









A disastrous consequence of the increase in density upon freezing.

What freezer temperature would have been safe?

Estimated tensile strength: 6 MPa Clapeyron slope: $\frac{dT}{dP} \approx 0.73 \frac{K}{MPa}$ In closed system expected: $\Delta T \approx -0.43$ K



- conservation of beer plus ice
- conservation of vapor (probably wrong)
- linear liquid/ice compressibilities
- neglect freezing point depression from alcohol
- rapid fluid and heat redistribution
- $P_0 = 10^5$ Pa, vapor volume fraction S_{v0}







When >91% of the pores of concrete are filled with water... [and] ... these molecules freeze, they expand by 9%, and because there is no room for their increased volume, the concrete distresses. The freeze can cause the bonds in cement around the aggregate to break ... As the seasons pass, concrete goes through the process of freezing and thawing, wearing out over time.

Images and text from civildigital.com

2007 Monthly freeze/thaw time series

Freeze Thaw Status -- Combined (CO) LEGEND: Blue 0=Frozen RED 1=Thawed OLIVE 2=Transitional GREEN 3=Inverse Transitional



ani_SSMI_37V_CO_FT_2007_day030.png

downloaded from: https://www.umt.edu/numerical-terradynamic-simulation-group/freeze-thaw/

Mechanisms:

volumetric
 expansion (should
 occur near 0°C, favored
 by low S_{v0})

2. thermal expansion cracking (more prominent with rapid *T* change, no direct dependence on *S*_{v0})

3. segregated ice
growth
(liquid flow towards
freezing sites, requires
T cold enough to
propagate cracks)



Matsuoka and Murton, 2008





Walder and Hallet, 1985

- stress intensity factor $K_I = \lim_{r \to 0} \sqrt{2\pi r} \sigma_{yy}(r, 0)$
- penny-shaped crack:

$$K_I = \sqrt{\frac{4c}{\pi}} P_i$$

- subcritical crack growth once K_I exceeds a "stress corrosion limit"
- extension rate determined by mass balance (partly by corrosion kinetics, and partly by water transport)



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Hales and Roering, 2007









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How does frost damage depend upon environmental factors and material properties over relevant scales in length and time? Suppose frost damage correlates with ice growth that extends cracks. How does porosity change when it is cold enough to crack?

1. Assume cracking once $K_I = K_c$ so for a single characteristic crack shape, require: $P_i = P_c$

2. Mass balance ($S_l = 1$ initially) and Darcy's law with a temperature-dependent permeability:

$$\frac{\partial n}{\partial t} = \nabla \cdot \left(\frac{k}{\mu} \nabla P\right) = \mu^{-1} \left(\frac{\mathrm{d}k}{\mathrm{d}T} \nabla T \cdot \nabla P + k \nabla^2 P\right)$$

- 3. Generalized Clapeyron equation with $P_i = P_c$ implies $\nabla P = \frac{\rho L}{T_m} \nabla T$ so $\frac{\partial n}{\partial t} = \frac{\rho L}{T_m \mu} \left[\frac{\mathrm{d}k}{\mathrm{d}T} (\nabla T)^2 + k \nabla^2 T \right]$
- 4. Ignore latent heat; assume cracking begins long after most water already frozen so $S_l \ll 1$

$$\frac{\partial n}{\partial t} \approx \frac{\rho L}{T_m \mu} \left[\frac{\mathrm{d}k}{\mathrm{d}T} \left(\nabla T \right)^2 + \frac{k}{\kappa} \frac{\partial T}{\partial t} \right]$$

5. Assume periodic annual thermal forcing

$$\Delta n(z, 1 \text{ year}) = \frac{\rho L}{T_m \mu} \int_{\Delta T > \Delta T_c} \frac{\mathrm{d}k}{\mathrm{d}T} \left(\frac{\partial T}{\partial z}\right)^2 \mathrm{d}t$$





How does the timing of crack-driven porosity increases vary with climate?

- only in winter if just cold enough
- most intense in fall if somewhat colder
- most intense in spring when still colder
- only in summer in coldest cases





Greatest depth-integrated deformation with largest seasonal amplitudes – especially if MAT close to crack growth onset temperatures.

"Swell depth" gets deeper as MAT decreases, but shallows once again when MAT below crack onset temperature.





The legacy of damage in past climates may influence near-surface crack patterns accessed by water today. Marshall et al., 2021



Scherler, 2013

Some evidence from landscape form for enhanced cracking at particular climatic thresholds. Note: location of damage relative to pre-existing joints likely important.



Other evidence of cracking doesn't appear to have the predicted low-temperature fall-off.





Procedure:

- 1. beaker-soak one-inch cores
- 2. wrap in plastic (avoid contact with glycol)
- 3. drop in glycol bath at fixed *T*
- 4. inspect frequently until cracks detected









Eugene formation sandstone expts.

Laura van Alst, MSc., 2011









Height [m]



A single crack size is likely too idealized. Evidence suggests coarsening with large cracks cannibalizing the ice formed initially in small cracks.

A statistical approach might be called for?

With unsaturated conditions the sizes of the largest saturated flaws should be reduced, increasing ΔT_{cu} .



Unsaturated conditions treated here by assuming small pores filled preferentially, limiting characteristic crack size so that:

$$\frac{\Delta n_u}{\Delta n} \sim \left(\frac{T_m - T_c}{T_m - T_{cu}}\right)^{\alpha + 1} \approx S^{(\alpha + 1)/(2\beta)}$$



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