing colloidal dispersions. *Langmuir*, *28*(48), 16512-16523.
Schollick, J. M., Style, R. W., Curran, A., Wettlaufer, J. S., Dufresne, E. R., Warren, P. B., ... & Aarts, D. G. (2016). Segregated ice growth in a suspension of colloidal particles. *The Journal of Physical Chemistry B*, *120*(16), 3941-3949.
Lay, R. D. (2005). *Development of a frost heave test apparatus*. Brigham Young University.
Wang, Y., Wang, D., Ma, W., Wen, Z., Chen, S., & Xu, X. (2018). Laboratory observation and analysis of frost heave progression in clay from the Qinghai-Tibet Plateau. *Applied Thermal Engineering*, *131*, 381-389.