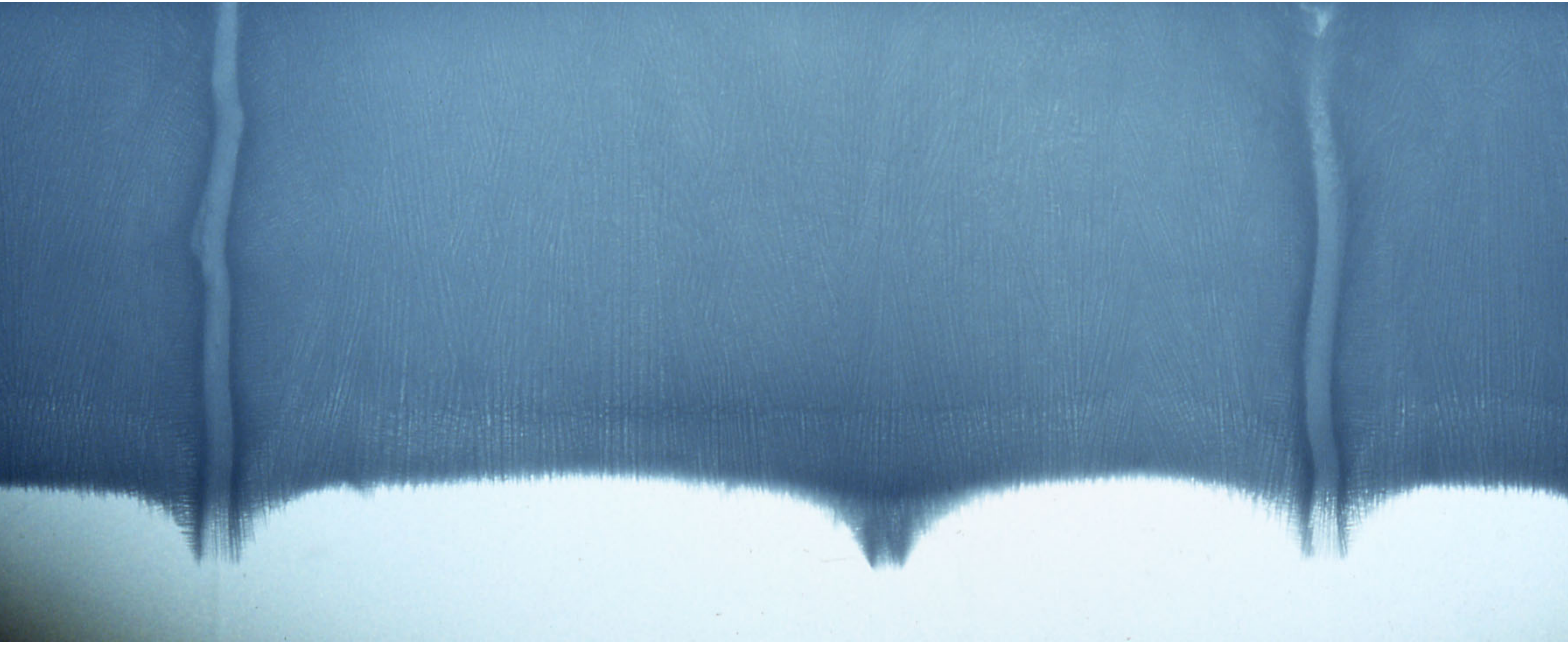
Brinicles of Death!

BBC Frozen Planet 2011

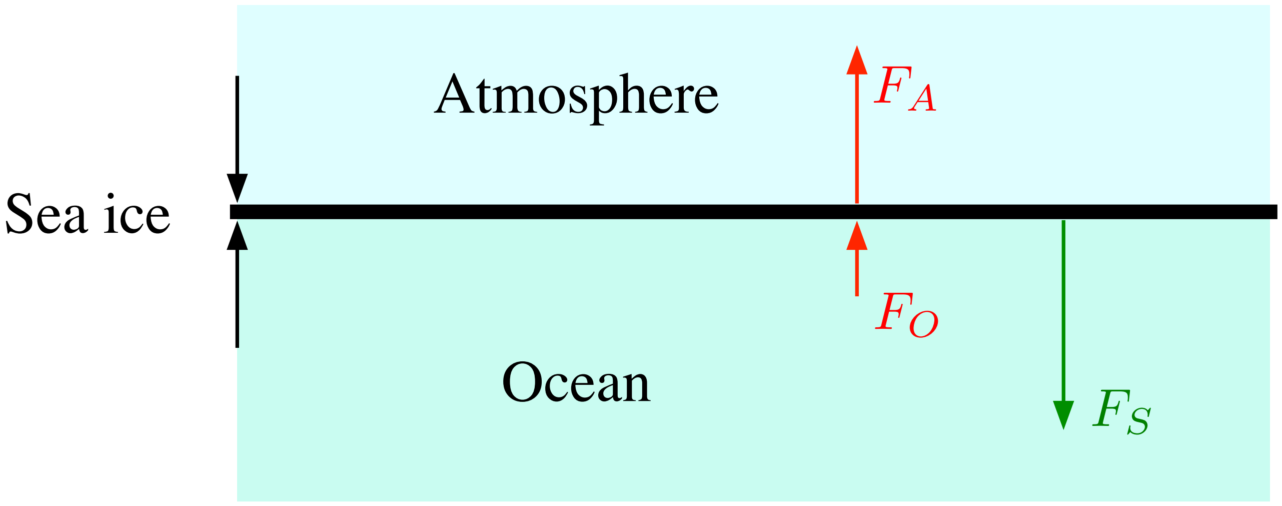
Brine Drainage from Sea Ice

GRAE WORSTER

DAMTP University of Cambridge



Sea Ice as an Interface



Atmosphere scale ~ 10^4 m

Sea-ice scale ~ 1 m

Ocean scale ~ 10^3 m

Simplest thermodynamic model

$$\rho L \phi \frac{dh}{dt} = F_A - F_O = k(\phi)\frac{T_O - T_A}{h} - F_O$$

is one-dimensional

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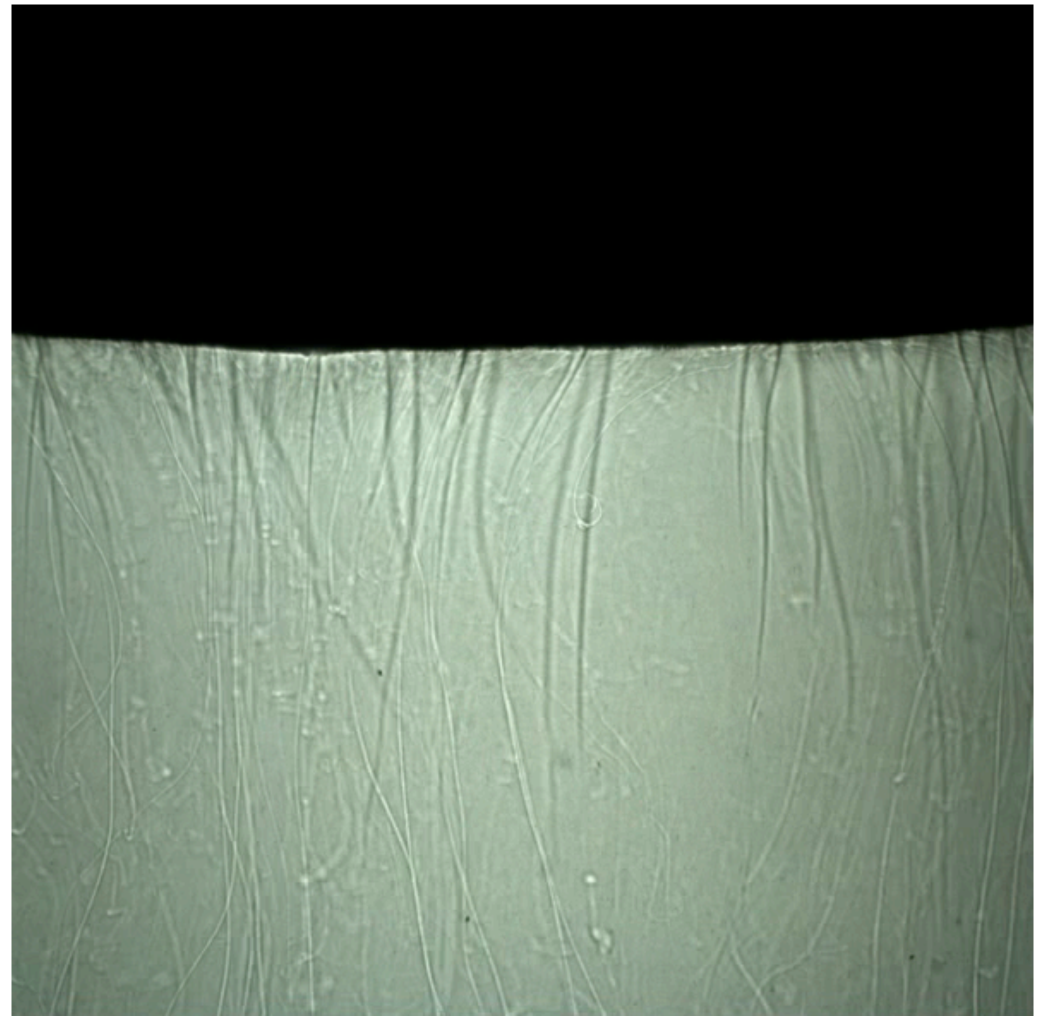
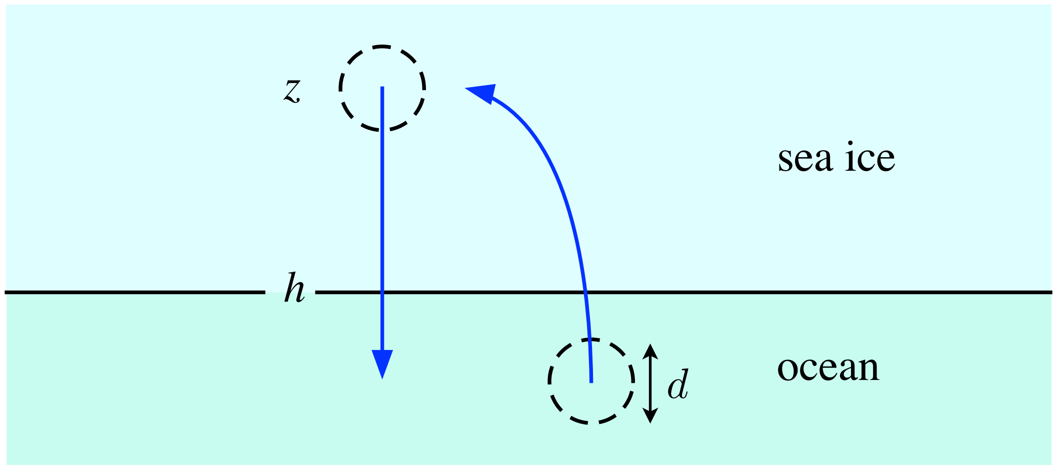


Fundamentals of Convection in Mushy Layers



Potential energy released $$PE \sim \rho_0 \beta [S_{br}(z) - S_0] d^3 g(h-z)$$

Dissipation $$D \sim \Delta p\, d^2 (h-z) \sim \frac{\mu U}{\Pi}(h-z) d^2 (h-z)$$

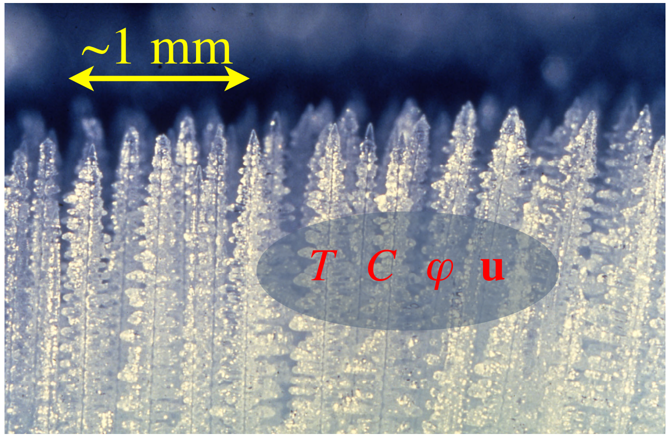
Parcel runs out of buoyancy if $$\tau \sim \frac{h}{U} \gtrsim \frac{d^2}{\kappa}$$

Optimal conversion of potential energy into kinetic energy (convection) if

$\tau \sim \dfrac{d^2}{\kappa}$ and $d \sim h - z$

$$\Rightarrow \; R_m \equiv \frac{\beta\,[S_{br} - S_0]\,g(h-z)\Pi}{\kappa\nu} \gtrsim 1$$

Convection in a Mushy Layer
(Sea Ice)



Conservation of heat
$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T + \frac{L}{c_p} \frac{\partial \phi}{\partial t}$$

Conservation of salt
$$(1-\phi)\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C = C \frac{\partial \phi}{\partial t}$$
ignore diffusion of salt

Liquidus constraint
$$T = T_m - mC$$

Darcy's equation
for flow in a porous medium
$$\mu \mathbf{u} = \Pi[-\nabla p + \rho \mathbf{g}]$$

Incompressibility
$$\nabla \cdot \mathbf{u} = 0$$

Constitutive relation
$$\rho = \rho_0[1 + \beta(C - C_0)]$$

Scaling

Suppose that system is solidifying at rate V and scale

velocities with V lengths with κ / V time with κ / V^2 pressure with with $\Delta\rho g\kappa / V$

Write $\theta = \frac{T - T_0}{\Delta T} = -\frac{C - C_0}{\Delta C}$ where $\Delta T = m\Delta C$

$$\frac{\partial\theta}{\partial t} + \mathbf{u}\cdot\nabla\theta = \nabla^2\theta + \mathcal{S}\frac{\partial\phi}{\partial t}$$

$$(1-\phi)\frac{\partial\theta}{\partial t} + \mathbf{u}\cdot\nabla\theta = (\theta - \mathcal{C})\frac{\partial\phi}{\partial t}$$

$$\mathbf{u} = R_m[-\nabla p + \theta\mathbf{k}]$$

$$\nabla\cdot\mathbf{u} = 0$$

$\mathcal{S} \gg 1, \quad \mathcal{C} \gg 1$ → $\frac{\mathcal{S}}{\mathcal{C}} = \mathrm{O}(1)$

$$\Omega\frac{D\theta}{Dt} = \nabla^2\theta$$

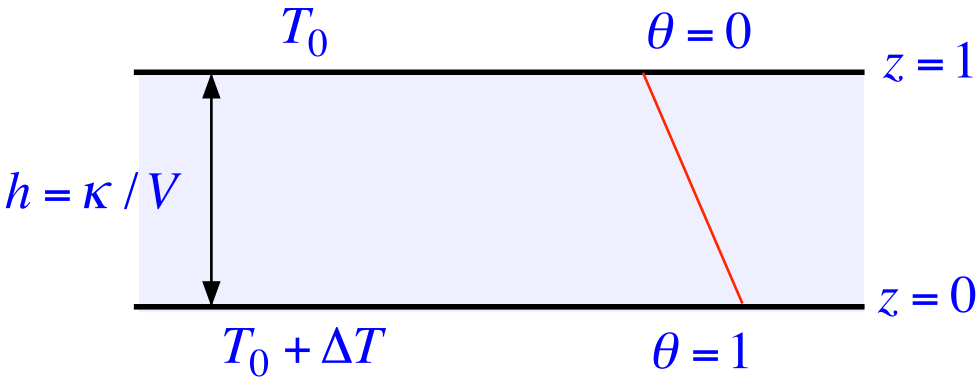
$$\frac{\partial\phi}{\partial t} = -\frac{1}{\mathcal{C}}\frac{D\theta}{Dt}$$

$$\mathbf{u} = R_m[-\nabla p + \theta\mathbf{k}]$$

$$\nabla\cdot\mathbf{u} = 0$$

$$\mathcal{S} = \frac{L}{c_p\Delta T} \qquad \mathcal{C} = \frac{C_0}{\Delta C} \qquad \Omega = 1 + \frac{\mathcal{S}}{\mathcal{C}} \qquad R_m = \frac{\beta\Delta C g\Pi(\kappa/V)}{\kappa\nu}$$

Indicative Stability Analysis



$$\Omega\left(\frac{\partial \theta}{\partial t} + \mathbf{u}\cdot\nabla\theta\right) = \nabla^2\theta$$

$$\mathbf{u} = R_m\left[-\nabla p + \theta\mathbf{k}\right]$$

$$\nabla\cdot\mathbf{u} = 0$$

Write $\quad \mathbf{u} = (u, w)$

$$\theta = 1 - z + \hat{\theta}(z)\, e^{i\alpha x + \sigma t}$$

$$w = \hat{w}(z)\, e^{i\alpha x + \sigma t}$$

Substitute, linearize and look for marginal (steady) states with $\sigma = 0$

$$D \equiv \frac{d}{dz}$$

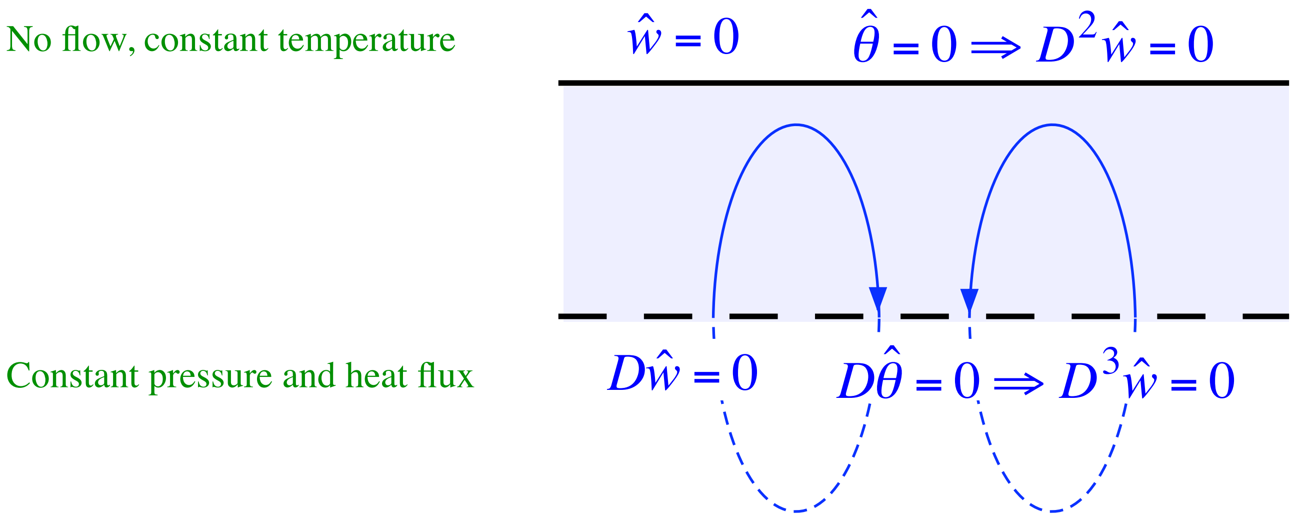
$$-\Omega\hat{w} = \left(D^2 - \alpha^2\right)\hat{\theta}$$

$$\left(D^2 - \alpha^2\right)\hat{w} = \alpha^2 R_m \hat{\theta}$$

$$\Rightarrow \quad \left(D^2 - \alpha^2\right)^2 \hat{w} = -\alpha^2 \Omega R_m \hat{w}$$

Marginal Stability Results

$$\left(D^2-\alpha^2\right)^2\hat{w}=-\alpha^2\Omega R_m\hat{w}$$



$$\hat{w}\propto\sin\left[\left(n+\tfrac{1}{2}\right)\pi(1-z)\right] \qquad \Omega R_m=\frac{\left[\left(n+\frac{1}{2}\right)^2\pi^2+\alpha^2\right]^2}{\alpha^2}$$

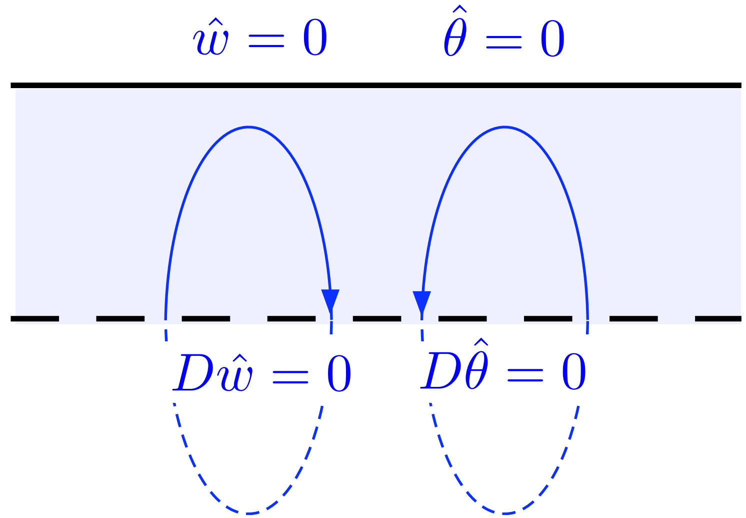
Lowest (most unstable) mode has $n=0$

$$\Omega R_m=\frac{\left[\frac{\pi^2}{4}+\alpha^2\right]^2}{\alpha^2}$$

Marginal Stability Results

No flow, constant temperature

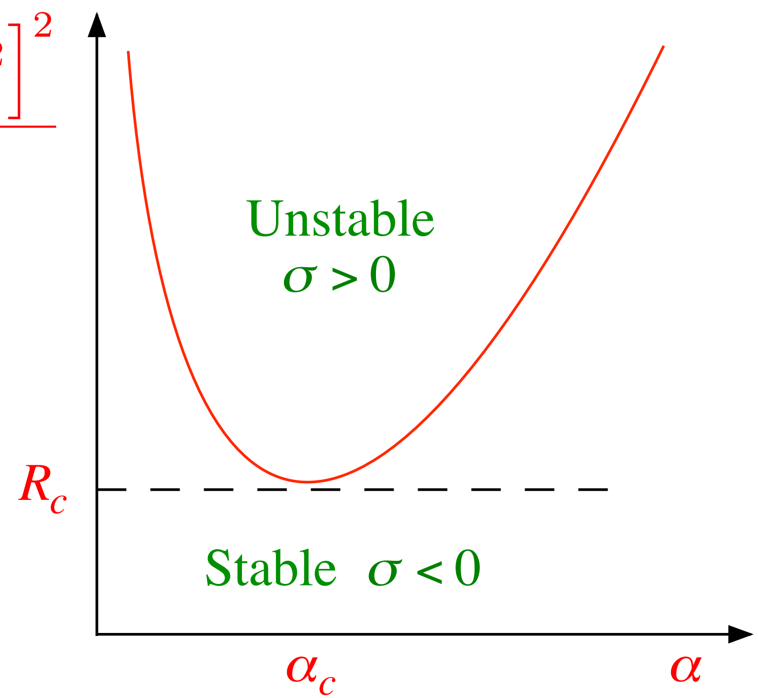
Constant pressure and heat flux



$$\theta = 1 - z + \hat{\theta}(z) e^{i\alpha x + \sigma t}$$

$$w = \hat{w}(z) e^{i\alpha x + \sigma t}$$

$$\Omega R_m = \frac{\left[\frac{\pi^2}{4} + \alpha^2\right]^2}{\alpha^2}$$

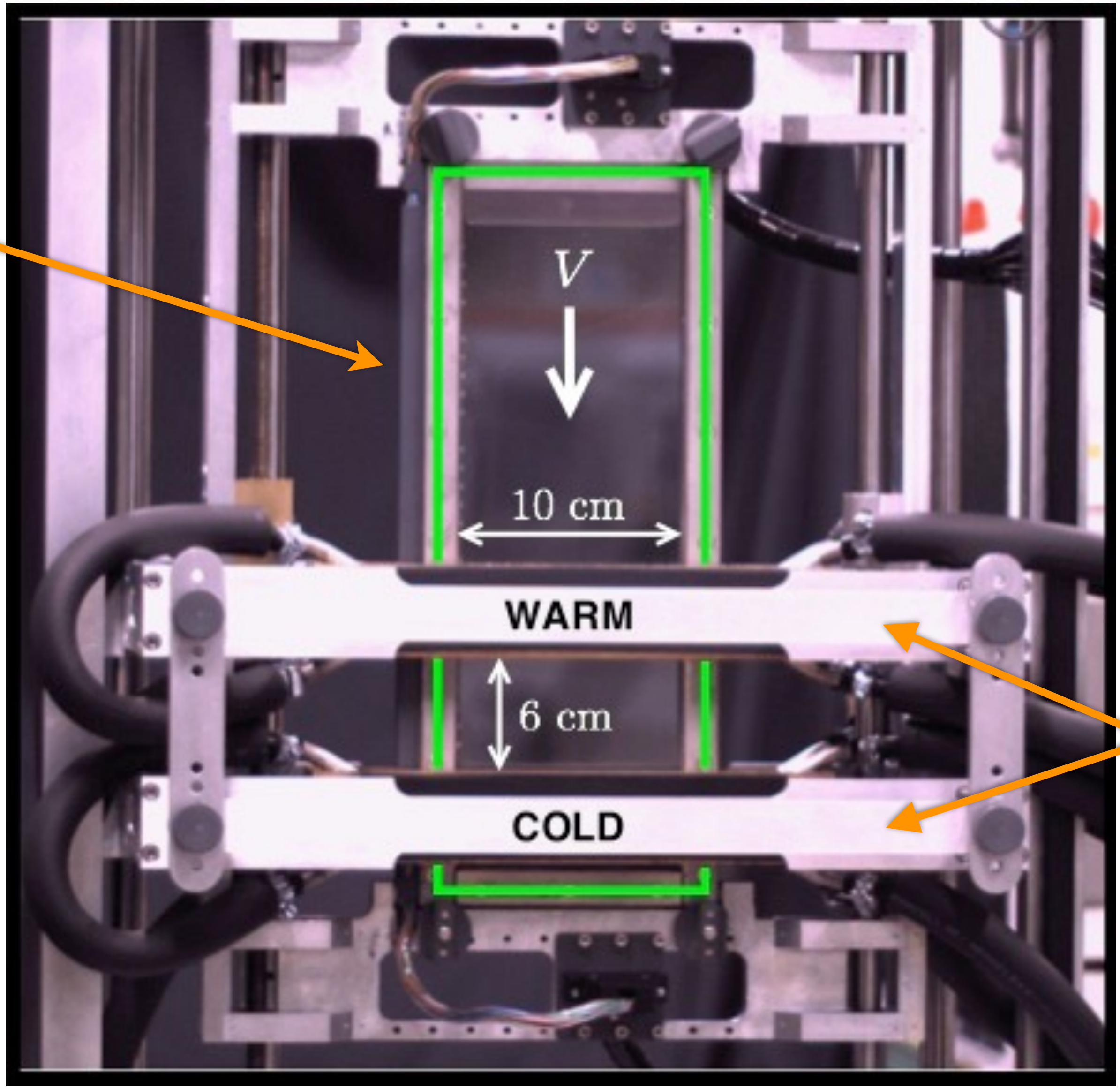


Instability if

$$\Omega R_m > R_c = \pi^2 \approx 10$$

Directional Solidification

(3 mm gap)

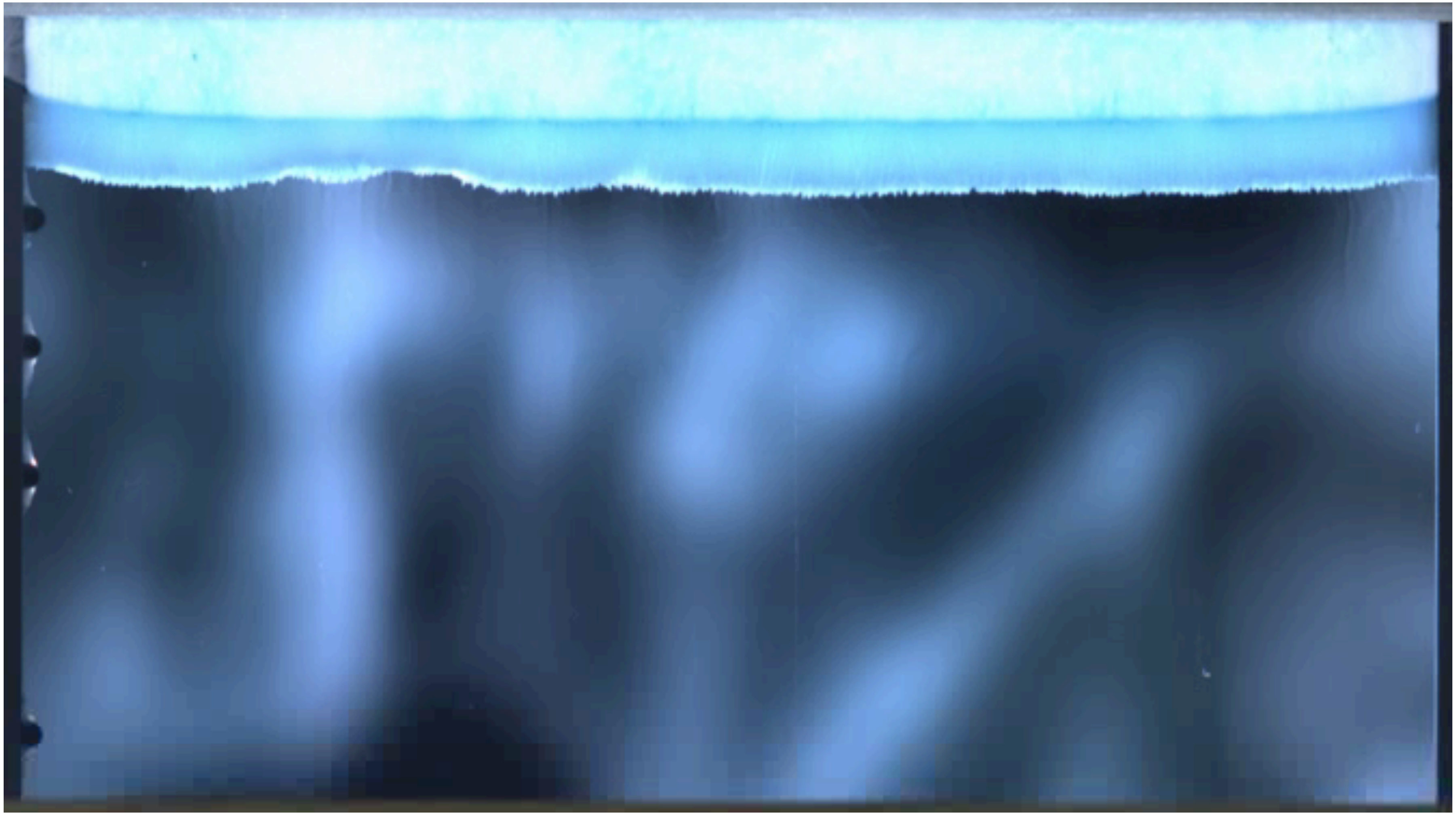


Fixed-temperature heat exchangers

Dynamic Control of Brine Rejection

$$R_m = \frac{\beta \Delta C \, g \, \Pi \, (\kappa/V)}{\kappa \nu}$$

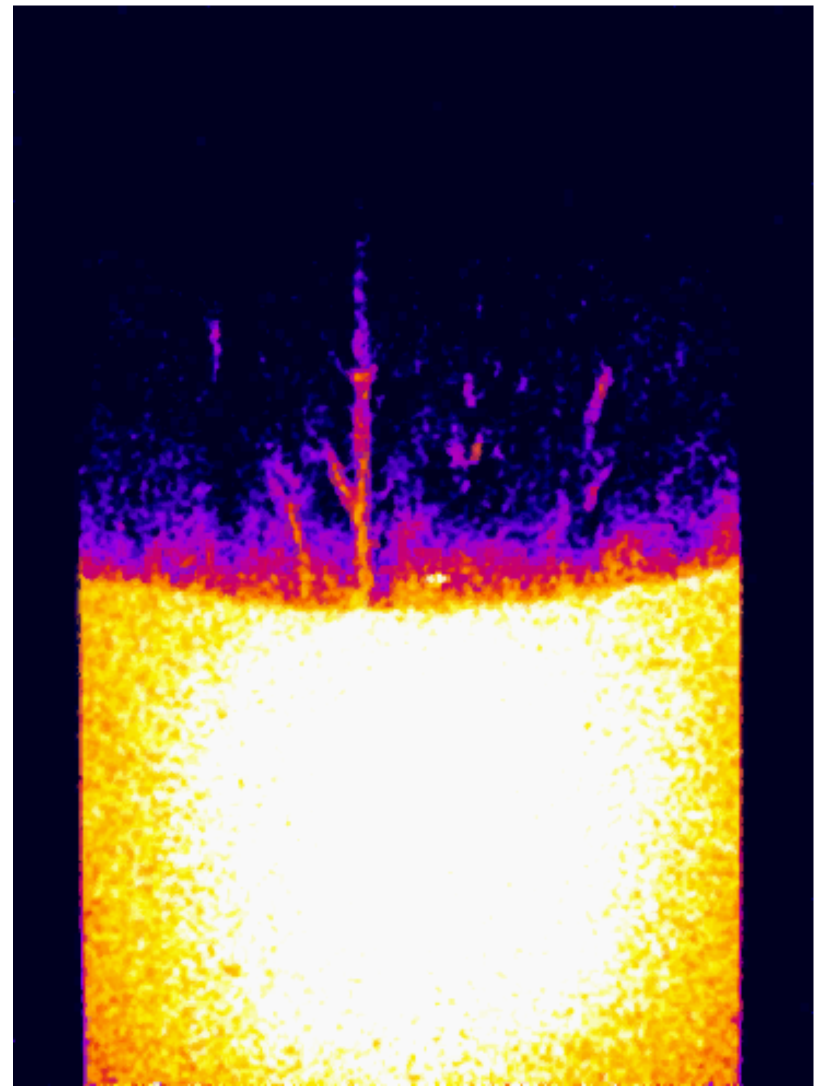
$$V = 2\mu\text{m s}^{-1} \rightarrow 0.5\mu\text{m s}^{-1} \rightarrow 2\mu\text{m s}^{-1}$$



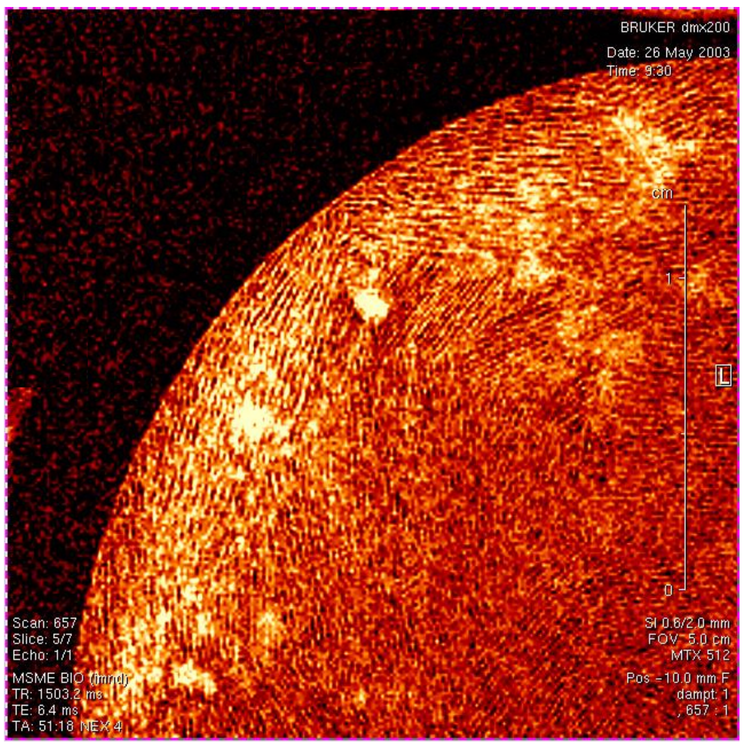
MRI study of structure and flow in convecting sea ice

Aussillous, Sederman, Gladden, Huppert & Worster

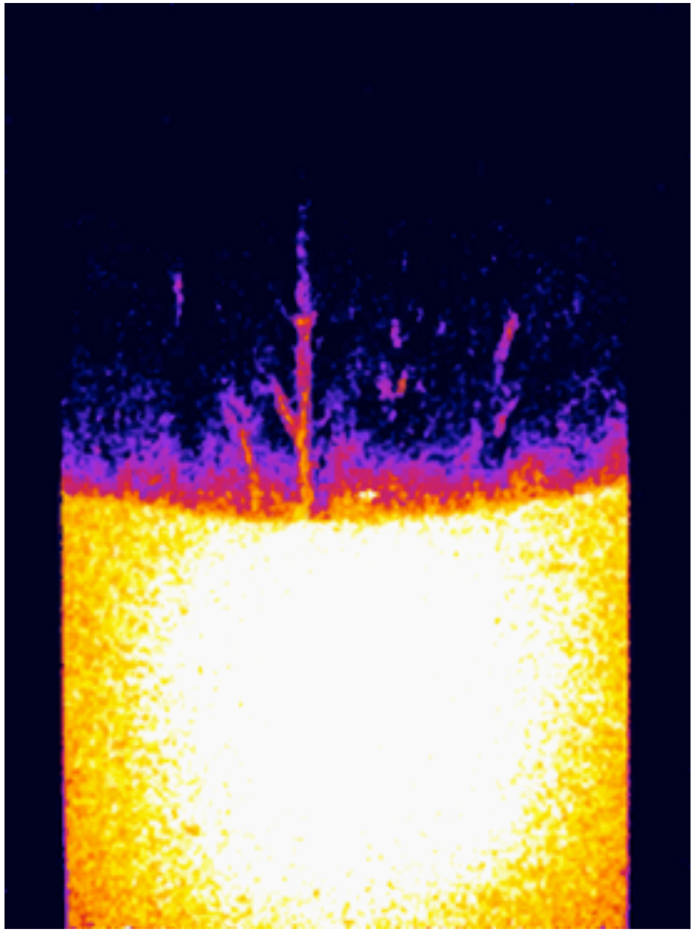
Vertical cross-section through an evolving brine channel

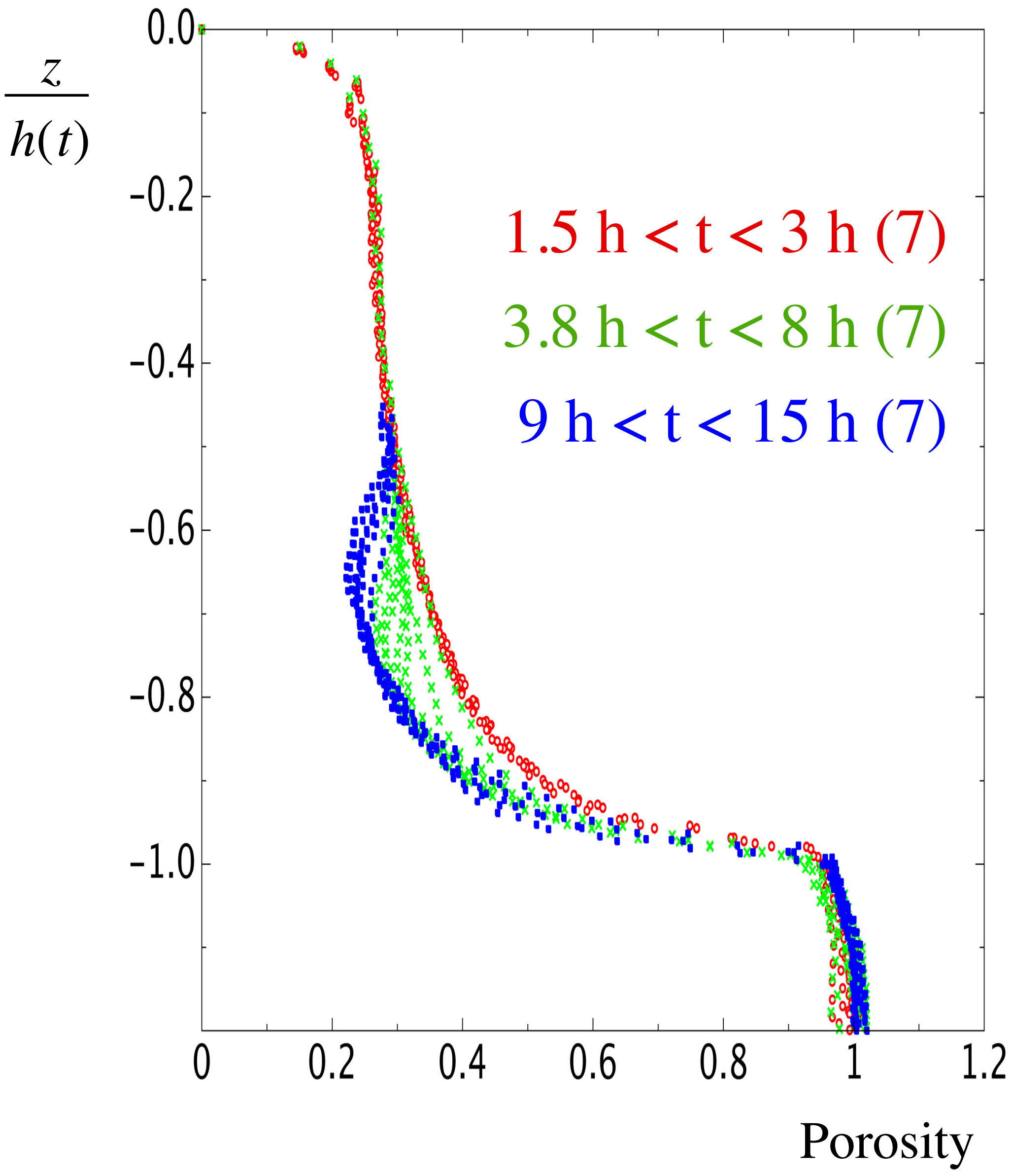


Horizontal cross-section of platelets and brine channels



Evolution of Solid Fraction

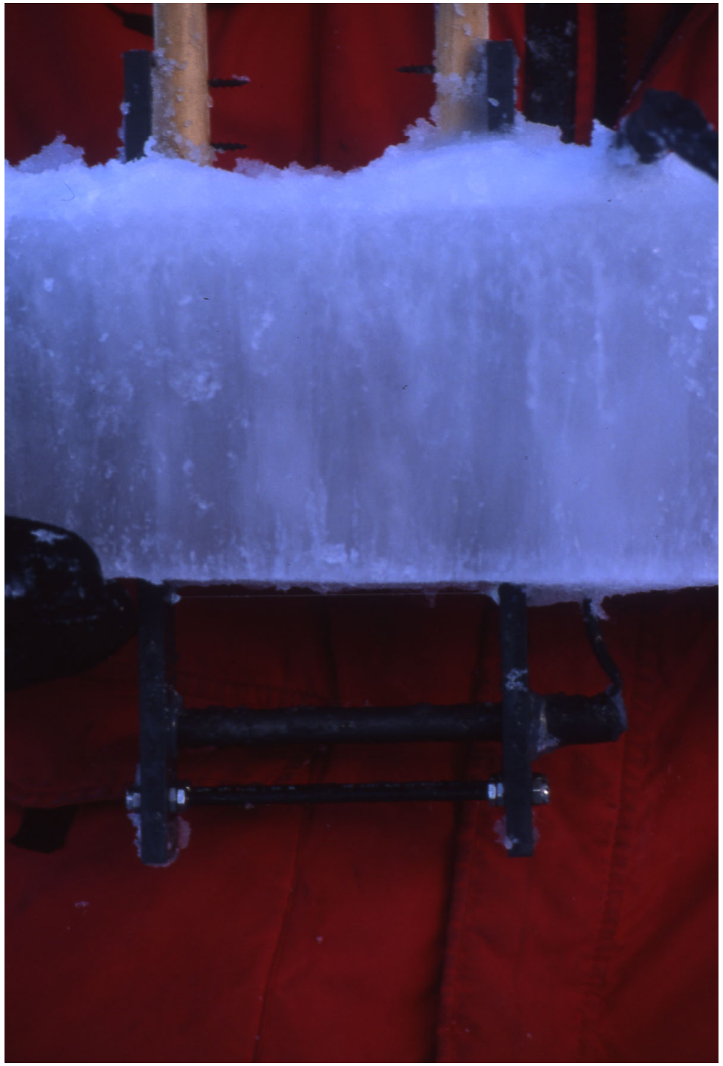




Van Mijen Fjord, Svalbard 2001

Field Experiments

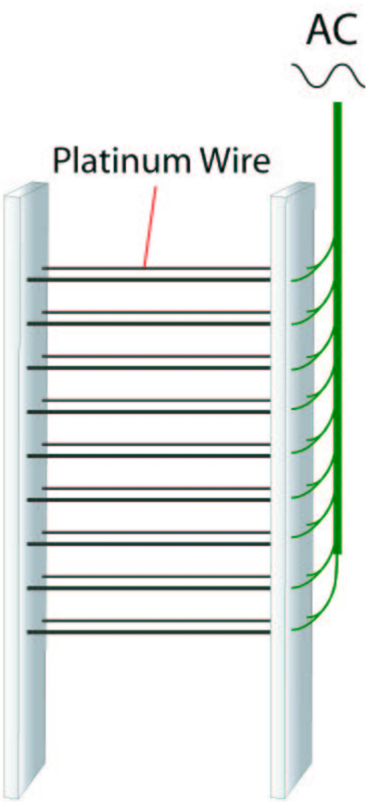
John Wettlaufer



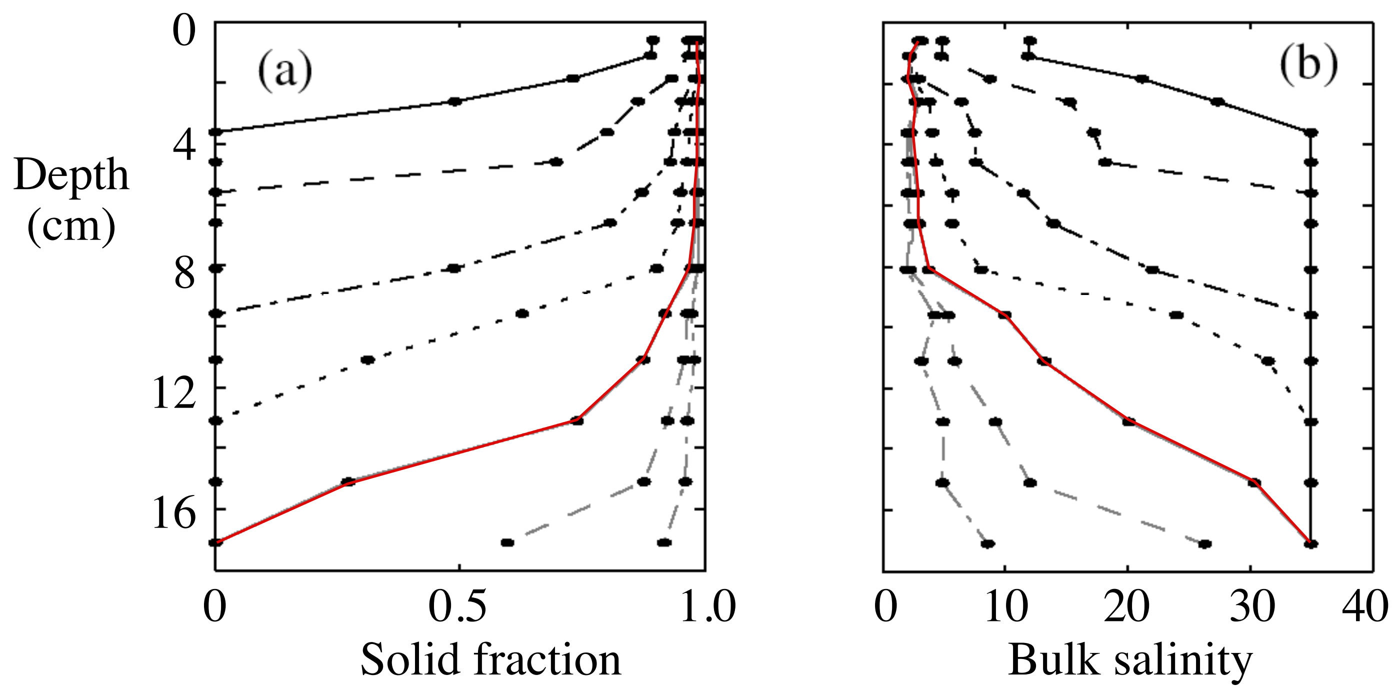
Dirk Notz

Field Measurements of Porosity and Salinity

Notz, Wettlaufer & Worster



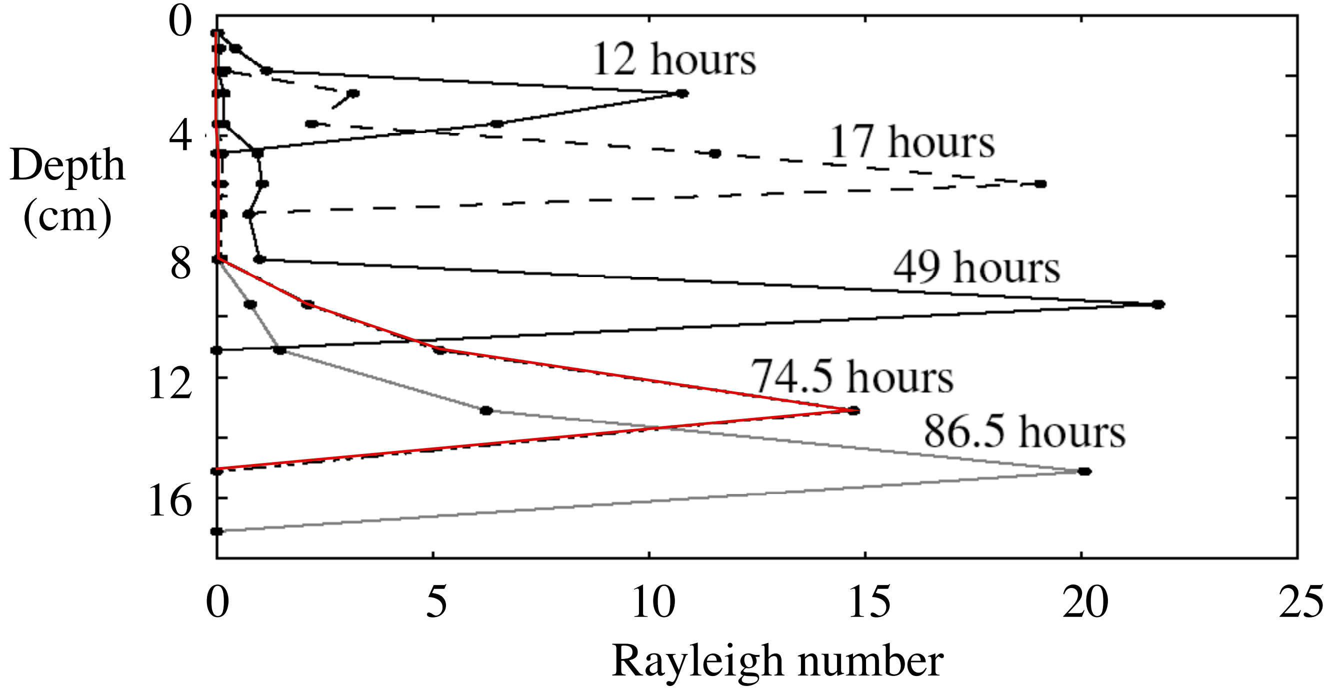
Impedance measurement of solid fraction



$$(1-\phi) \propto \text{conductance} \qquad C_B = (1-\phi)\left(\frac{T_m - T}{m}\right)$$

Derived Measurements of Local Rayleigh Number

$$R_m = \frac{\beta g[C(z) - C_0]\Pi(z)h}{\kappa\nu}$$

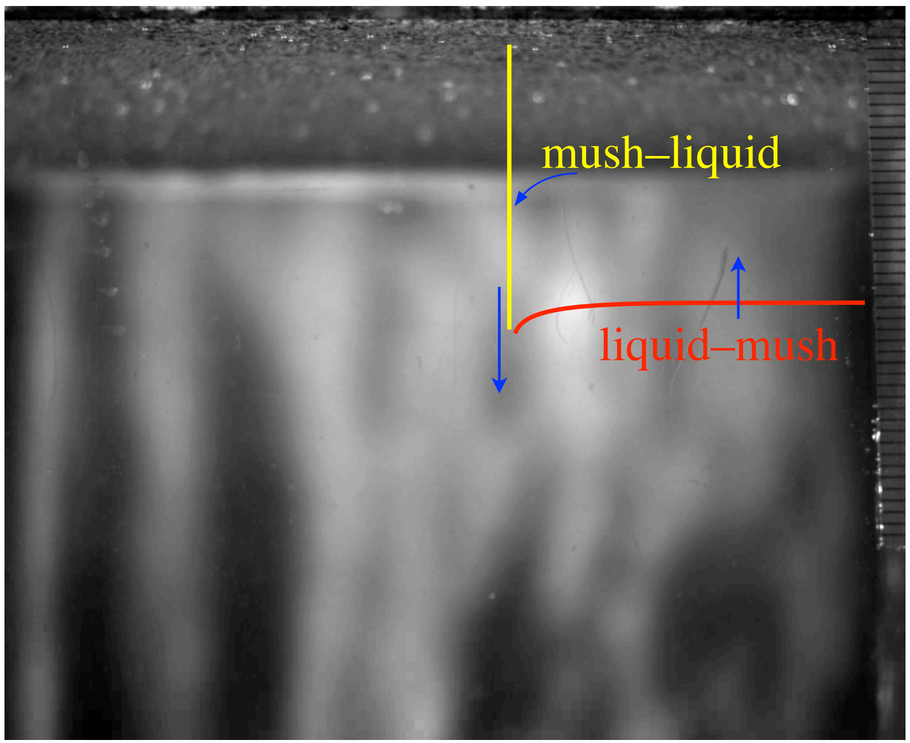


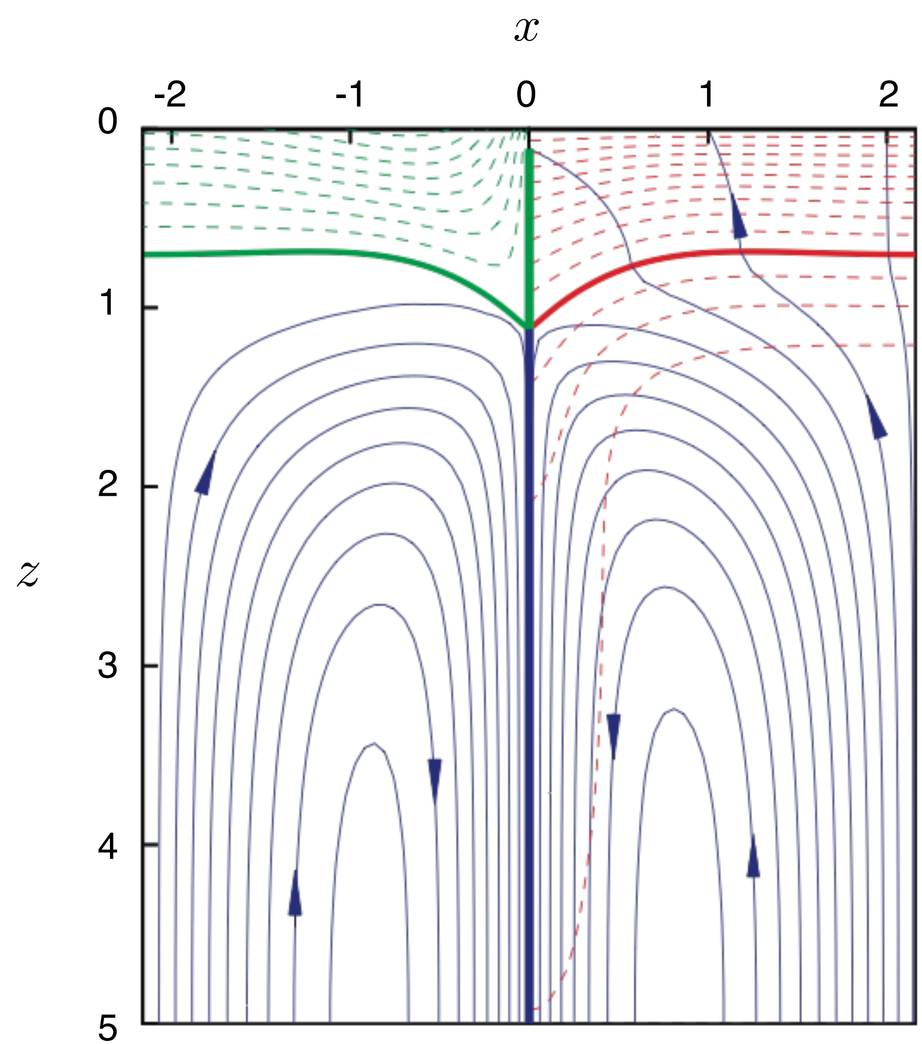
2D Numerical Modelling

$$\frac{\partial \phi}{\partial t} = -\frac{1}{\mathcal{C}} \frac{D\theta}{Dt}$$

Cold

Warm

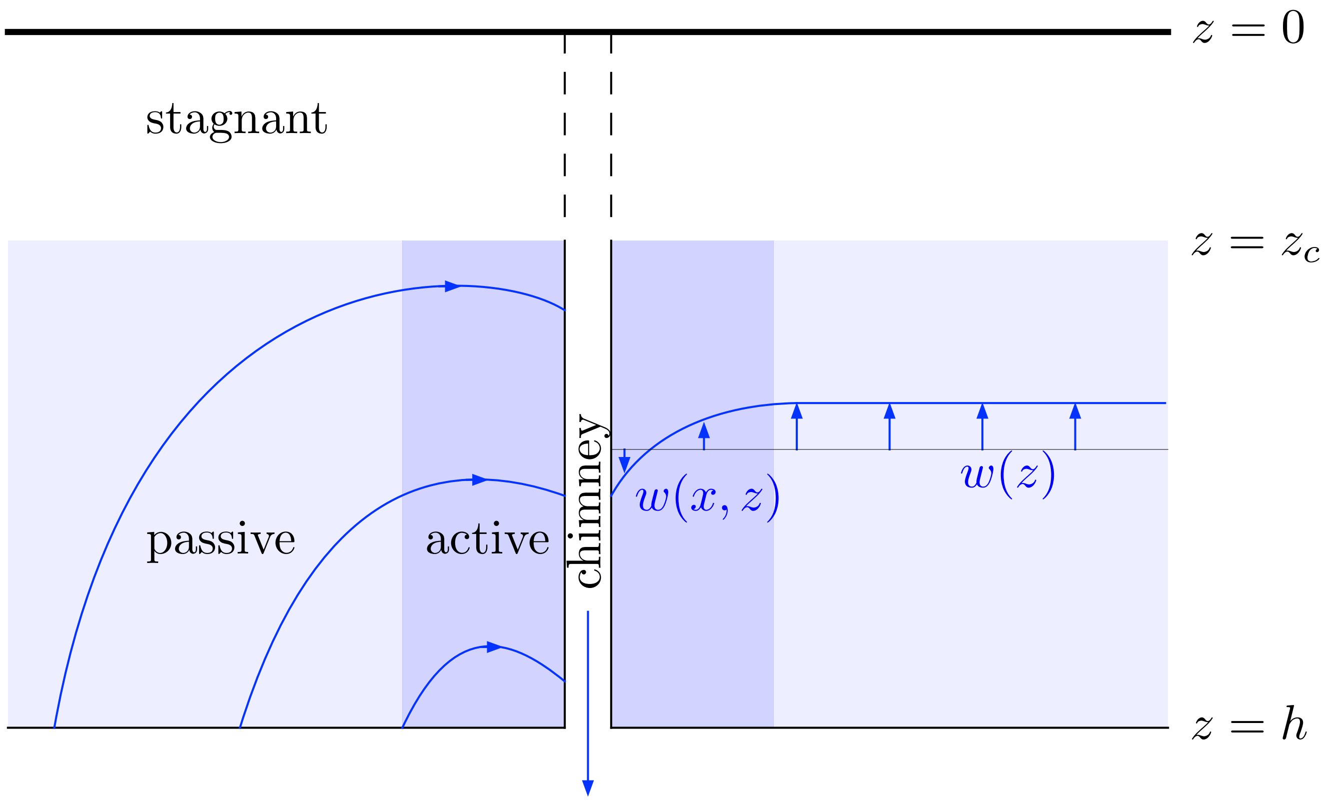




Chung & Worster 2002

CAP Model of Convection with Brine Channels

Rees Jones & Worster

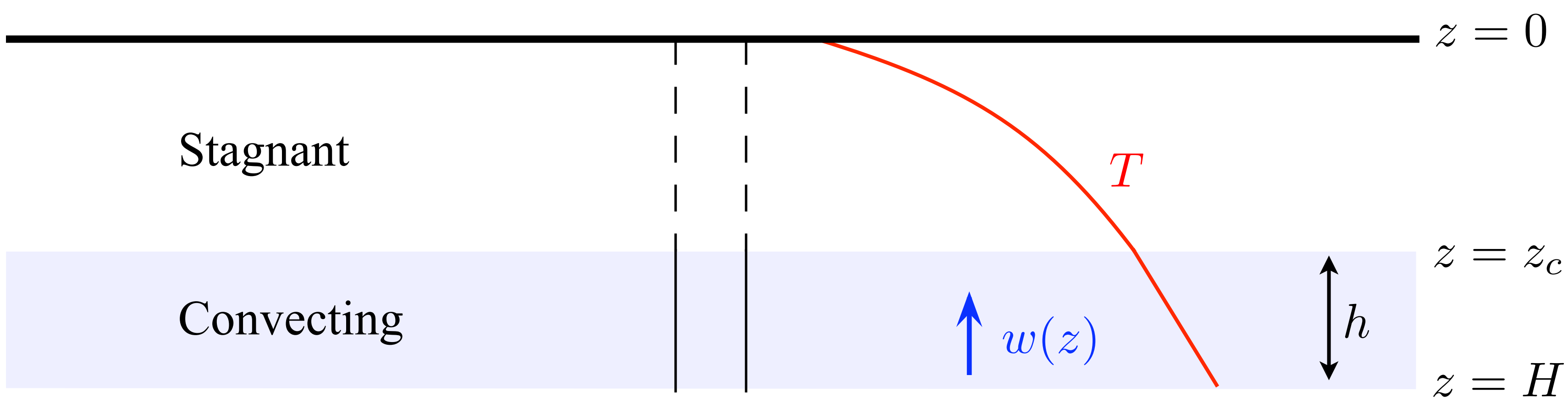


Determine channel spacing by optimising the buoyancy flux (Wells & Wettlaufer)

Which gives

$$w = \gamma \frac{\beta g \Pi}{\nu} \left[S_{br}(z_c) - S_0 \right] \frac{z - z_c}{h - z_c}$$

One-Dimensional Model of Convecting Sea Ice



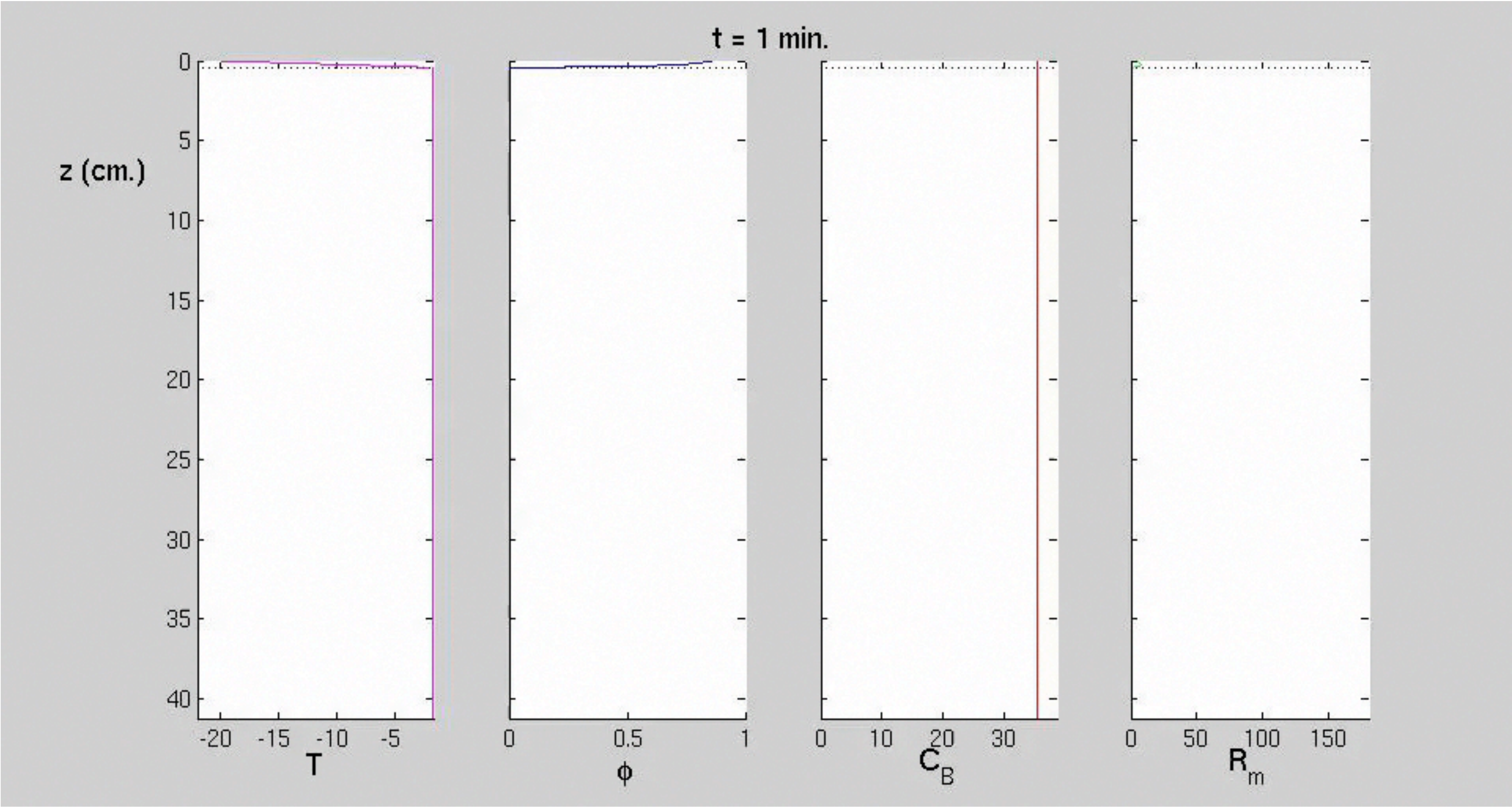
Depth of convecting region set by critical Rayleigh number

$$\frac{\beta g [C(z_c) - C_0] \langle \Pi \rangle h}{\kappa \nu} = R_c$$

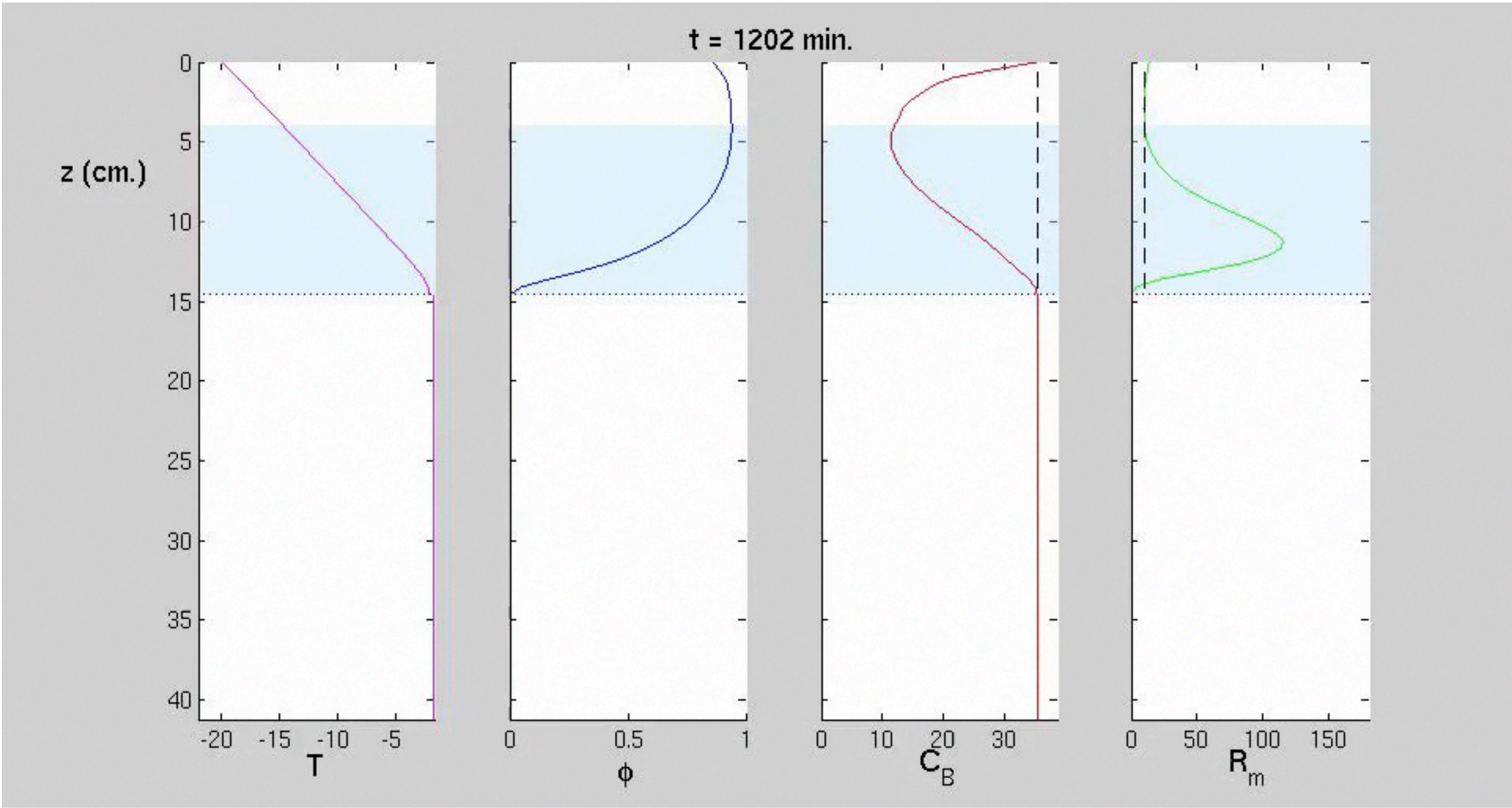
One-dimensional modelling of convecting mushy layer

$$c \frac{\partial T}{\partial t} + \rho_{br} c_{br} w \frac{\partial T}{\partial z} = \nabla \cdot (k \nabla T)$$

Evolution of Convecting Sea Ice



Evolution of Convecting Sea Ice



Capabilities and Characteristics of 1-D Convection Schemes

Buoyancy flux determined dynamically – not thermodynamically tied to growth rate.

Capture delay in onset of convection seen in some experiments.

Predict enhancement or re-initiation of brine drainage during periods of warming.

Straightforwardly adjust to different ocean salinities.

Summary

Convection begins at a critical value of the Rayleigh number

Convective fluxes (eg brine fluxes) are best parameterized in terms of a Rayleigh number

The Rayleigh number for a mushy layer (sea ice) depends on permeability and solutal buoyancy but thermal diffusivity

Convection in sea ice causes formation of brine channels by dissolution

Convection in a mushy layer is confined to a region near the ice–ocean interface.