Turbulence in Fusion Plasmas

with reduced models

Laboratoire de Physique des Plasmas

Ö.D. Gürcan
Laboratoire de Physique des Plasmas
Ecole Polytechnique, France

Plasma Descriptions (i.e. fusion)

- Trajectories of N-particles (Newton + Maxwell) : Klimontovich
- N very large → BBGKY + closure to keep only the 1-particle pdf : Vlasov, Boltzmann
- Strong B→ guiding center description : Gyrokinetics, Drift-kinetics.
- $\delta B/B$ too small → Electrostatic.
- ℓ_m "short" → finite number of moments : Gyrofluid etc.
- Note that Gyrofluid models can be obtained from Drift-reduced Braginskii formulation as well.
- Hasegawa-Wakatani model is a toy model for "edge" turbulence in fusion devices.
- Probably the simplest gyrofluid model.
- A modified version of the model can describe zonal flows reasonably correctly.
- Not really a credible model for fusion devices, but for basic plasma devices (e.g. ToriX @ LPP - Ecole Polytechnique or CSDX @ UCSD) it is a somewhat realistic model.

Dissipative Drift Waves

- Consists of an equation of vorticity and an equation of continuity

Hasegawa-Wakatani

$$\partial_t \nabla^2 \Phi + \hat{\mathbf{z}} \times \nabla \Phi \cdot \nabla \nabla^2 \Phi = C(\Phi - n) + D_\phi$$

$$\partial_t n + \hat{\mathbf{z}} \times \nabla \Phi \cdot \nabla n + \kappa \partial_y \Phi = C(\Phi - n) + D_n$$

- κ is the diamagnetic drift velocity (wave motion).
- C is the "adiabaticity parameter" (or normalized electron conductivity).
- $D_\phi = (-1)^\alpha \nabla^{4+2\alpha} \Phi$ and $D_n = (-1)^\alpha \nabla^{2+2\alpha} n$ are the (hyper-)dissipation terms.

- Same nonlinear structure with passive scalar, convection and many other problems.
- C is the critical parameter of interest:
  - $C \ll 1$: Hyrdodynamic limit (decoupling).
  - $C \gg 1$: The adiabatic limit (strong coupling) : Charney-Hasegawa-Mima equation.
- Note that the equation has the form of Potential Vorticity (PV) conservation:

$$q = \left(\frac{\Omega + \frac{eB}{mc}}{n_i} \right) \cdot \nabla s \rightarrow \nabla^2 \Phi - n$$

$s = P/n^\Gamma$ is the specific entropy.

Linear Solution:

general solution

$$\gamma_{\mathbf{k}} = -A \pm \sqrt{\frac{\left(B^2 + \frac{C^2}{k^2}\right)}{2} + \frac{1}{2}\sqrt{\left(B^2 + \frac{C^2}{k^2}\right)^2 + C^2\kappa^2 k_y^2/k^4}}$$

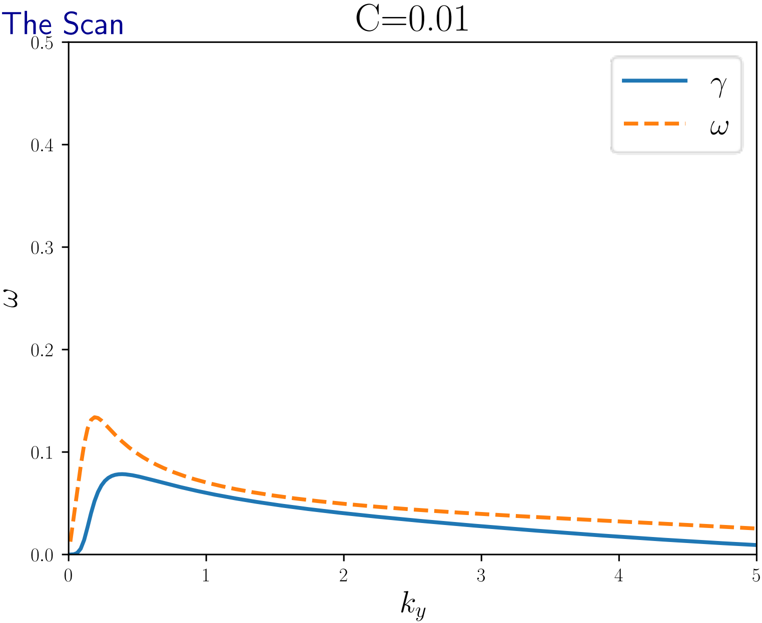
$$\omega_{r\mathbf{k}} = \pm\sqrt{\frac{1}{2}\sqrt{\left(B^2 + \frac{C^2}{k^2}\right)^2 + C^2\kappa^2 k_y^2/k^4} - \frac{\left(B^2 + \frac{C^2}{k^2}\right)}{2}}$$

where $\boxed{A = \frac{1}{2}\left[\left(Dk^2 + C\right) + \left(\frac{C}{k^2} + \nu k^2\right)\right]}$ and $\boxed{B = \frac{1}{2}\left[\left(Dk^2 + C\right) - \left(\frac{C}{k^2} + \nu k^2\right)\right]}$.

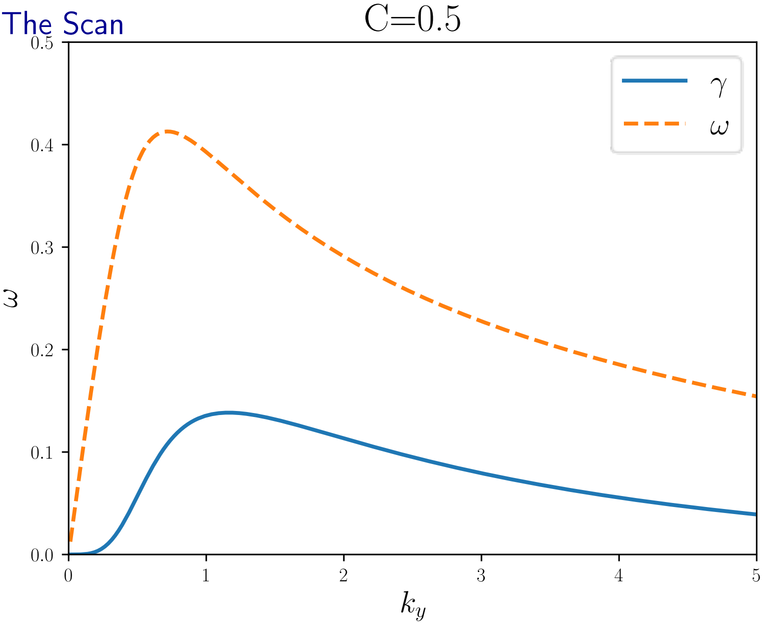
- The Hydrodynamic limit (i.e. $C \ll 1$): $\gamma \sim \omega \approx \pm\sqrt{\frac{1}{2} C\kappa k_y/k^2}$
- The adiabatic limit (i.e. $C \gg 1$) $\boxed{\omega \approx \frac{\kappa k_y}{1+k^2}}$, $\gamma^+ \approx \frac{1}{C}\frac{\kappa^2 k_y^2 k^2}{(1+k^2)^3}$

LPP

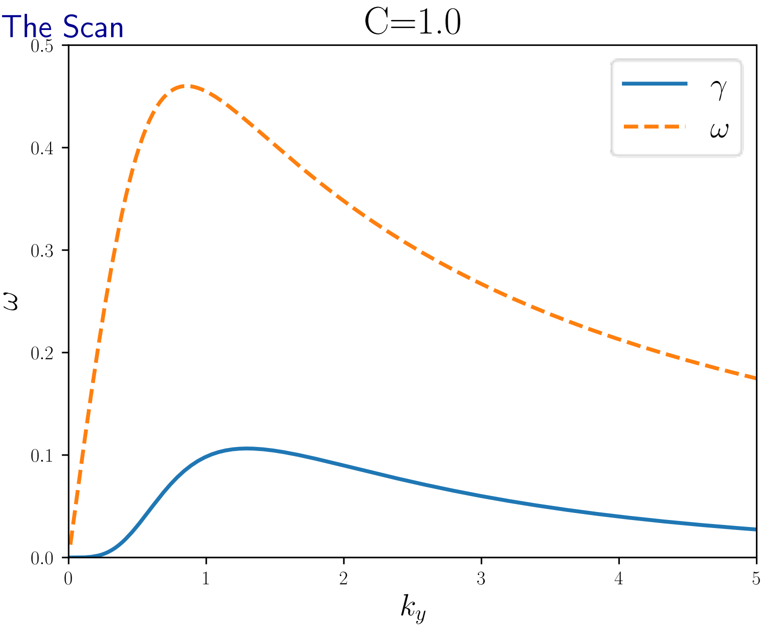
The Scan



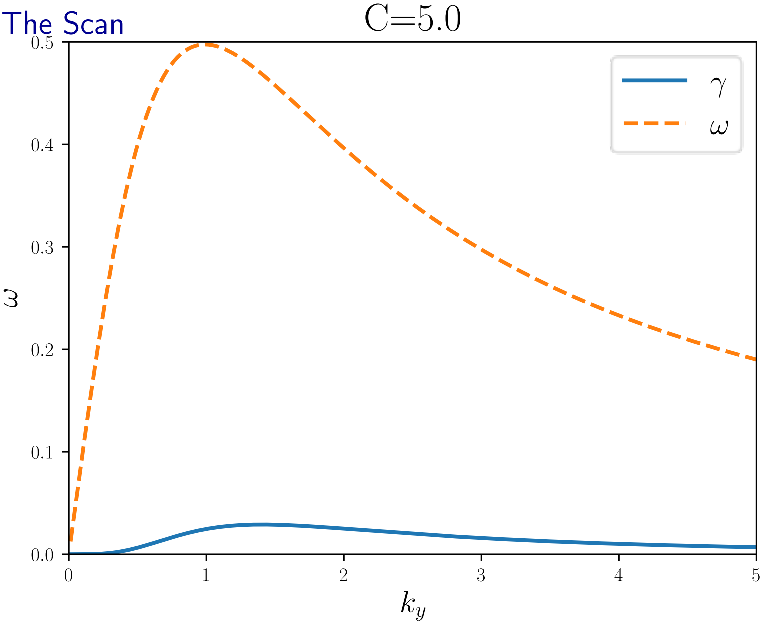
The Scan



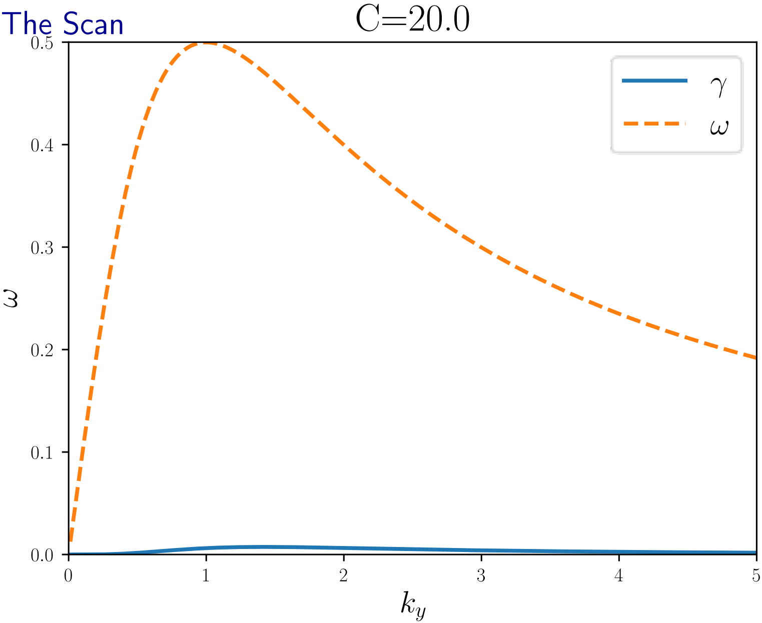
The Scan



The Scan



The Scan



Energy budget

$$\partial_t E(k) + \frac{\partial}{\partial k}\Pi(k) - 2C\left[E(k) - \frac{H(k)}{k}\right] = \mathscr{E}_E(k)$$

$$\partial_t F(k) + \frac{\partial}{\partial k}\Pi(k) + 2\kappa\Gamma(k) - 2C\left[\frac{H(k)}{k} - F(k)\right] = \mathscr{E}_F(k)$$

where

$$E(k) = \int |\Phi_{\mathbf{k}}|^2 k^3 d\alpha_k\,, \quad F(k) = \int |n_{\mathbf{k}}|^2 k d\alpha_k$$

$$H(k) = \int \mathrm{Re}\left[\Phi^*_{\mathbf{k}} n_{\mathbf{k}}\right] k^2 d\alpha_k\,, \quad \Gamma(k) = \int \sin\alpha_k \mathrm{Im}\left[\Phi^*_{\mathbf{k}} n_{\mathbf{k}}\right] k^2 d\alpha_k$$

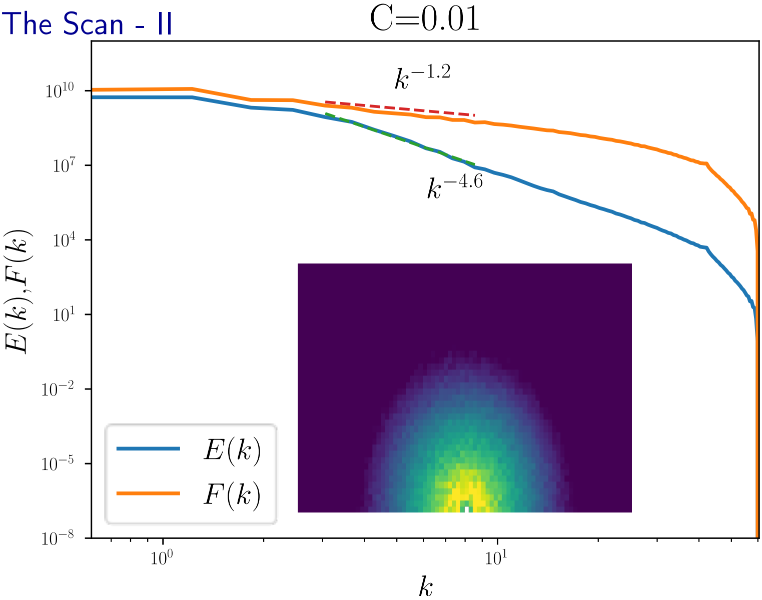
Usual passive scalar solution for $\kappa \sim C \ll 1$.

- Other well known solutions and relations between $F(k)$, $\Gamma(k)$, etc. are available (e.g. [Ghantous K. and Gürcan Ö. D. Phys. Rev. E. 2015])
- WTT in Hasegawa-Mima limit (\w ZFs) [Connaughton C., Nazarenko S. and Quinn B. Physics Reports 2015]

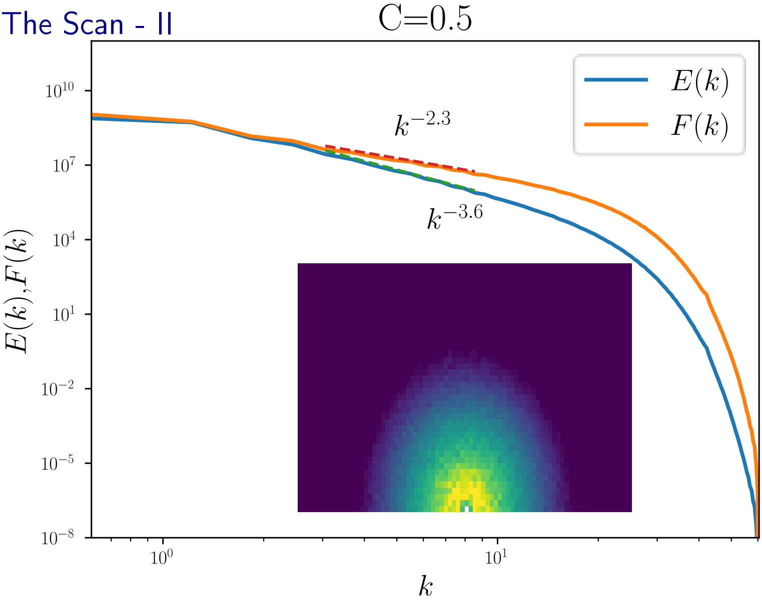
Results

file:///home/ogurcan/Videos/hw_standard.mp4

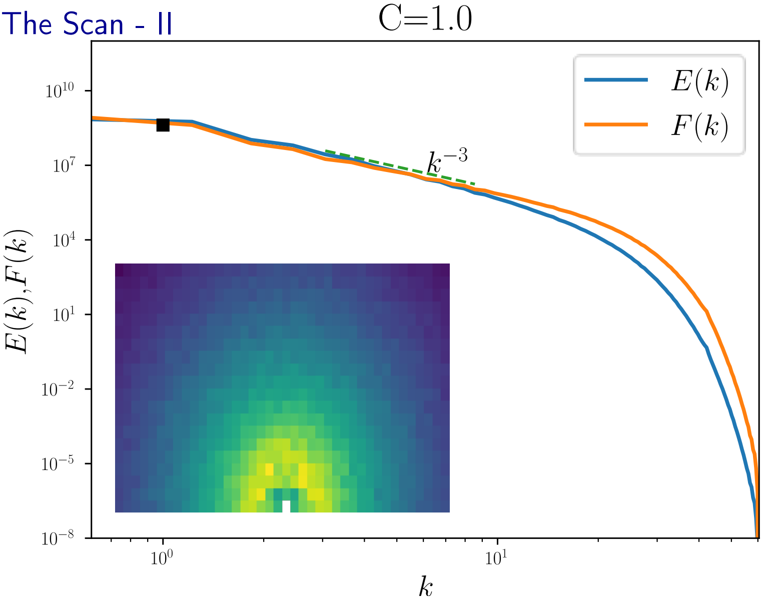
The Scan - II



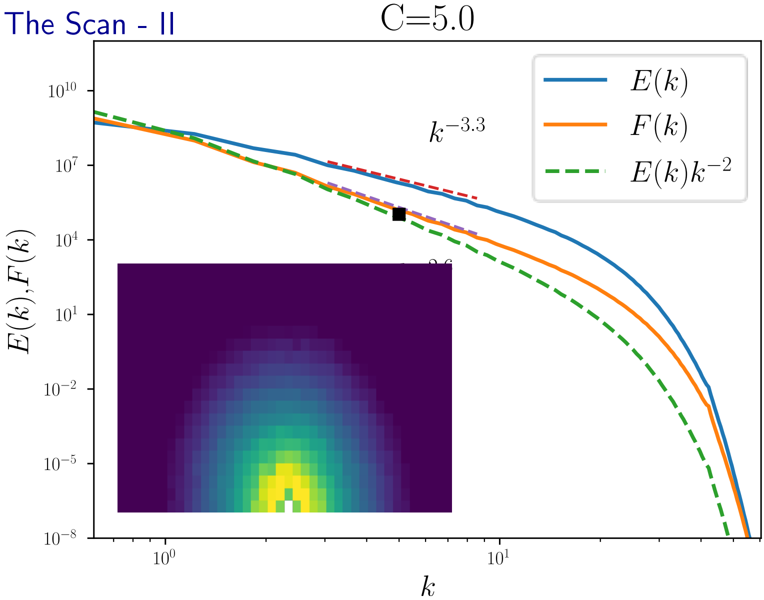
The Scan - II



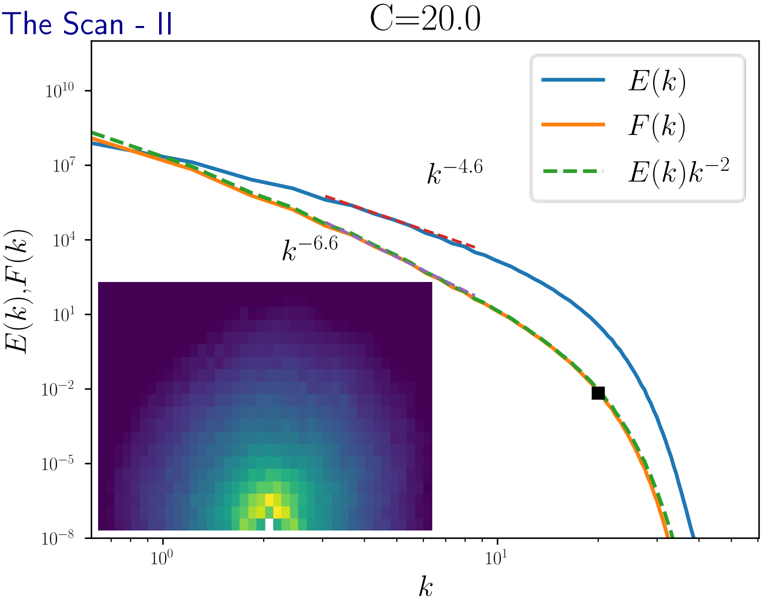
The Scan - II



The Scan - II



The Scan - II



C=20.0
$E(k)$
$F(k)$
$E(k)k^{-2}$
$k^{-4.6}$
$k^{-6.6}$
$E(k),F(k)$
k
10^{10}
10^{7}
10^{4}
10^{1}
10^{-2}
10^{-5}
10^{-8}
10^{0}
10^{1}

Zonal Flows

Hasegawa-Wakatani

$$\partial_t \nabla^2 \Phi + \hat{\mathbf{z}} \times \nabla \Phi \cdot \nabla \nabla^2 \Phi = C(\Phi - n) + D_\phi$$

$$\partial_t n + \hat{\mathbf{z}} \times \nabla \Phi \cdot \nabla n + \kappa \partial_y \Phi = +C(\Phi - n) + D_n$$

Modified Hasegawa-Wakatani

$$\partial_t \nabla^2 \Phi + \hat{\mathbf{z}} \times \nabla \Phi \cdot \nabla \nabla^2 \Phi = C(\tilde{\Phi} - \tilde{n}) + D_\phi$$

$$\partial_t n + \hat{\mathbf{z}} \times \nabla \Phi \cdot \nabla n + \kappa \partial_y \Phi = +C(\tilde{\Phi} - \tilde{n}) + D_n$$

- where $\tilde{\Phi} = \Phi - \langle \Phi \rangle$, and $\tilde{n} = n - \langle n \rangle$.
- $\langle \Phi \rangle = \int \Phi(x,y)\, dy$ or simply $\langle \Phi \rangle_k = \Phi_{k_y=0,k_x}$.
- Because, in fact $C \propto k_\parallel^2$, so for zonal modes, we have $k_y = 0$ and $k_\parallel = 0$, so $C = 0$ for them.
- Physically, this is because the electrons can not respond to axisymetric perturbations.
- Implies:

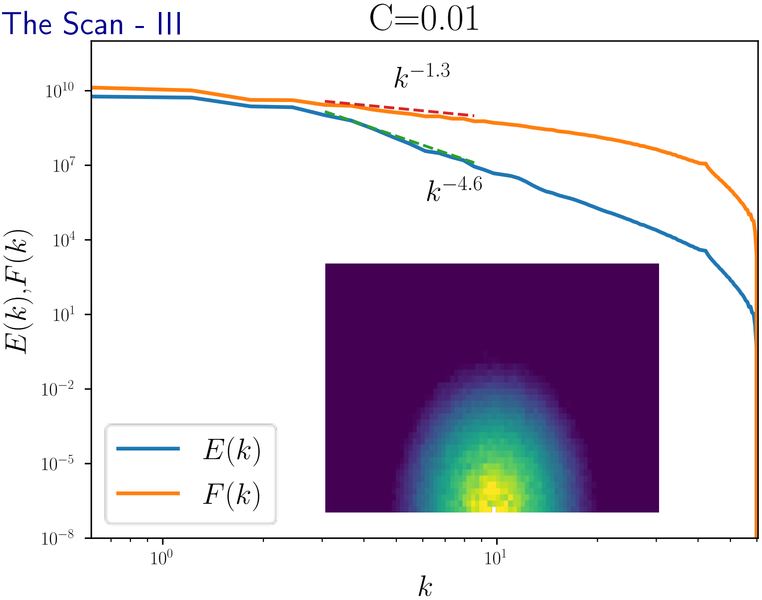
$$\partial_t \left\langle \nabla^2 \Phi \right\rangle + \left\langle \hat{\mathbf{z}} \times \nabla \Phi \cdot \nabla \nabla^2 \Phi \right\rangle = D_{\langle \phi \rangle}$$

LPP

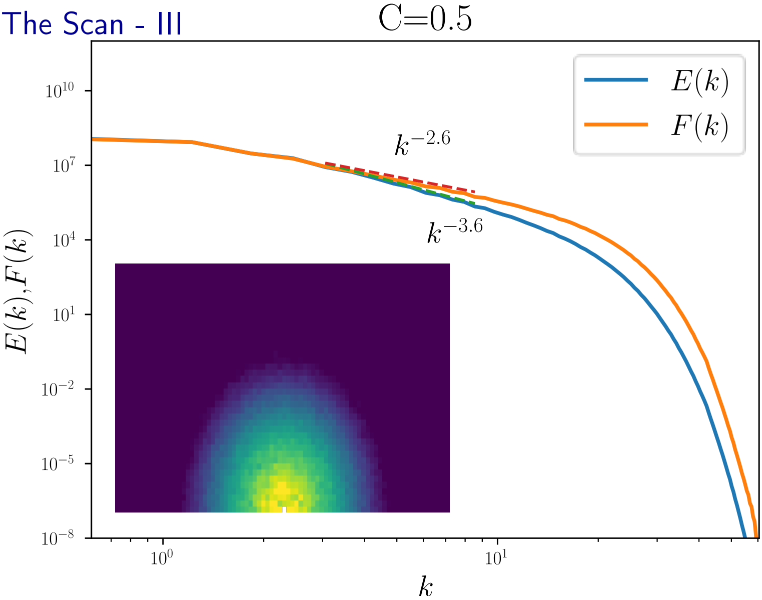
Results

```
file:///home/ogurcan/Videos/hw_mod.mp4
```

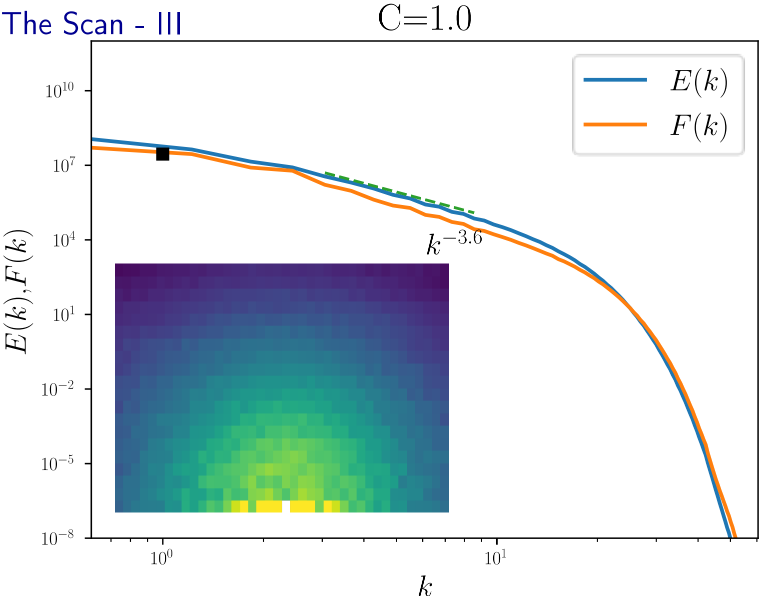
The Scan - III



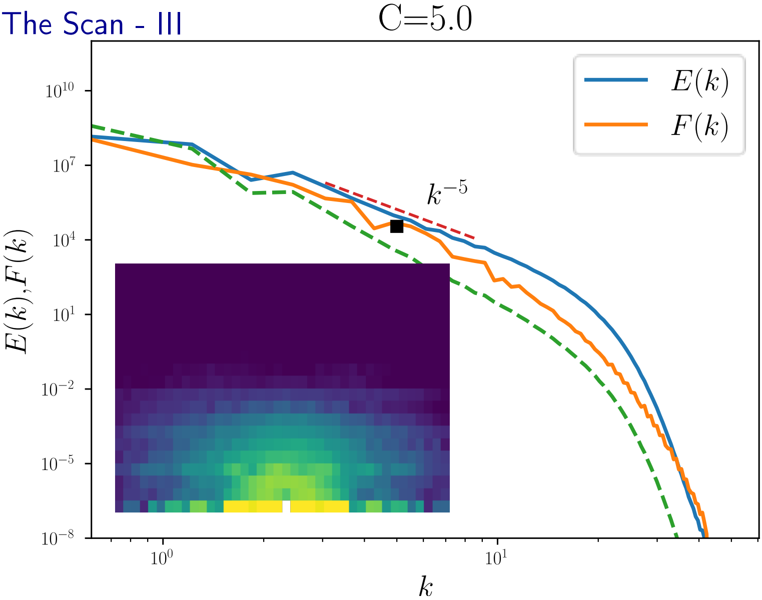
The Scan - III



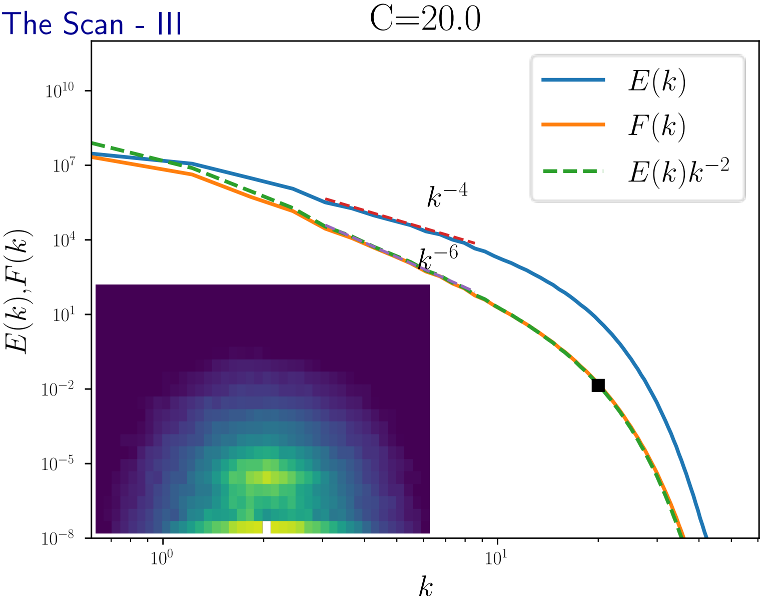
The Scan - III



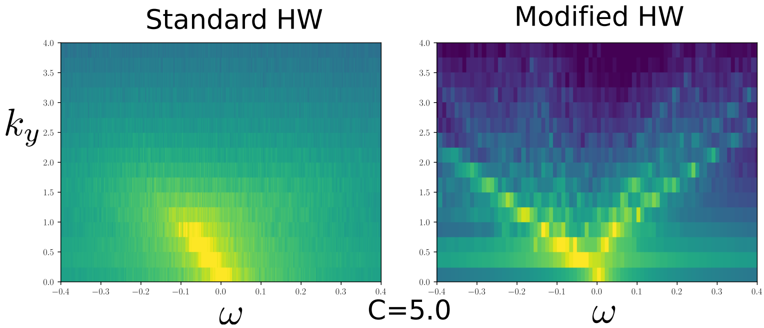
The Scan - III



The Scan - III

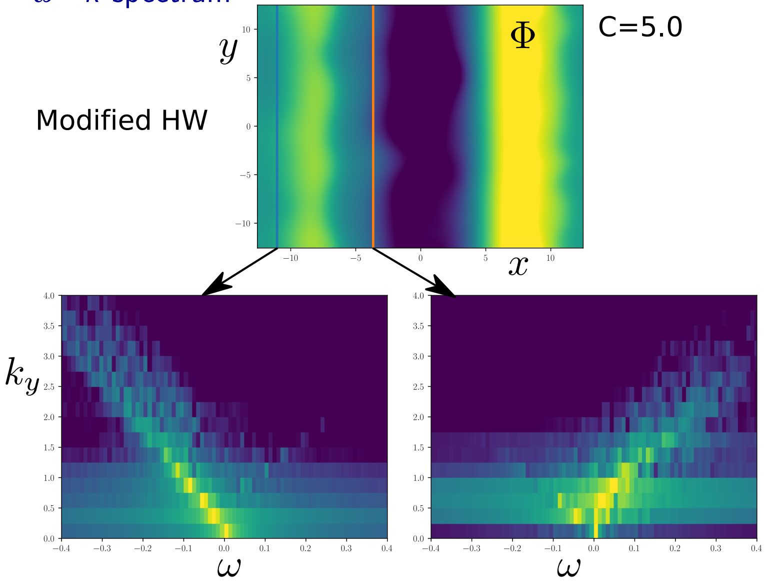


$\omega - k$ spectrum



$\omega - k$ spectrum

Modified HW



Predator-Prey Evolution

- Turbulence drive zonal flows.
- Zonal Flows reduce turbulence intensity.
- In many cases, we have Zonal Flow damping (friction, or drag).
- This leads to diminishing of the Zonal Flows. But when the zonal flows diminish, turbulence goes up.
- The cycle repeats.
- Examples from gyrokinetics [Kobayashi S. and Gürcan Ö. D. Phys. Plasmas. 2015] or basic experiment [Donnel P. et al. Phys. Plasmas. 2018]

Conclusion

- Hasegawa-Wakatani system has rich/interesting behaviour where linear drive/wave dynamics/nonlinear dynamics all compete.
- While there are certain simple limiting cases, there are also cases where multiple terms compete.
- Large scale structures clearly play a role. The waves appear around structures which establish themselves as an "evolving background"
- One can define various scales, in particular the $k_c = C/\kappa$ where the dynamics transits from adiabatic to non-adiabatic. (parallel electron conduction time equals eddy turnover time)