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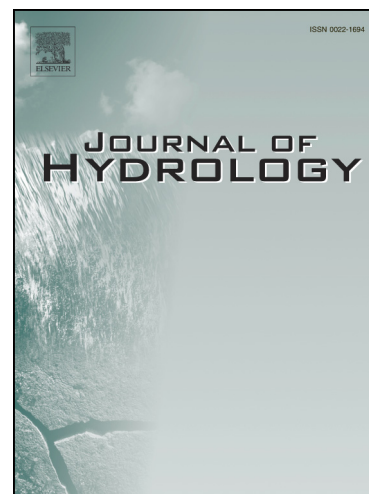
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# Modeling rain-driven overland flow: empirical versus analytical friction terms in the shallow water approximation

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## Abstract

Modeling and simulating overland flow fed by rainfall is a common issue in watershed surface hydrology. Modelers have to choose among various friction models when defining their simulation framework. The purpose of this work is to compare the simulation quality for the Manning, Darcy-Weisbach, and Poiseuille friction models on the simple case of a constant rain on a thin experimental flume. Results show that the usual friction law of Manning is not suitable for this type of flow. The Poiseuille friction model gave the best results both on the flux at the outlet and the velocity and depth profile along the flume. The Darcy-Weisbach model shows good results for laminar flow. Additional testing should be carried out for turbulent cases.

**Keywords:** shallow water, Saint Venant, overland flow, rain, friction, Poiseuille

## 1. Introduction

The rain falling on agricultural fields produces overland flows, which lead to soil erosion (Moss et al. (1979), Morgan et al. (1999)), pollutant transport (Cai et al. (2007), Benkhaldoun et al. (2007)) and flood events downstream (Cea et al. (2010), An et al. (2015)). To prevent and understand these often undesirable effects, rain-induced flows have to be modeled accurately, thanks in particular to numerical simulations. As long as the flows have a horizontal length scale larger than the vertical one, the vertical velocity profile can be integrated, leading to a 2D system of equations, called the shallow-water equations (de Saint-Venant (1871)). Such shallow-water equations are commonly used for modeling overland flow (*e.g.* Smith et al. (2007)), tsunamis (*e.g.* Popinet (2011)), dam breaks and flood

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18 events (*e.g.* An et al. (2015)) or river flooding (*e.g.* Bates et al. (2010)), which are generally flows at high Reynolds  
 19 numbers. Because numerical simulations of such systems play a significant role in government decision-making to  
 20 prevent or control inundation risks, it is crucial to properly model the underlying physical mechanisms as well as  
 21 develop accurate and validated numerical schemes.

22 One of the key points in the shallow-water framework is the effective friction term which depends on the assump-  
 23 tion made for the vertical velocity profile. This friction term depends on several parameters, but principally on the  
 24 dynamical characteristics of the flow (*i.e.* laminar or turbulent). In general, because the flows are at high Reynolds  
 25 numbers and also because of complex topography and scale effects (see for instance Smith (2014)), empirical laws are  
 26 used, in particular the Darcy-Weisbach and the Manning models (see for instance Chow, V. (1959), Smith et al. (2007),  
 27 Viollet et al. (1998), Chanson (2004) and An et al. (2015)). However, it is important to notice that for rain-induced  
 28 flow, the thin liquid films involved have small Reynolds numbers. Hence, the use of turbulent modeling is question-  
 29 able, compared to the classical laminar friction term deduced from a Poiseuille velocity profile. Moreover, quantitative  
 30 experiments are still rare (Esteves et al. (2000)), underlying the need for systematic quantitative comparisons between  
 31 numerical models and experimental measures.

32 In this paper, we focus on an “ideal rain” over a rough impermeable substrate. Experimental laboratory results are  
 33 compared with numerical results of the shallow-water equations using both empirical (Darcy-Weisbach and Manning  
 34 models) and a laminar (Poiseuille model) friction terms. We will show that in this case, the laminar version of the  
 35 shallow-water equations is the suitable model for overland flows that can be generalized using a Darcy-Weisbach  
 36 approach.. The configuration studied is presented in the next section as well as the experimental setup. The numer-  
 37 ical methods are described in section III, as well as validating cases. The numerical results are compared with the  
 38 experimental measurements in section IV, and a general discussion is then given.

## 39 2. Materials and methods

### 40 2.1. The “ideal rain” case

The numerical simulations of the shallow-water equations are compared with experimental measurements on an  
 ideal configuration of overland flow produced by rain. Real cases in nature are complicated to model for various  
 reasons: firstly the topography is often complex and not always well-known; then rainfall is usually not measured  
 everywhere; finally many different physical mechanisms are imbricated in nature (rain, erosion, infiltration, *etc.*).  
 Dedicated experiments where these different effects can be isolated then need to be designed. We focus here on an  
 ideal case of rain falling on a flat impermeable surface as shown in Fig. 1. The same experimental setup was used  
 before to evaluate the validity of numerical schemes in Delestre et al. (2009). The flat topography is tilted by an angle  
 $a$  and a constant rain intensity equal to  $I$  ( $mm.h^{-1}$ ) is imposed. The flume has a length  $L = 4.04$  m (direction  $x$ ) and  
 width  $l = 11.5$  cm (direction  $y$ ), and is initially dry. The rain leads to an overland flow which is characterized by  
 $h_{2D}(x, y, t)$  the water depth and  $u_{3D}(x, y, z, t)$  the velocity profile, and finally  $S_0 = \tan(a)$  is the absolute value of the

flume slope. We also define the transverse averaged water depth profile:

$$h(x, t) = \frac{1}{l} \int_{-l/2}^{l/2} h_{2D}(x, y, t) dy,$$

and the transverse and depth averaged velocity profile:

$$u(x, t) = \frac{1}{lh(x, t)} \int_{-l/2}^{l/2} \int_0^{h(x, t)} u_{3D}(x, y, z, t) dy dz.$$

41 The rain intensity  $R(x, t)$  is taken homogenous in space and constant during a duration  $t_{stop}$  yielding:

$$R(x, t) = \begin{cases} I & \text{if } t \in [0, t_{stop}] \\ 0 & \text{if } t > t_{stop} \end{cases} \text{ for } x \in [0, L]. \quad (1)$$

42 Three dynamical regimes can thus be identified on the measured outflow discharge:

- 43 • between  $t = 0$  s and a time  $t_s$ , the water depth in the flume is increasing as well as the outflow discharge: it is  
44 the transient, or rising stage,
- 45 • between  $t_s$  and  $t_{stop}$  the flow is in its steady stage, and
- 46 • for  $t > t_{stop}$  the rain event is finished and the outflow discharge decreases: it is the recessing stage.

47 This ideal configuration will be studied both experimentally and numerically in order to investigate and validate  
48 an effective rainflow overland model.

## 49 2.2. Experimental setup

### 50 2.2.1. Overall design

51 These experiments were carried out at the Rainfall Simulation Hall of the French Institute for Agricultural Re-  
52 search (INRA, Orléans, France). The test bench is a 4.04 m long and 11.5 cm wide flat flume having a rectangular  
53 section (Fig. 2). A sheet of glued printing paper is added on the flume for its hydrophilic property, avoiding the for-  
54 mation of threaded flow. The varying parameters of this experiment are the channel slope  $S_0$  and the rainfall intensity.  
55 The slope of the panel can be adjusted and is measured using a spirit level (accuracy:  $0.5 \text{ mm.m}^{-1}$ ) and a stainless  
56 steel rule. The rainfall is produced by a nozzle-type rainfall simulator based on the design of Foster et al. (1979) and  
57 located above the channel. Water pressure is set to 90 kPa. Five oscillating nozzles are uniformly distributed over the  
58 flume (1.1 m between them). Using a combination of nozzles with slightly varying openings (Veejet 6540, 6550 and  
59 6560; Spraying System Corp.), a coefficient of variation limited to 8.5% for the spatial variability of the rain intensity  
60 is obtained. Before each experiment, the channel is pre-wetted. A frequency of 55 sweeps per minute is used for the  
61 prescribed  $50 \text{ mm.h}^{-1}$  rainfall intensity (half for the  $25 \text{ mm.h}^{-1}$ ).

62 The experimental cases differences are based on the prescribed rainfall intensity (25 or  $50 \text{ mm.h}^{-1}$ ) and slope (2%  
63 or 5%). The three cases considered thereafter are:

- 64 • 25 mm.h<sup>-1</sup> and 2%,
- 65 • 25 mm.h<sup>-1</sup> and 5%,
- 66 • 50 mm.h<sup>-1</sup> and 2%.

### 67 2.2.2. Measurements

68 The data of these measurements can be found in the supplementary material section.

#### 69 *Outflow hydrograph.*

70 The outflow discharge is recorded during the whole run, including both the rising limb of the hydrograph (at the  
71 beginning of the rainfall) and its recessing limb (after the end of the rainfall). The outflow discharge is collected in a  
72 bucket by a funnel as schematized in Fig. 1. The outlet of the funnel is custom-made to direct the water flow laterally,  
73 avoiding flow pressure to be transmitted to the scale. The cumulative weight of the bucket was recorded using an  
74 electronic scale (30 kg range, with a 1 g resolution) at a rate of about 10 Hz. The outflow discharge measurement is  
75 replicated six times. The hydrographs (*i.e.* the derivative of the cumulative weight) are quite noisy, because of the  
76 high measurement frequency for a small weight increment (maximum flow rate of about 7 g.s<sup>-1</sup>). To make the outflow  
77 hydrograph data more readily usable, they are processed by first calculating a moving average over two seconds on  
78 each replicate. This duration is long enough to reduce the noise while still being much shorter than the durations of  
79 the rising or recessing limbs (which are of several minutes). Then, the median values over the replicates are taken and  
80 a Kalman filter (see for instance Kalman (1960)) is applied to smooth the hydrograph.

#### 81 *Rain intensity.*

82 During the experimental runs, rainfall intensity is measured by two independent methods:

- 83 • using a set of fourteen beakers positioned along the channel sides and weighted before and after the run,
- 84 • using the flow discharge at steady-state.

#### 85 *Depth and velocity.*

86 Flow depths and velocities are measured at the middle of the flume width at steady state at up to seven positions along  
87 the channel, during one of the replicates. Flow depths are measured using a dial indicator by taking the difference  
88 between the reading at the bottom and at the surface. Each flow depth measurement is replicated twice. Flow velocities  
89 are measured with the automated salt-tracing gauge described in Planchon et al. (2005) using a salt gauge with a 3  
90 cm spacing between the upstream and downstream electrodes. The measurement is carried out for a few minutes at  
91 each location, with one reading every ten seconds. At each location, for both depth and velocity, the mean value and  
92 the standard deviation of the measurements are calculated. This will allow for the comparison between measurements  
93 and simulation results.

94 2.3. Numerical method

95 2.3.1. Governing equations

96 As stated above overland flows are well-described by the Saint-Venant equations, introduced in de Saint-Venant  
97 (1871), known also as the non-linear shallow-water equations. These equations are deduced by averaging the Navier-  
98 Stokes equations over the water depth, assuming horizontal length scales much larger than the vertical one. In the  
99 “ideal rain” case considered here, the 1D system of Saint-Venant is strictly equivalent of the 2D one because:

- 100 • the topography is constant over the flume width, and
- 101 • the friction on the walls are not described by the equations.

102 Neglecting the influence of drop impacts on the momentum, the resulting 1D equations of mass and momentum  
103 conservation are:

$$\partial_t h(x, t) + \partial_x q(x, t) = R(x, t), \quad (2)$$

$$\partial_t q(x, t) + \partial_x \left( \frac{q(x, t)^2}{h(x, t)} + \frac{g}{2} h(x, t)^2 \right) = gh(x, t)(S_0 - S_f), \quad (3)$$

105 where  $h(x, t)$  and  $q(x, t)$  are respectively the local flow depth and the local depth-averaged flux,  $R(x, t)$  the rainfall  
106 intensity,  $g$  the acceleration of gravity,  $S_0 = -\partial_x Z_b$  the opposite of the slope (with  $Z_b$  the topography) and  $S_f$  the  
107 friction coefficient in its kinematic form. The derivation of the Saint-Venant equations with rain as the first numerical  
108 simulations using this system can be found in Zhang and Cundy (1989). We define the maximal Reynolds number  $Re$   
109 with respect to the experimental conditions:

$$Re = \frac{\cos(a)IL}{\nu}, \quad (4)$$

110 which characterizes the behavior of the fluid : laminar (resp. turbulent) for  $Re < 500$  (resp.  $Re > 2000$ ), where  $\nu$  is the  
111 kinematic viscosity of the fluid (typically  $10^{-6} m^2 \cdot s^{-1}$  for water) and  $a$  is the angle of the flume with the horizontal.  
112 We define the local Reynolds number with respect to the local value of the numerical 1D fields :

$$Re_l(x, t) = \frac{q(x, t)}{\nu} \quad (5)$$

113 We also introduce the Froude number  $Fr$  which characterizes the relative speed of the waves in the flow. The flow  
114 is sub-critical (resp. supercritical) when the liquid velocity is slower (resp. faster) than the surface waves, for  $Fr < 1$   
115 (resp.  $Fr > 1$ ). The local Froude number is:

$$Fr = \frac{u(x, t)}{\sqrt{gh(x, t)}}. \quad (6)$$

116 Different friction terms have been proposed in the literature depending on the flow properties. We will consider here  
117 the three main friction models: the Darcy-Weisbach model (e.g. Darcy (1857)), the Manning model (see for instances  
118 Gauckler (1867) and Manning et al. (1890)), and the Poiseuille model (e.g. Igawaki (1955)). The Darcy-Weisbach  
119 and Manning models were empirically deduced while the Poiseuille model was obtained analytically.

120 The Manning model was designed for open channel flows driven by gravity. The friction coefficient follows:

$$S_f^M = n^2 \frac{q(x, t) |q(x, t)|}{h(x, t)^{10/3}}, \quad (7)$$

where  $n$  is the Manning coefficient. This coefficient is usually found by a trial and error calibration run.

For a laminar flow, the vertical velocity profile is given by a Poiseuille flow. Denoting  $u_{2D}(x, z, t)$  the 2D local velocity for a 2D Poiseuille flow and

$$u(x, t) = \frac{1}{h(x, t)} \int_{z_b}^{h(x, t)} u_{2D}(x, z, t) dz$$

121 the local depth-averaged horizontal velocity, we can express the 2D local velocity as:

$$u_{2D}(x, z, t) = \frac{3}{2} \frac{u(x, t)}{h^2(x, t)} z(2h(x, t) - z). \quad (8)$$

122 A well-known analytical solution of the Poiseuille coefficient  $S_f^P$ , without any free parameter, can be then deduced  
123 from the Navier-Stokes equations:

$$S_f^P = \frac{\nu}{gh(x, t)} \partial_z u_{2D}(x, z = 0, t) = \frac{3\nu}{g} \frac{q(x, t)}{h^3(x, t)}. \quad (9)$$

124 Note that in contrast with the Manning models, the Poiseuille friction model does not contain any empirical/adjustable  
125 parameter (other than the fluid viscosity which is set to that of water for the case of an ideal rain).

126 The Darcy-Weisbach model was initially designed for turbulent flows inside pipes, but it is generally used because  
127 the coefficient  $f$  can be deduced from the Moody diagram (*e.g.* Bell et al. (1989)). The friction coefficient for this law  
128 can be written in kinematic form as:

$$S_f^{DW} = \frac{f}{8g} \frac{q(x, t) |q(x, t)|}{h(x, t)^3}, \quad (10)$$

129 where  $f$  is the Darcy-Weisbach coefficient. We can find in the literature different laws giving the coefficient  $f$  with  
130 respect to the local Reynolds number, see for instance the Henderson version (Henderson (1996)) of the Colebrook-  
131 White formulae (Colebrook and White (1937)), but such laws are not designed to be used for such low Reynolds  
132 flows. Here, we propose a simple law for the coefficient  $f$  :

$$f = \begin{cases} \frac{24}{Re_l} & \text{if } Re_l < 48, \\ 0.5 & \text{if } Re_l \geq 48. \end{cases} \quad (11)$$

133 In the low Reynolds region (*i. e.*  $Re_l \leq 48$ ), this law mimics the Poiseuille Model (Equ. (9)). In the "high" Reynolds  
134 region, the value of  $f = 0.5$  is chosen to be the highest possible for a smooth surface (see Paraschivoiu et al. (2003)  
135 p.317 for details), in order to have an influence in this setup.

### 136 2.3.2. Numerical scheme

137 Numerical simulations are performed using well-known tested codes that implement the following numerical  
138 scheme (*i.e.* Popinet (2013) and Delestre et al. (2014)). The shallow-water system of partial derivative equations  
139 (PDE) writes under the vectorial form

$$\partial_t U + \partial_x F(U) = S(U), \quad (12)$$

140 with

$$U = \begin{pmatrix} h(x, t) \\ q(x, t) \end{pmatrix}, F(U) = \begin{pmatrix} q(x, t) \\ \frac{gh(x, t)^2}{2} + \frac{q(x, t)^2}{h(x, t)} \end{pmatrix}, S(U) = \begin{pmatrix} R \\ gh(x, t)(S_0 - S_f) \end{pmatrix}. \quad (13)$$

141 This is a set of conservation laws, where the first equation represents the mass conservation and the second one  
 142 represents the momentum balance. Thus a finite volume method is used which is by construction a conservative  
 143 method. It consists in integrating the equations on cells  $[x_{i-1/2}, x_{i+1/2}] \times [t^n, t^{n+1}]$ , where  $[x_{i-1/2}, x_{i+1/2}]$  is centered on  
 144 point  $x_i$ . We have  $x_{i+1/2} - x_{i-1/2} = \Delta x$  and  $t^{n+1} - t^n = \Delta t$ . After calculations on these cells, with the homogeneous  
 145 system (*i.e.* with no rain, no friction and no topography), we get the following explicit in time finite volume scheme

$$\begin{cases} \frac{h_i^{n+1} - h_i^n}{\Delta t} + \frac{F_{1i+1/2}^n - F_{1i-1/2}^n}{\Delta x} = 0 \\ \frac{q_i^{n+1} - q_i^n}{\Delta t} + \frac{F_{2i+1/2}^n - F_{2i-1/2}^n}{\Delta x} = 0 \end{cases} \quad (14)$$

146 where  $F_{1i+1/2}^n$  (resp.  $F_{2i+1/2}^n$ ) is the approximation of the first component (resp. the second component) of the flux  
 147 function  $F(U)$  at the cells interface located at point  $x_{i+1/2}$ . The CFL stability criteria ensure that the scheme is stable  
 148 for :

$$\Delta t \leq 0.5 \frac{\Delta x}{a} \quad \text{with } a = \max(ap, -am) \quad (15)$$

149 where  $a$  is the magnitude of the velocity of waves,  $ap$  the maximum value of  $u_i + \sqrt{(G * h_j)}$  and  $am$  the minimum value  
 150 of  $u_i - \sqrt{(G * h_j)}$  for  $j \in \{i-1; i; i+1\}$  and  $\forall i$  (see Courant et al. (1928) for details). The topographic term is treated  
 151 inside the flux term thanks to a well-balanced scheme (*i.e.* it captures lake at rest solutions), which is preserving the  
 152 non-negativity of the water depth (Audusse and Bristeau (2005), Kurganov and Petrova (2007)). The friction source  
 153 term is treated semi-implicitly (Bristeau and Coussin (2001)), the accuracy of the scheme is improved in space with a  
 154 MUSCL reconstruction (Van Leer (1979)) and in time with a generic second order method (Williamson (1980)).

### 155 2.3.3. Numerical cases

156 We simulate a one dimension channel with a fixed slope  $S_0$ , as presented in Fig 1. Its horizontal length is  $L_x =$   
 157  $\frac{L+2}{\sqrt{1+S_0^2}}$  with  $L = 4.04 \text{ m}$  and we shift the origin at  $X = -1 \text{ m}$  to avoid effects of the rain source term at the left boundary.  
 158 At the right boundary, we put a water tank of 1 meter width and 1 meter depth to reproduce the experimental setup.  
 159 We set closed boundary condition at the left of the slope ( $X = -1 \text{ m}$ ) and at the right ( $X = 5.04 \text{ m}$ ). The rain source is  
 160 equal to zero for  $X < 0$  and equal to (11) for  $X > 0$ . We chose a reasonably small cell size:  $\Delta x = \frac{L_x}{2096} = 0.00288 \text{ m}$ .  
 161 The largest time step  $\Delta t_{max}$  verifying the CFL condition is automatically chosen by the solver, following the equation  
 162 (15). We start the simulation at  $t_{start} = 0$  and we stop it at  $t_{end} = 1000 \text{ s}$ . The rain is stopped at  $t_{stop} = 600 \text{ s}$ .

163 The first stage was to ensure the convergence of simulations. Simulations using the case “ $I = 25\text{mm.h}^{-1}$  and  
 164  $S_0 = 5\%$ ” with different numbers of cells were performed to compute the following error norms at the steady stage  
 165 (taken at  $t = 599 \text{ s}$ ):

$$\|e_1(N)\| = \frac{\int_0^L |h_N(x) - h_{max}(x)| dx}{L}, \quad (16)$$



$$\|e_2(N)\| = \frac{\sqrt{\int_0^L (h_N(x) - h_{max}(x))^2 dx}}{L}, \quad (17)$$

$$\|e_{max}(N)\| = \max_x (h_N(x) - h_{max}(x)), \quad (18)$$

with  $h_N(x)$  the water depth profile with  $N$  cells and  $h_{max}(x)$  the water depth profile with the maximum number of cells 2096. We can see in Fig. 3 that our simulations converge. The rate of convergence of  $e_{max}$ , *i.e.* the maximum error, is of order one. It is the best convergence rate we can have due to the presence of the shock at the wet-dry transition upstream (Godunov's theorem).

The second stage prescribes the parameters of the three friction terms. For the Poiseuille friction term, the typical kinematic viscosity  $\nu = 10^{-6} \text{ m}^2 \cdot \text{s}^{-1}$  (water) was considered. As described above, the Poiseuille friction coefficient does not include any calibrated value and the Darcy-Weisbach coefficient depends mainly on the Reynolds number. For the Manning coefficient, a calibration was performed on the experimental case " $I = 50 \text{ mm} \cdot \text{h}^{-1}$  and  $S_0 = 2\%$ ". The best possible fit was assessed by trial-and-error. This led to a Manning coefficient of  $n = 0.025 \text{ s} \cdot \text{m}^{-1/3}$ . Thereafter, this value is used for the two other experimental cases.

### 3. Results and discussion

The parameters relevant to each case are summarized in the Table 1. For the numerical cases, the rain intensity (*Num. rain*) was chosen to fit the experimental outflow during the steady stage. We also list the values of the Reynolds number and the Froude number computed numerically with the Poiseuille friction term during the steady stage ( $t = 599 \text{ s}$ ) at the bottom of the slope ( $X = 4.04 \text{ m}$ ). Note that the Reynolds number depends only on the experimental conditions. We can see that the flows are always laminar and subcritical. The "*Exp. Outflow*" entry in the table is the mean of the discharge measured at the end of the slope during the steady stage for the experimental cases.

#### 3.1. Hydrographs

We compute numerically the flow rates at the bottom of the slope for the three different friction terms for a channel width of 0.115 meter filled with water and we compare them to the experimental measurements. The resulting hydrographs for each case are shown on Fig. 4.

To illustrate the dynamics of the rising limb, we define two times

- $t_b$  as the time when the hydrograph reaches 1/10 of the steady value  $q_s$ , and
- $t_s$  as the time when hydrograph reaches its first local maximum, corresponding to the steady state equilibrium.

We note on Fig. 4b the times  $t_b$  and  $t_s$  for the experimental case. It is clear that  $t_b$  can be considered as the starting time of the rising limb of the hydrograph, and  $t_s$  as the beginning of the steady stage. We report on Table 2 the values of  $t_b$  and  $t_s$  for each friction term in numerical simulations and for the experimental hydrographs. For the

195 starting time  $t_b$ , the simulations using the Manning term leads to values much smaller than the experimental value in  
 196 all cases, while the simulations using the Poiseuille coefficient are much closer. We can see that the simulations using  
 197 the Darcy-Weisbach model gives similar results than the Poiseuille term, since the local Reynolds number almost  
 198 never exceeds the critical value (48) of the model. Only for the case  $S_0 = 2\%$  and  $I = 50\text{mm}$  this critical value is  
 199 reached leading to a small variation only in the results. For the time  $t_s$  it is for instance slightly larger than for the  
 200 Poiseuille model, and no general conclusion can be drawn given such a small effect. For the beginning of the steady  
 201 stage  $t_s$ , the simulations using the Manning term lead to values smaller than expected for the cases “ $I = 25\text{mm.h}^{-1}$   
 202 and  $S_0 = 2\%$ ” and “ $I = 25\text{mm.h}^{-1}$  and  $S_0 = 5\%$ ”, and to values slightly too high for the case “ $I = 50\text{mm.h}^{-1}$   
 203 and  $S_0 = 2\%$ ”. Simulations using the Poiseuille and Darcy-Weisbach friction terms give the closest estimate of  $t_s$   
 204 for the three experimental cases. Hence, it is clear that the Poiseuille friction term is the best to model the dynamic  
 205 of the rising stage. Basically, the Manning terms leads to a too early initiation of the rising limb (Fig. 4) while the  
 206 Darcy-Weisbach term is mimicking the Poiseuille term in such experiments, except again for the case  $S_0 = 2\%$  and  
 207  $I = 50\text{mm}$  where only a small difference is observed at the end of the rise. For the steady stage ( $t_s < t < t_{stop}$ ), the  
 208 experimental data shows small oscillations around a mean value because of the water movement in the tank collecting  
 209 the water flux at the bottom of the slope. The simulated discharges for the three friction terms are strictly equals,  
 210 because at the steady stage the friction terms do not affect the water flux at the outlet.

211 Focusing on the decreasing limb ( $t > 600\text{s}$ ), we observe that, at first, the outflow for Poiseuille decreases faster  
 212 than for Manning. Then the outflow for Poiseuille becomes higher than for Manning. The Darcy-Weisbach term gives  
 213 same results as Poiseuille term. However, due to the noise in the experimental hydrographs, it is not really clear which  
 214 friction term is the best at modeling this stage.

### 215 3.2. Velocity and water depth

We now look at the velocity profiles for each case during the steady stage ( $t = 599\text{s}$ ). An important methodological  
 difference is that experimental velocities are measured at the free surface in the middle of the flume, while the 1D  
 numerical profiles can be seen as the transverse averaged values of the 3D field. We therefore need to perform some  
 transformations on the velocity field before comparison. Denoting the full 3D local velocity field  $u_{3D}(x, y, z, t)$ , the 1D  
 velocity profile computed numerically can be expressed

$$u(x, t) = \frac{1}{h(x, t)l} \int_{-l/2}^{+l/2} \int_0^{h(x, t)} u_{3D}(x, y, z, t) dy dz.$$

216 For the 3D velocity profile, we chose as hypothesis a bi-parabolic profile to take into account the influence of walls:

$$u_{3D}(x, y, z, t) = 9 \frac{u(x, t)}{h^2(x, t)l^2} \left( \frac{l^2}{4} - y^2 \right) z (2h(x, t) - z). \quad (19)$$

217 We can finally express the experimental measurement of the velocity with respect to the 1D transverse averaged one  
 218 as:

$$u_{3D}(x, y = 0, z = h(x, t), t) = \frac{9}{4} u(x, t). \quad (20)$$

219 We present on Fig. 5 the velocity profiles computed numerically and the mean and standard deviation of experimental  
 220 measurements normalized by  $\frac{9}{4}$ . Firstly, we can see that the normalized velocity profile is in good agreement with our  
 221 numerical results independently from the friction law, validating the hypothesis made on the 3D velocity profiles in  
 222 (19). However, the Manning velocities are always too large compared to the experimental values. In all three cases,  
 223 the velocities computed using the Poiseuille term are the closest to the experimental values. To compare the water  
 224 depth of the numerical simulations against the experimental results, we compute the averaged value of the water depth  
 225 as: denoting  $U_{exp}(X_{bot})$  the closest velocity measurement at the bottom of the slope ( $X_{bot} = 3.72 \text{ m}$ ),  $\bar{U}_{exp}(X_{bot})$  its  
 226 transverse averaged value following (20) and  $h_{exp}(X_{bot})$  the measurement of the water depth at the same coordinates.  
 227 We compute the flow rates at  $X_{bot}$  as:  $q_c(X_{bot}) = \bar{U}_{exp}(X_{bot}) \times h_{exp}(X_{bot})$ . We can extrapolate the values at the end  
 228 of the slope  $q_c(L)$ . During the steady stage,  $\partial_t h(x, t) = 0$ , then solving Equ. (2) leads to  $q(x) = R \times x$ , so that  $q_c(L)$   
 229 is found using:  $q_c(L) = q_c(X_{bot}) \times \frac{L}{X_{bot}}$ . As already said, we measure the discharge at the end of the slope with the  
 230 balance and we denote  $q_{exp}$  its value during the steady stage. Finally, we normalize the field  $h_{exp}$  by a factor:  $\frac{q_{exp}}{q_c(L)}$  to  
 231 find the transverse averaged water depth. With this method, we can extrapolate directly the water depth profile as long  
 232 as the averaged velocity profile is correct. For the water depth profiles (Fig. 5), the Manning term leads to values too  
 233 low. As for the velocities, the graphics comparison shows that the Poiseuille term gives the best match for all three  
 234 cases, still with a D-W correction for the case  $S_0 = 2\%$  and  $I = 50\text{mm}$ . In this case, we can see at  $X = 3.75 \text{ m}$  that  
 235 both water depth and velocity profiles stop to follow the Poiseuille model and start following the Manning model, a  
 236 trend that is consistent given the experimental results available.

237 To make a quantitative assessment of the numerical results, we define for each friction model a water depth index  
 238  $Ind_h$  and a velocity index  $Ind_u$  as follows:

$$Ind_h = \frac{1}{N} \sum_{i=1}^N \frac{\sqrt{(h_{num}(X_i) - h_{exp}(X_i))^2}}{h_{exp}(X_i)}, \quad (21)$$

$$Ind_u = \frac{1}{N} \sum_{i=1}^N \frac{\sqrt{(u_{num}(X_i) - u_{exp}(X_i))^2}}{u_{exp}(X_i)}, \quad (22)$$

240 with  $N = 6$  the number of experimental measurements,  $X_i$  the position on the flume of the experimental measure-  
 241 ments,  $h_{num}$  and  $u_{num}$  the numerical results for the water depth and the velocity, respectively, at the position  $X_i$  for the  
 242 corresponding friction model (Darcy-Weisbach, Manning or Poiseuille) and  $h_{exp}$  and  $u_{exp}$  the mean of the water depth  
 243 and velocity, respectively, measured experimentally at the position  $X_i$ . A zero value for these indexes means that the  
 244 numerical result fits perfectly the experimental measurements.

245 Because the experimental measurements are done at left of  $X = 3.75 \text{ m}$ , the Poiseuille and Darcy-Weisbach indices  
 246 are equals. For the water height, the index is the smallest when the Poiseuille term is used (Table 3). Only in the case  
 247 “ $I = 50\text{mm.h}^{-1}$  and  $S_0 = 2\%$ ” the Manning term gives a result as good as the Poiseuille term. For the velocity, the  
 248 index is always the lowest with the Poiseuille term. Hence, it is clear that the Poiseuille friction term is the best to  
 249 model both the water depth and the velocity profiles at steady state.

250 Overall, for a smooth surface with a rain-fed, laminar and subcritical flow, the Poiseuille term leads consistently  
251 to the best match for the water flux at the outlet during the initiation of the hydrograph, for the water depth profile at  
252 steady state and for the velocity profile at steady state. Hence, the Poiseuille term could be used for inter-rill overland  
253 flow, a condition commonly encountered in watershed surface hydrology. The adequacy of this term needs however  
254 to be evaluated on field data in the future.

255 Compared to the empirical Manning term, the Poiseuille term has the advantage to be defined analytically and to  
256 have no parameter to be calibrated. In watershed surface hydrology, issues of over-calibration, *i.e.* the use of codes  
257 requiring the calibration of numerous parameters based on limited data set, have been leading to equifinality cases and  
258 to a limited confidence in the simulation quality, as mentioned in Beven (2008). The use of the Poiseuille term could  
259 help in achieving a parsimonious parametrization, improving the overall quality of hydrologic simulations.

#### 260 4. Conclusion

261 Three different friction terms in the Saint-Venant equations have been examined: the commonly used Manning  
262 and Darcy-Weisbach models which are empirical and the Poiseuille term, which is deduced directly from the laminar  
263 Navier-Stokes equations. The Manning model investigated in this study is using a constant Manning coefficient chosen  
264 thanks to a previous trial-and-error run. The Darcy-Weisbach coefficient is following a well-known laminar law at low  
265 Reynolds number and a constant value at high Reynolds number, which is set thanks to literature. The Poiseuille term  
266 does not depend on any free parameter (aside from the fluid viscosity). The “ideal rain” case has been reproduced in  
267 laboratory and numerical simulations of these events have been performed for these friction terms. The simulation  
268 results have been compared with the experimental results. For both the discharge at the end of the flume and for the  
269 velocity and water depth profiles along the flume, we have shown that the Poiseuille friction term appears to be the  
270 most relevant to reproduce such laboratory experiments. We noted that the Darcy-Weisbach coefficient reproduces the  
271 laminar cases investigated here as well as the Poiseuille model. Only small differences are observed for the highest  
272 local Reynolds situations for which no quantitative conclusions can be drawn. However, such D-W model offers an  
273 interesting simple approach able to deal with the variation of the flow structure and should be studied in the future for  
274 more turbulent film-flow. On the other hand, the Poiseuille friction term that has been shown to correctly account for  
275 laminar film flow needs to be investigated on complex 2D bathymetry for which local slope variations could perturb  
276 the laminar approach. Finally, we would like to emphasize that by investigating firstly a simple laminar flow for which  
277 both experimental and numerical results could be quantitatively compared, our work paves the road for a systematic  
278 approach of complex rain-driven overland flows.

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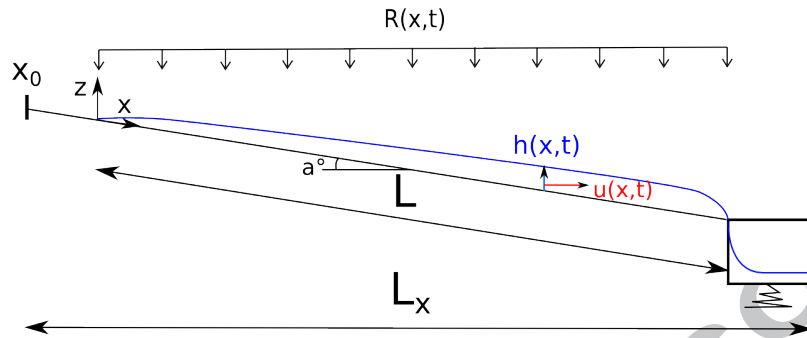


Figure 1: The “ideal rain” case: an homogeneous rain is falling on a tilted flume, producing overland flow.

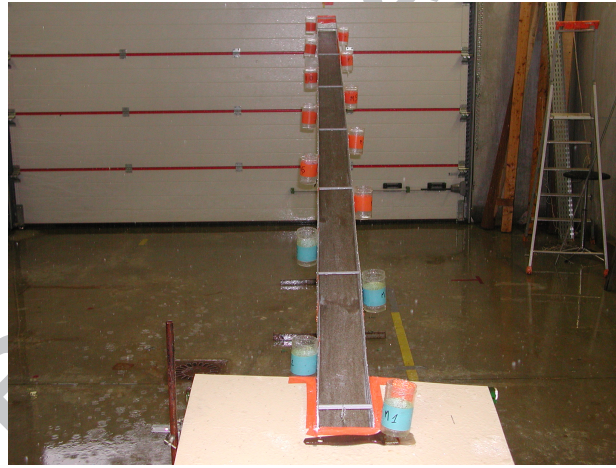


Figure 2: Front picture of the flume in the Rainfall Simulation Hall

Tar. Rain ( $mm.h^{-1}$ )	Slope (%)	Num. Rain ( $mm.h^{-1}$ )	Reynolds	Froude	Exp. Outflow ( $g.s^{-1}$ )
25	2	22	24	0.4	2.8
25	5	23.5	26	0.65	3.0
50	2	45.5	54	0.6	5.8

Table 1: Main quantities for each studied case.

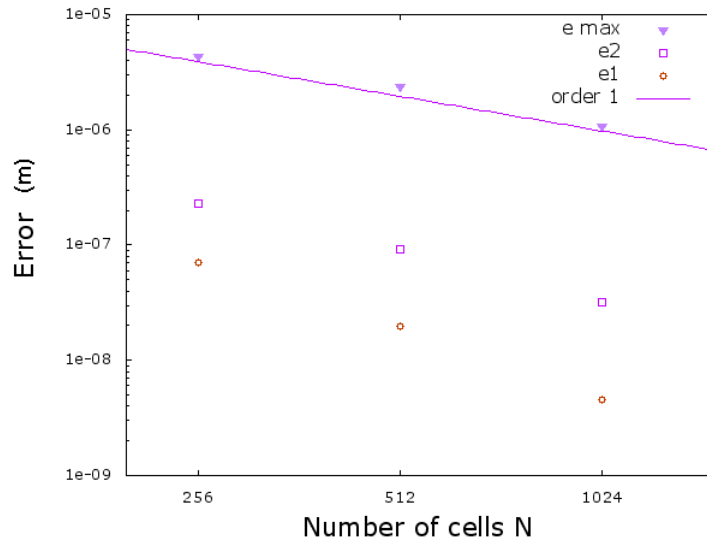


Figure 3: Error norms defined in Equ. 16, 17 and 18 with respect to the number of cells of the simulation calculated for the case " $I = 25\text{mm.h}^{-1}$  and  $S_0 = 5\%$ " for the Darcy-Weisbach friction term. Results shown in log-log scale. The straight line is a guide for the eyes of an order 1 curve.

Rain and Slope	Num. or Exp. Cases	$t_b$ (s)	$t_s$ (s)
$25\text{ mm.h}^{-1}$ and 2 %	Exp.	55	115
	Poiseuille	55	120
	Darcy-W.	55	120
	Manning	30	105
$50\text{ mm.h}^{-1}$ and 2 %	Exp.	30	75
	Poiseuille	35	75
	Darcy-W.	35	80
	Manning	20	80
$25\text{ mm.h}^{-1}$ and 5 %	Exp.	45	85
	Poiseuille	40	85
	Darcy-W.	40	85
	Manning	20	75

Table 2: Values of  $t_b$  and  $t_s$  in each case.



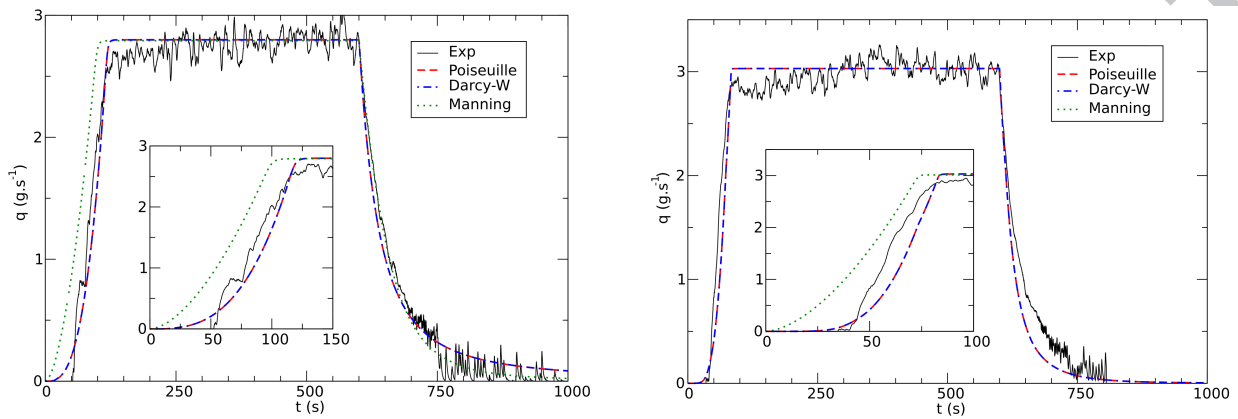
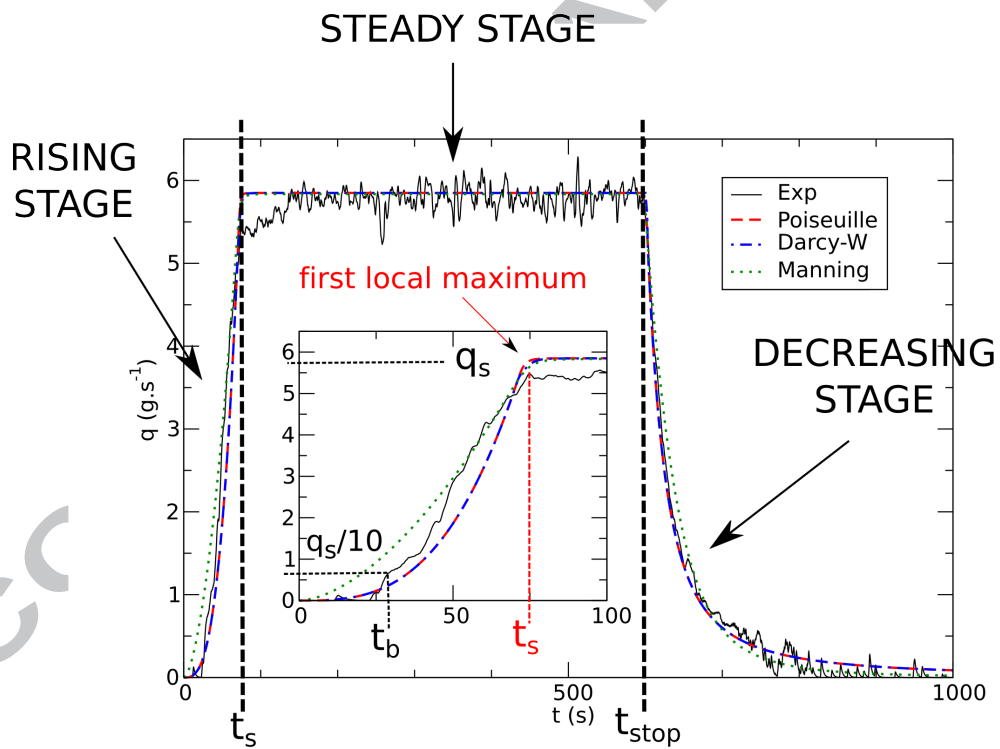
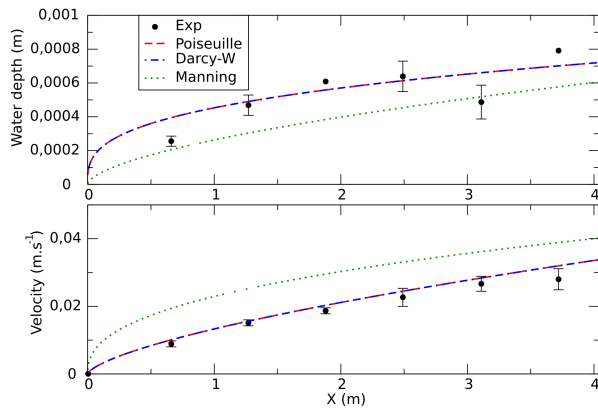
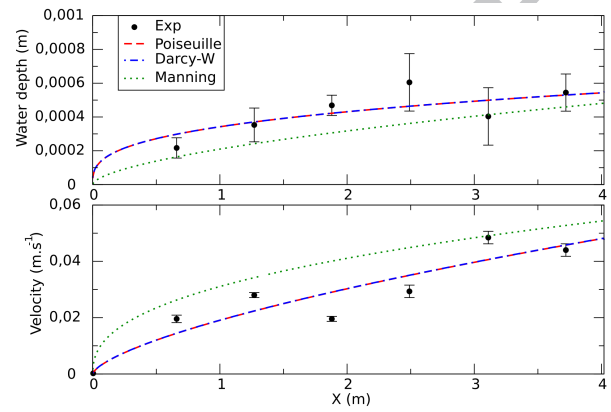
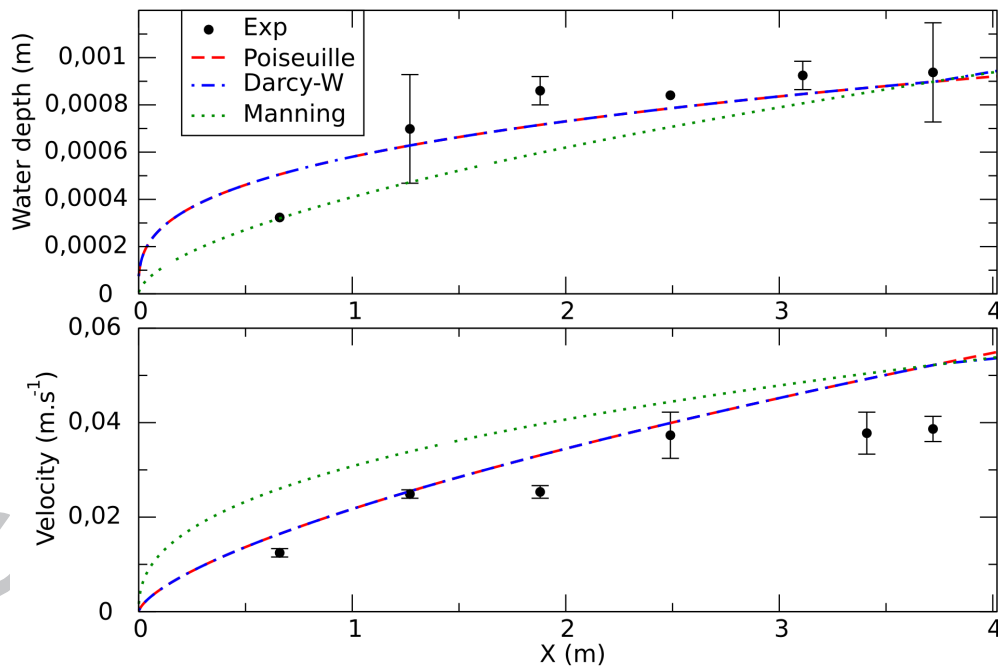
(a) Slope = 2 %, Rain = 25  $\text{mm}\cdot\text{h}^{-1}$ (b) Slope = 5 %, Rain = 25  $\text{mm}\cdot\text{h}^{-1}$ .(c) Slope = 2 %, Rain = 50  $\text{mm}\cdot\text{h}^{-1}$ . Definition of  $t_b$ ,  $t_s$ ,  $t_{stop}$  and the three stages of the hydrograph. In inset, the Darcy-Weisbach model stops following Poiseuille model at  $t = 70$  s to follow the Manning model.

Figure 4: Numerical results with different friction terms and experimental discharge at the end of the slope versus time for different slopes and rain intensities. Zoom of the rising limb in inset.

(a) Slope = 2 %, Rain = 25  $mm.h^{-1}$ (b) Slope = 5 %, Rain = 25  $mm.h^{-1}$ (c) Slope = 2 %, Rain = 50  $mm.h^{-1}$ . At  $X = 3.75$  m, the Darcy-Weisbach model stops following the Poiseuille model and starts following the Manning model.Figure 5: Water depth (top) and velocity (bottom) profiles along the slope at the steady stage ( $t = 599$  s). Error bars are standard errors.

Rain and Slope	Friction model	$Ind_h$	$Ind_u$
25 mm.h <sup>-1</sup> and 2 %	Poiseuille	0.20	0.09
	Darcy-W.	0.20	0.09
	Manning	0.28	0.61
50 mm.h <sup>-1</sup> and 2 %	Poiseuille	0.17	0.22
	Darcy-W.	0.17	0.22
	Manning	0.17	0.47
25 mm.h <sup>-1</sup> and 5 %	Poiseuille	0.17	0.23
	Darcy-W.	0.17	0.23
	Manning	0.25	0.37

Table 3: Values of  $Ind_h$  and  $Ind_u$  in each case. The closer to zero the index is, the closer to the experimental measurements the simulation is.

A case of rain falling on a flume is performed in laboratory.

The case is reproduced numerically thanks to the shallow-water equations.

Three friction terms are tested : the terms of Manning, Darcy-Weisbach and Poiseuille.

Numerical results and experimental measurements are compared.

The Poiseuille and Darcy-Weisbach friction terms are the best to reproduce such laminar experiments.