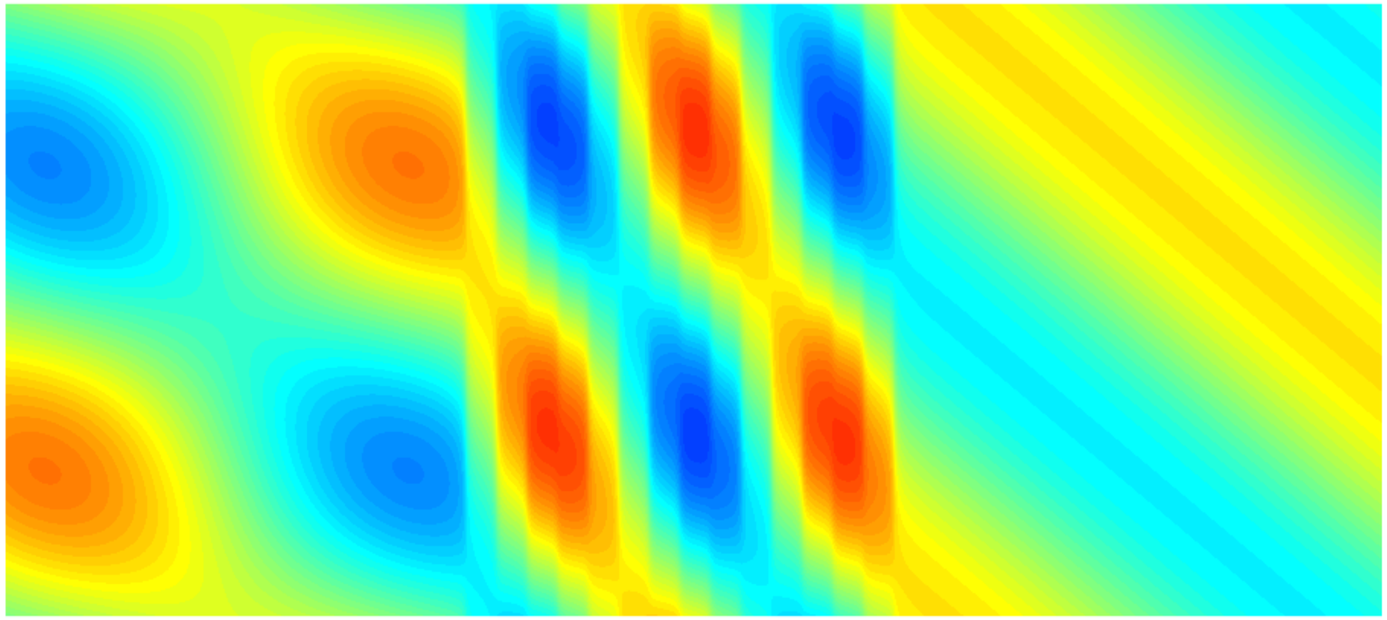
Propagation of water waves over structured ridges



Agnès Maurel, I. Langevin/ESPCI - Paris

Jean-Jacques Marigo, LMS/Polytechnique, Palaiseau

Kim Pham, UME/ENSTA, Palaiseau

Vincent Pagneux, LAUM, Le Mans

Philippe Petitjeans, PMMH/ESPCI, Paris

Pablo Cobelli, DFI, Buenos-Aires

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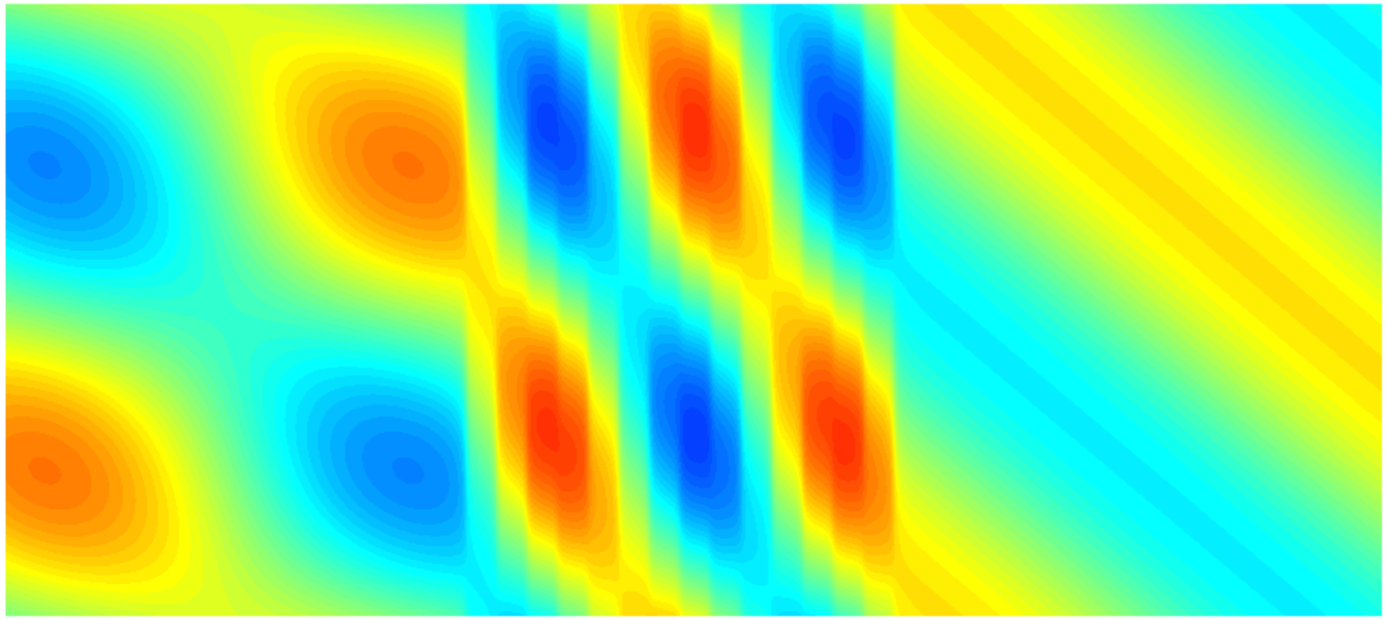

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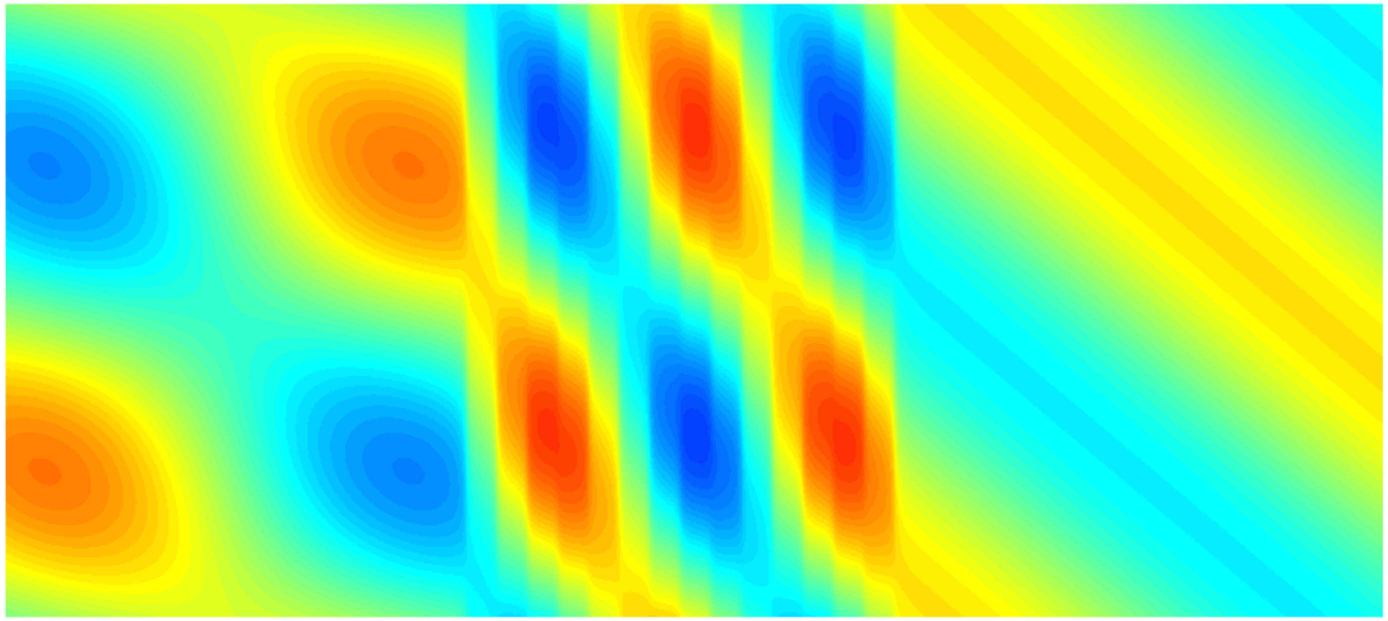

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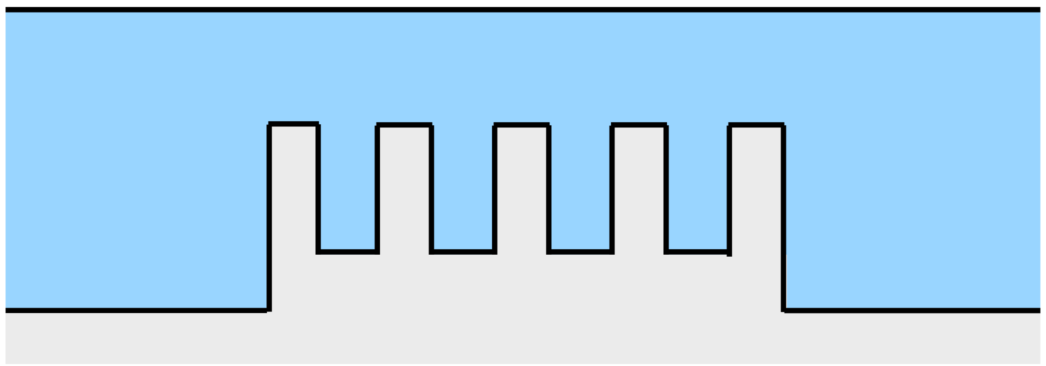

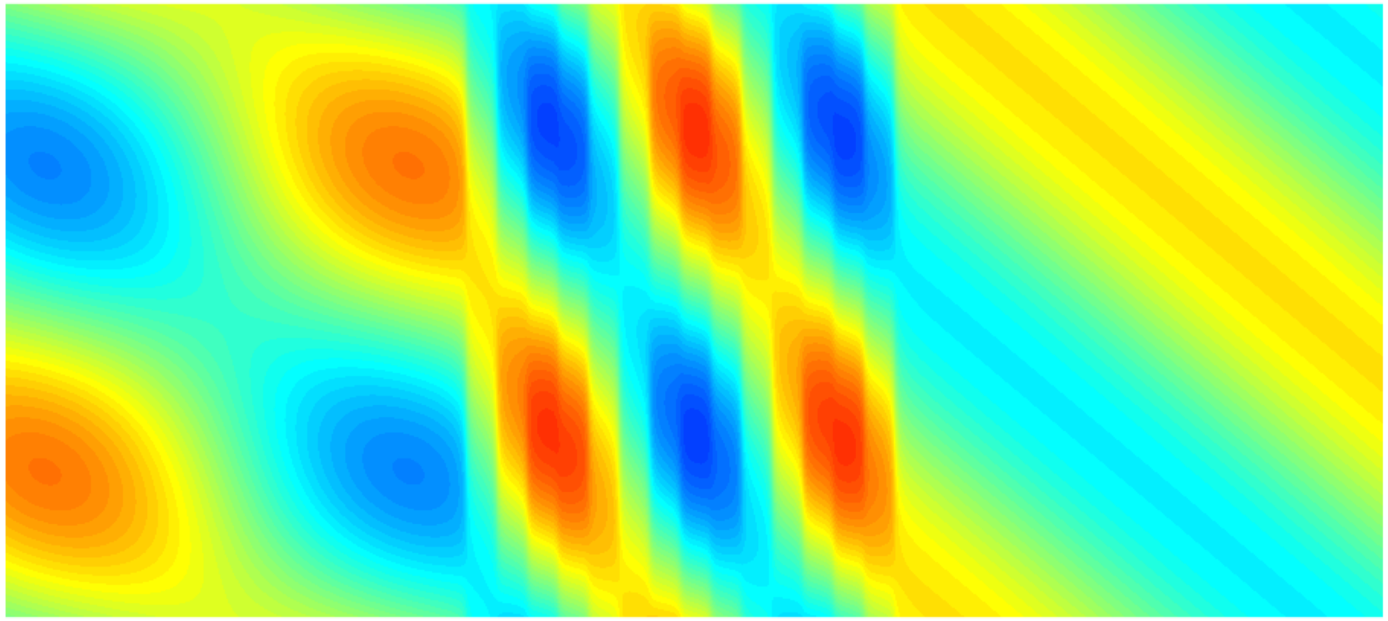

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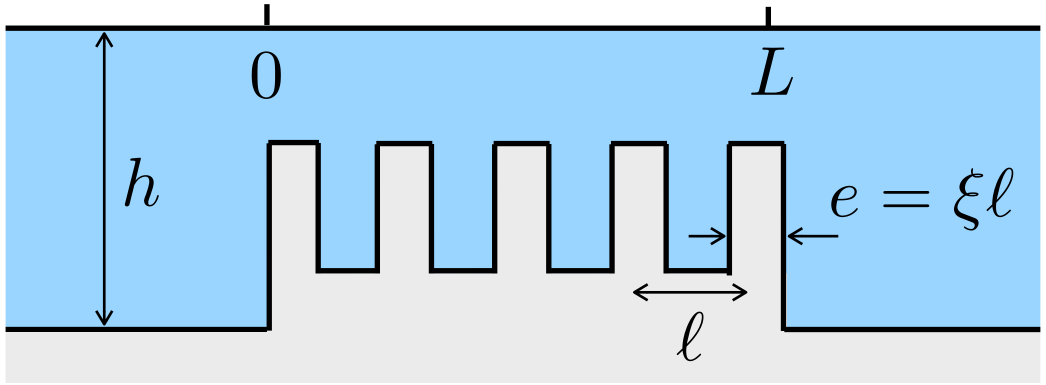

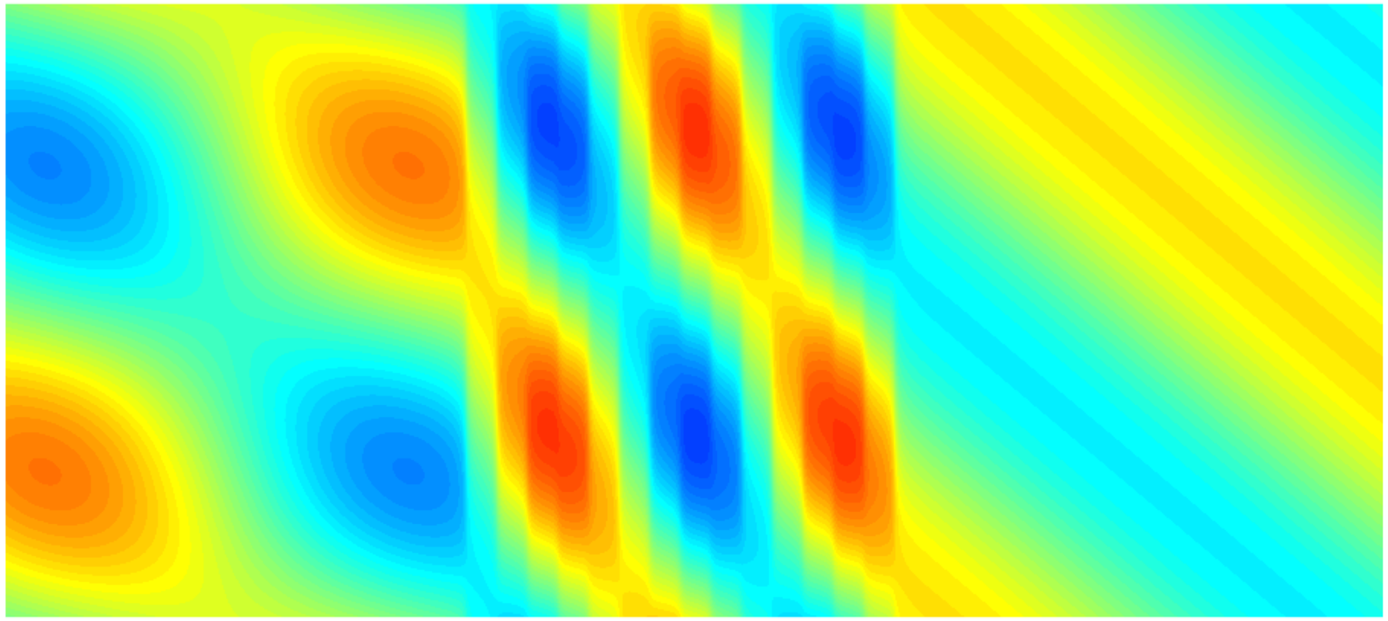

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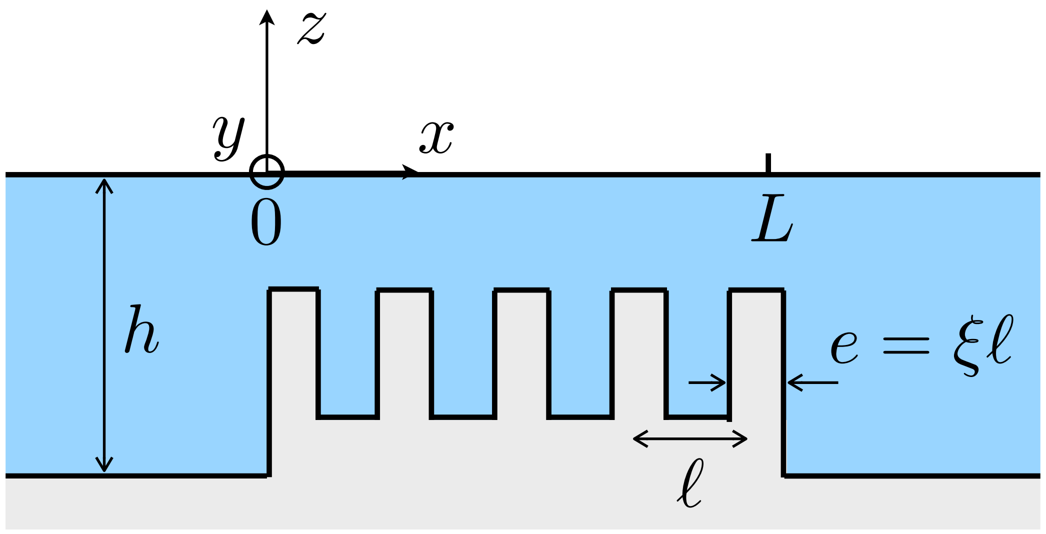

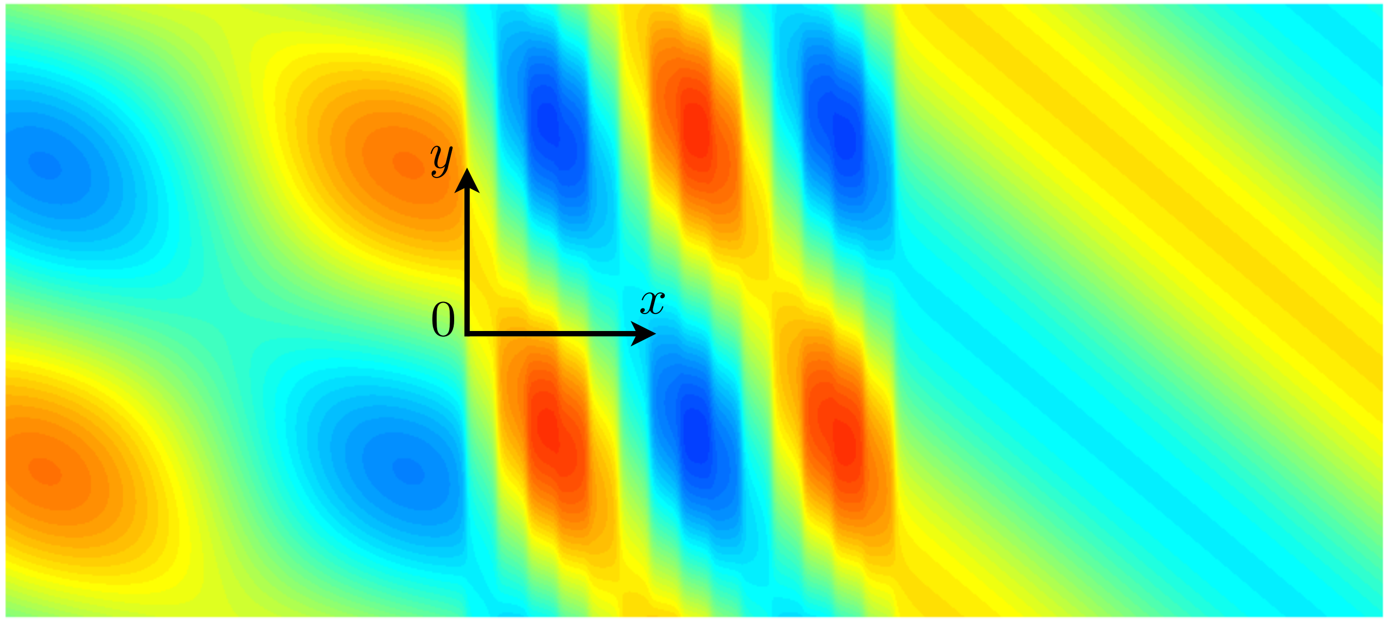

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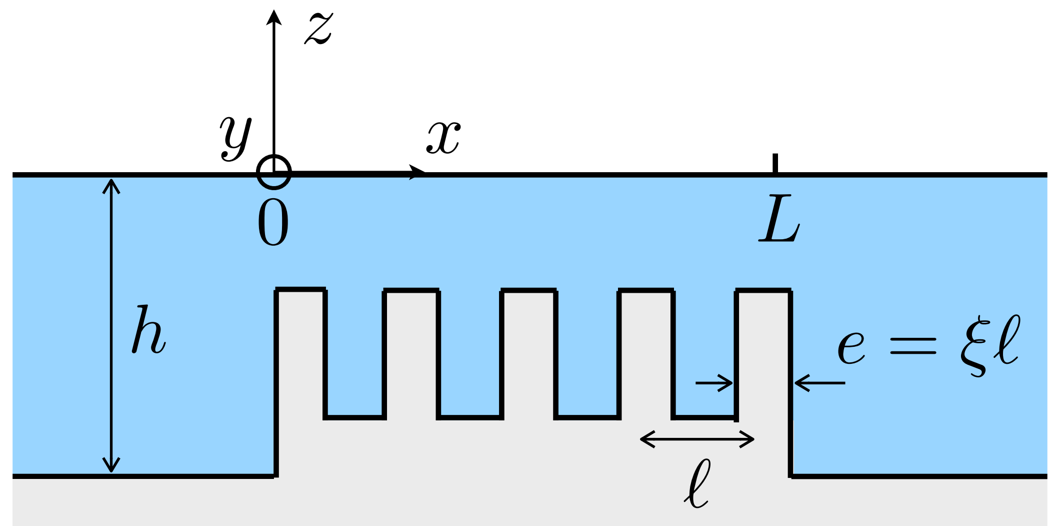

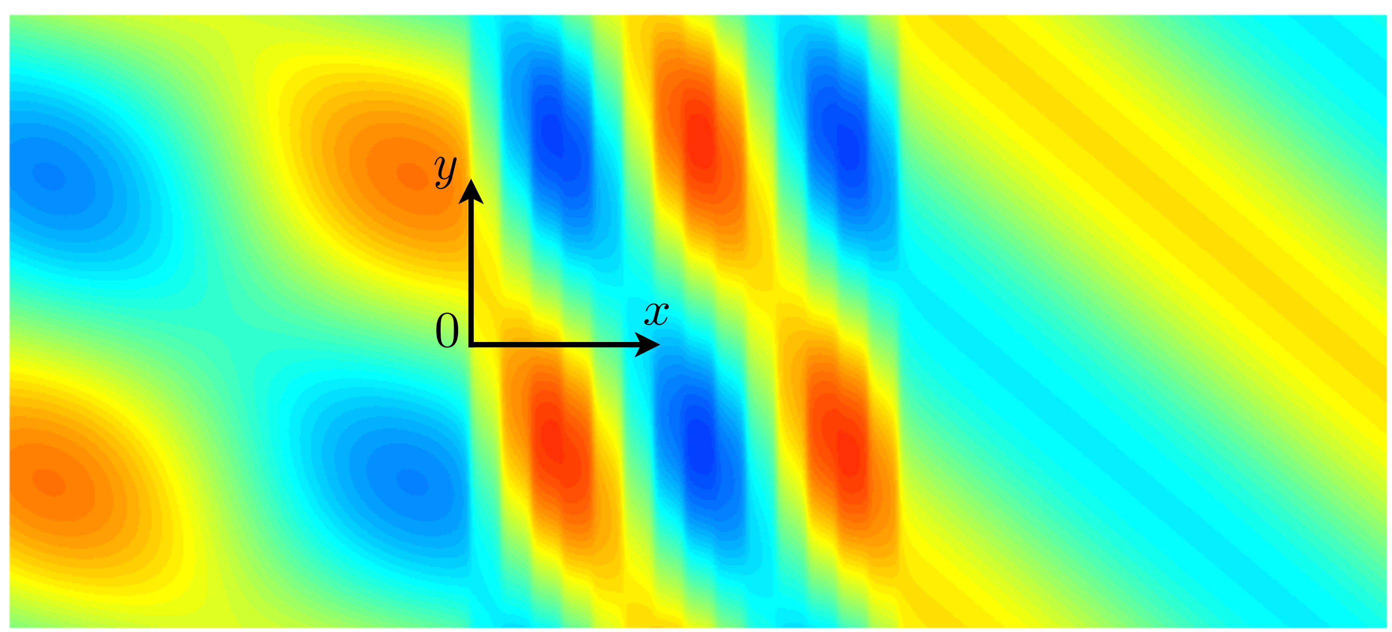

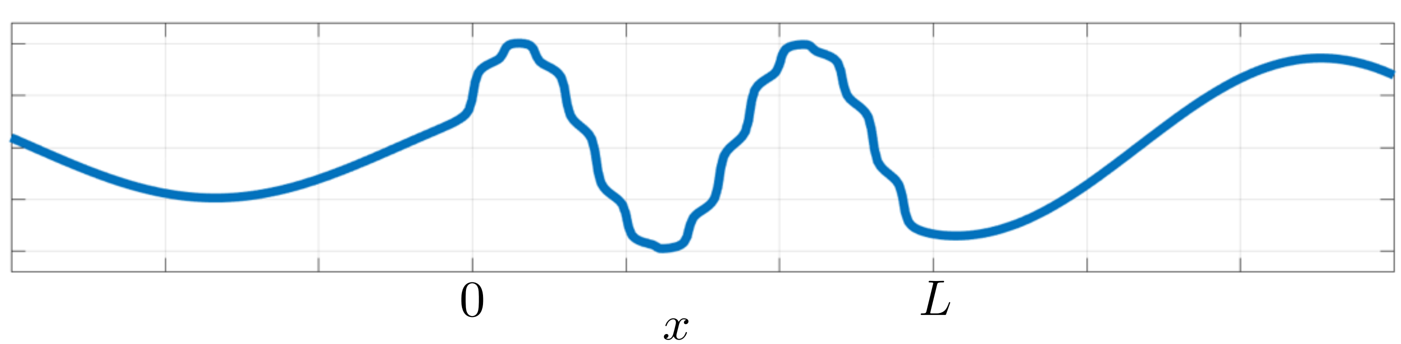

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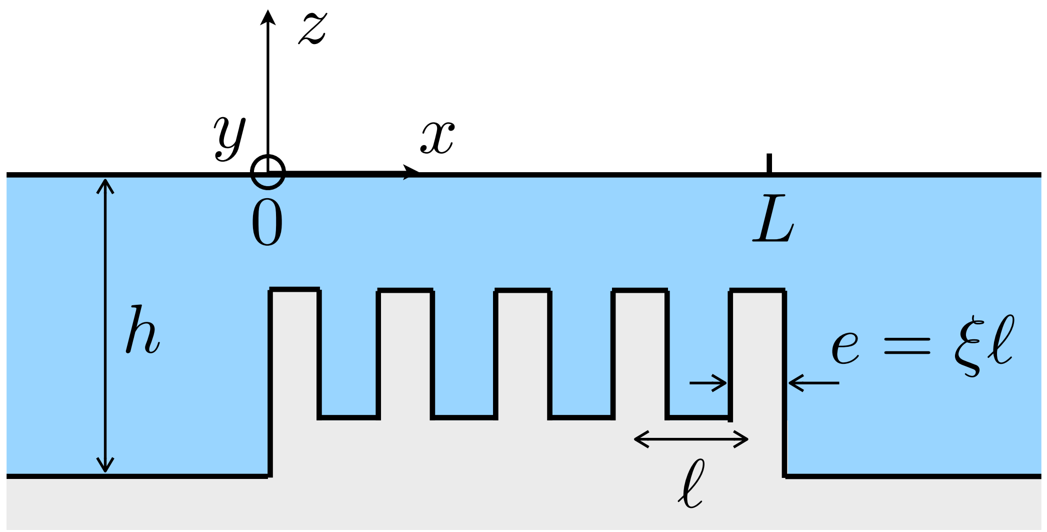
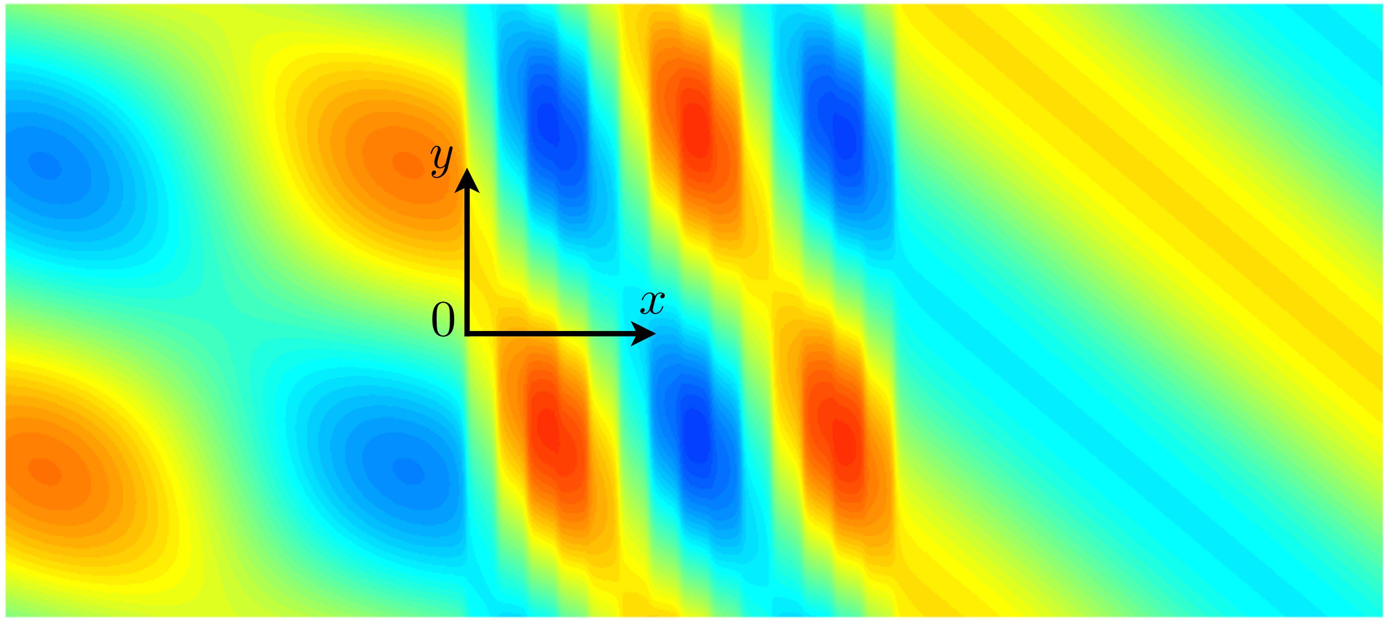
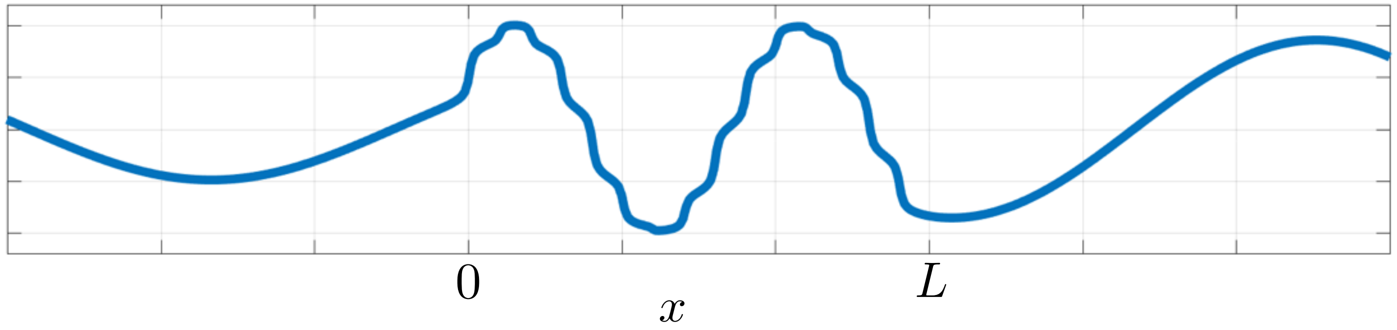


$kh \ll 1$

$k\ell \ll 1$

k typical wavenumber





Propagation of water waves over structured ridges

Community of fluid mechanics

Propagation of water waves over structured ridges

Community of fluid mechanics

J. Fluid Mech. (2011), *vol.* 687, *pp.* 461–491. © Cambridge University Press 2011 461
doi:10.1017/jfm.2011.373

Long waves through emergent coastal vegetation

Chiang C. Mei[1,2]†, **I-Chi Chan**[2], **Philip L.-F. Liu**[2,3], **Zhenhua Huang**[4] **and Wenbin Zhang**[4]

[1] Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

[2] School of Civil and Environmental Engineering, Cornell University, Ithaca, NY 14853, USA

[3] Institute of Hydrological and Oceanic Sciences, National Central University, Jhongli, Taiwan, 32001

[4] Earth Observatory of Singapore and Department of Civil and Environmental Engineering, Nanyang Technological University, Singapore 639398

(Received 26 December 2010; revised 14 July 2011; accepted 5 September 2011; first published online 14 October 2011)

We study the effects of emergent coastal forests on the propagation of long surface waves of small amplitude. The forest is idealized by an array of vertical cylinders. Simple models are employed to represent bed friction and to simulate turbulence generated by flow through the tree trunks. A multi-scale (homogenization) analysis similar to that for seepage flows is carried out to deduce the effective equations on the macro-scale. The effective coefficients are calculated by numerically solving the micro-scale problem in a unit cell surrounding one or several cylinders. Analytical and numerical solutions for wave attenuation on the macro-scale for different bathymetries and coastal forest configurations are presented. For a transient incident wave, analytical results are discussed for the damping of a leading tsunami. For comparison series of laboratory data for periodic and transient incident waves are also presented. Good agreement is found even though some of the measured waves are short or nonlinear.

Key words: shallow water flows, surface gravity waves, wave–turbulence interactions

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Surface water waves over a shallow canopy

Benlong Wang[1], **Xiaoyu Guo**[1] **and Chiang C. Mei**[1,2,]†

[1]Ministry of Education Key Laboratory of Hydrodynamics, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, 200240 Shanghai, China

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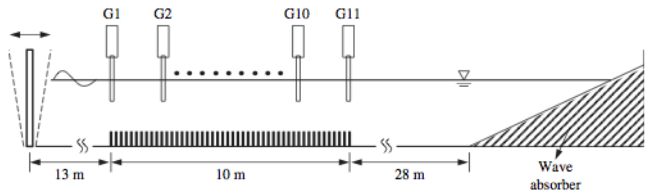

FIGURE 4. The set-up of the wave flume and canopy.

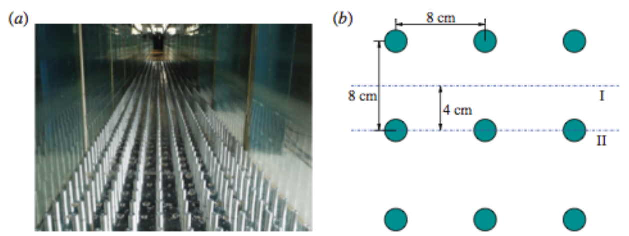

FIGURE 5. (Colour online) (*a*) Front view of the canopy. (*b*) Top view of the cylinder array. Station II marks the light sheet for photography.

Propagation of water waves over structured ridges

Community of fluid mechanics

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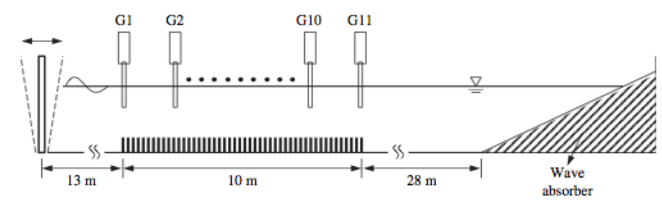

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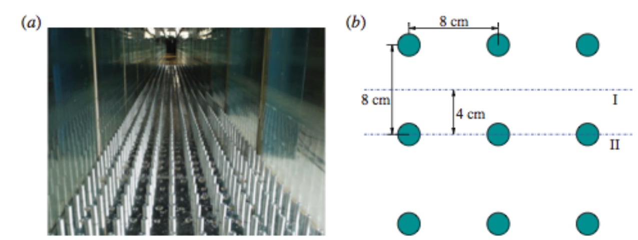

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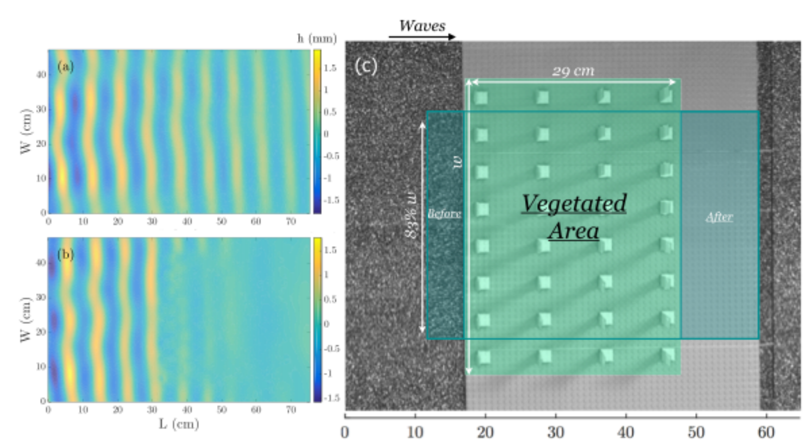
Bioinspir. Biomim. 13 (2018) 036006 https://doi.org/10.1088/1748-3190/aaae8c

Surface wave energy absorption by a partially submerged bio-inspired canopy

C Nové-Josserand[1], F Castro Hebrero[2], L-M Petit[1], W M Megill[3], R Godoy-Diana[1] and B Thiria[1]

[1] Laboratoire de Physique et Mécanique des Milieux Hétérogènes (PMMH), CNRS UMR 7636, ESPCI Paris—PSL Research University, Sorbonne Universités—Université Pierre et Marie Curie—Paris 6, Université Paris Diderot—Paris 7, 10 rue Vauquelin, 75005 Paris, France
[2] Laboratorio de Fluidodinámica (LFD), Facultad de Ingeniería, Universidad de Buenos Aires (CONICET), Av. Paseo Colón 850, C1063ACV, Buenos Aires, Argentina
[3] Faculty of Technology and Bionics, Rhine Waal University of Applied Sciences, Kleve, Germany



Propagation of water waves over structured ridges

Community of metamaterials

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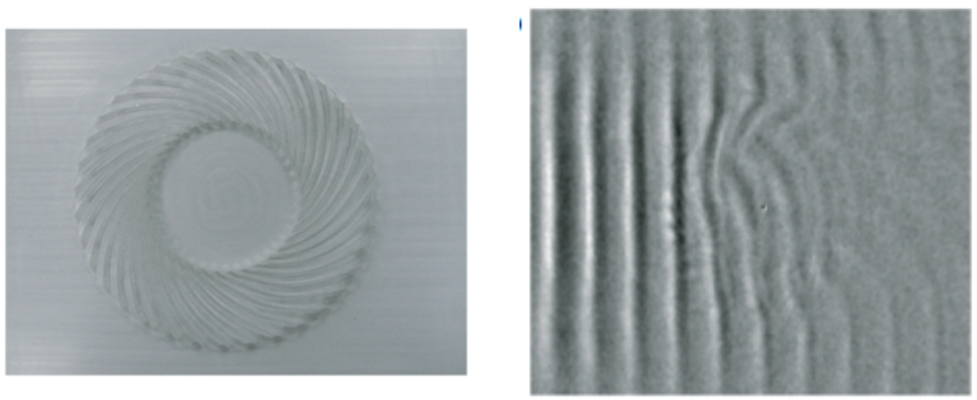
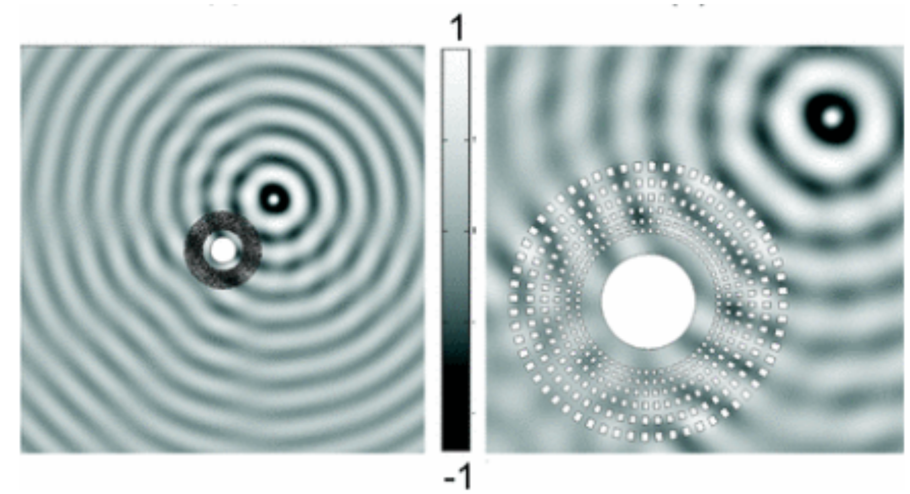
Community of metamaterials

control of the wave propagation: rotator



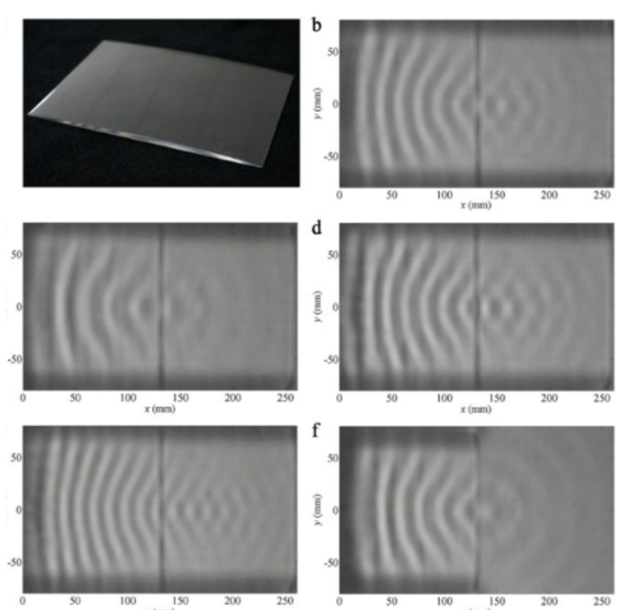
Transformation media for linear liquid surface waves
EPL, 85 (2009) 24004 doi: 10.1209/0295-5075/85/24004

cloaking effect



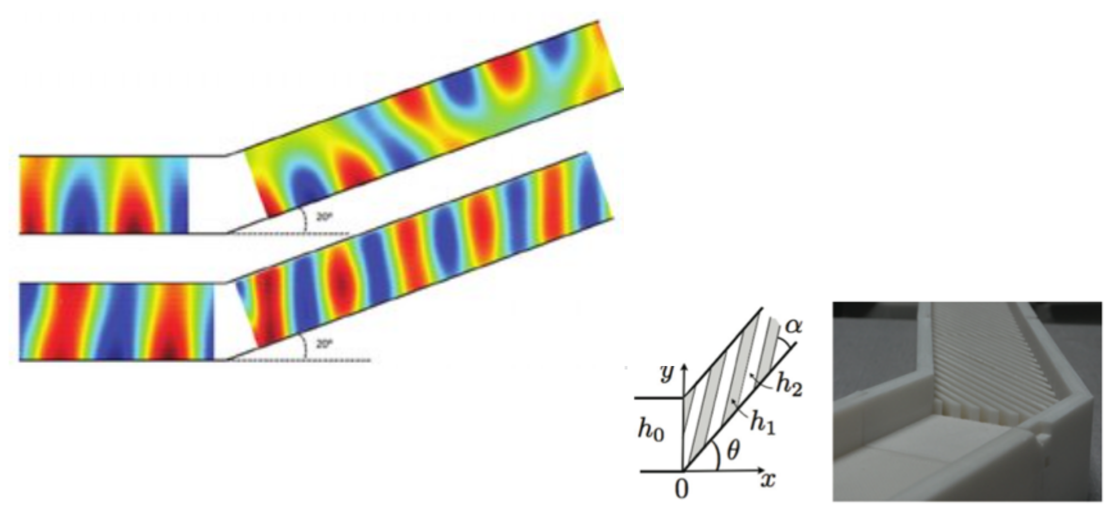
Numerical and experimental study of an invisibility carpet in a water channel
Phys. Rev. E 91, 023010 – 2015

focusing with varying bathymetry



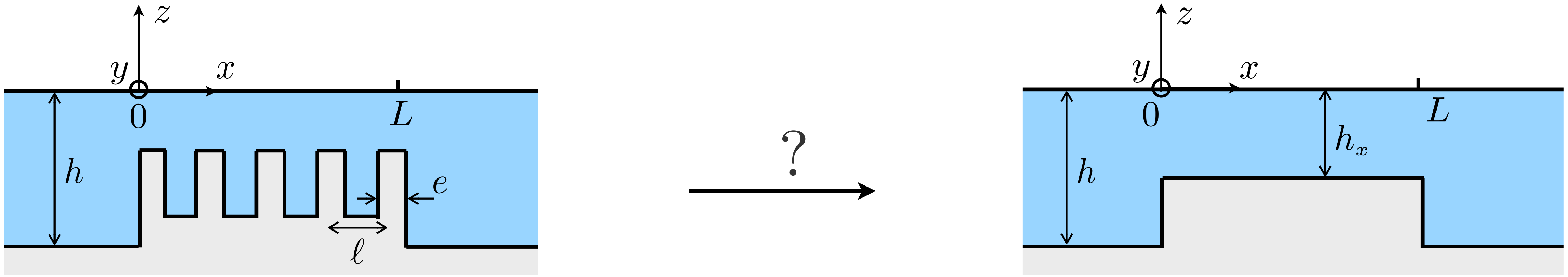
Manipulating Water Wave Propagation via Gradient Index Media
Scientific Reports 16846 (2015)

reflection-less shifter



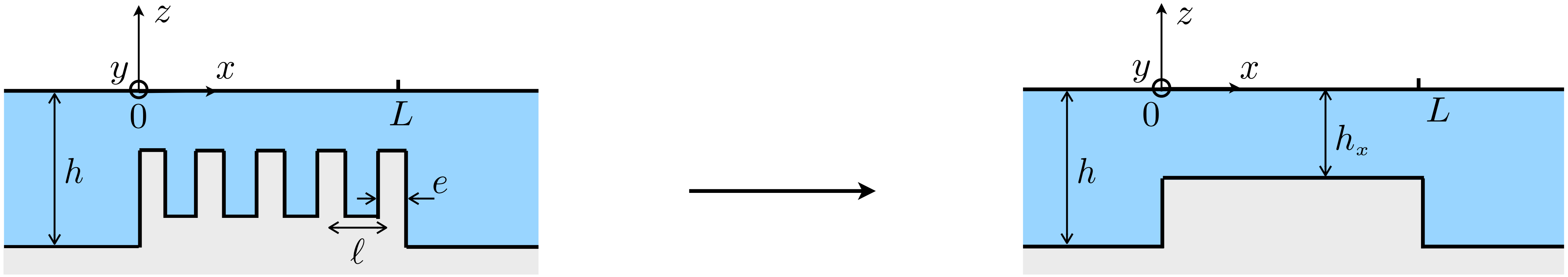
Experimental realization of a water-wave metamaterial shifter
PHYSICAL REVIEW E 88, 051002(R) (2013)

Propagation of water waves over structured ridges



- Shallow water: $kh \ll 1$
- microstructured ridge: $k\ell \ll 1$

Propagation of water waves over structured ridges



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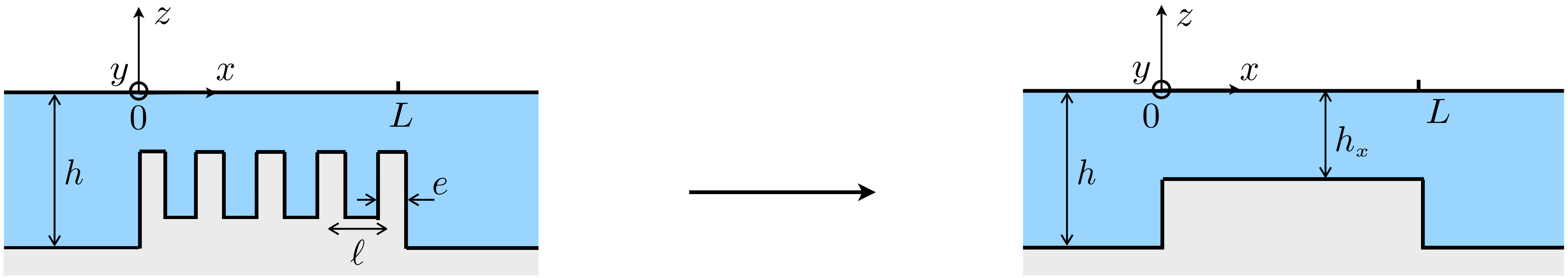

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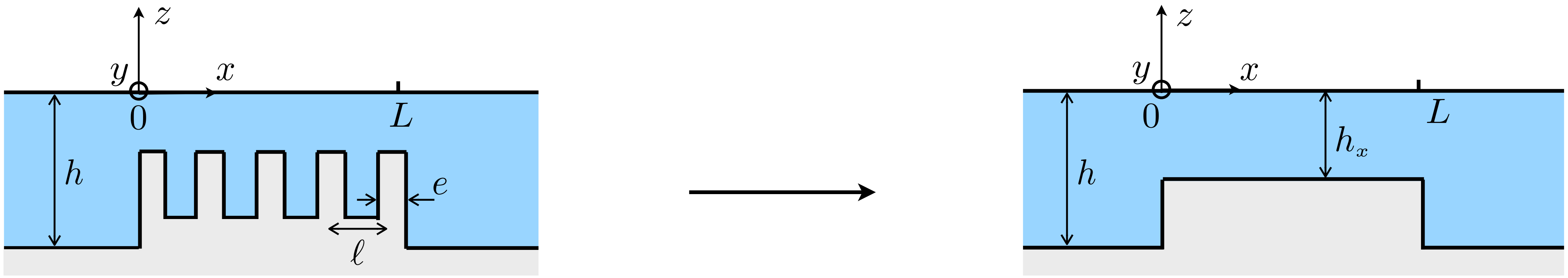
A tempting approach (largely used)

Propagation of water waves over structured ridges



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actual 3D problem

$$\Phi(x, y, z)$$

$$\Delta\Phi = 0,$$

$$\nabla\Phi \cdot \mathbf{n} = 0 \qquad \partial_z \Phi = -\frac{1}{g}\partial_{tt}\Phi$$

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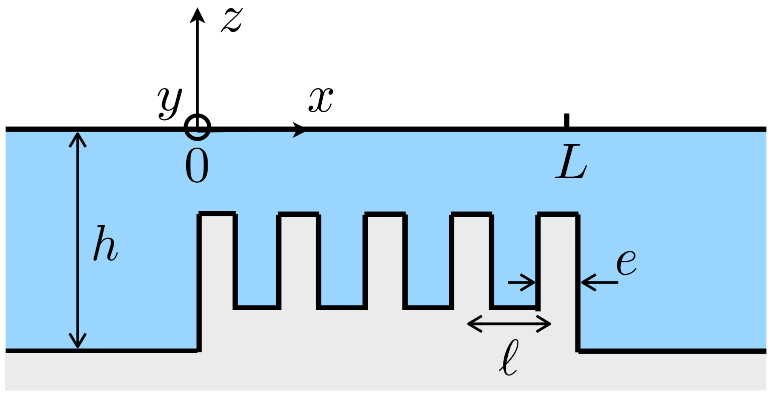

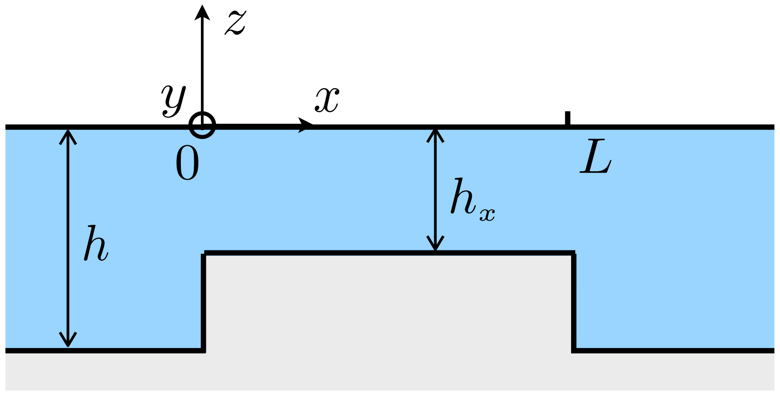

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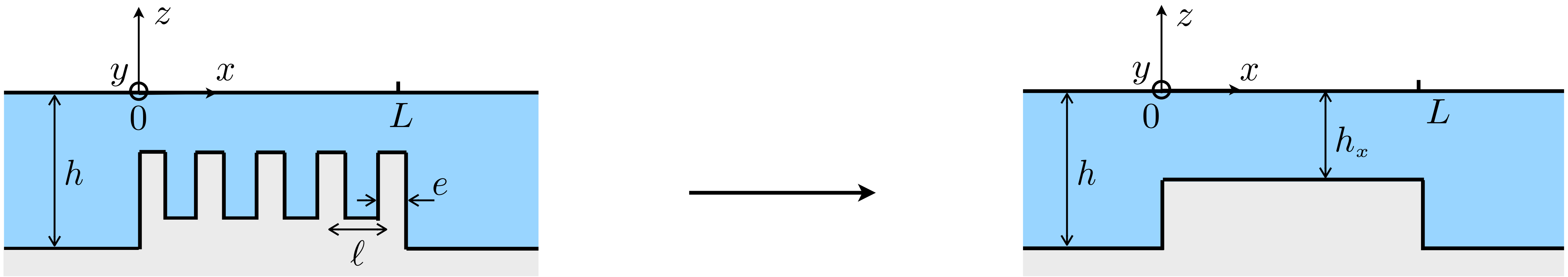
↓ 3D to 2D reduction

2D problem (shallow water app.)

$$\phi(x, y)$$

$$\nabla \cdot (h(x)\nabla\phi) - \frac{1}{g}\partial_{tt}\phi = 0$$

Propagation of water waves over structured ridges



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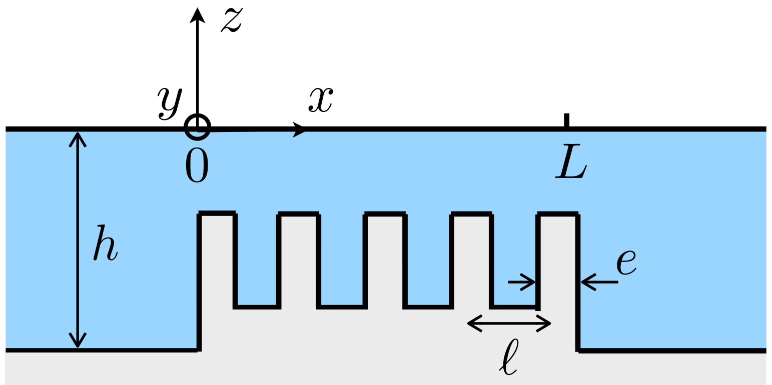
→ homogenization

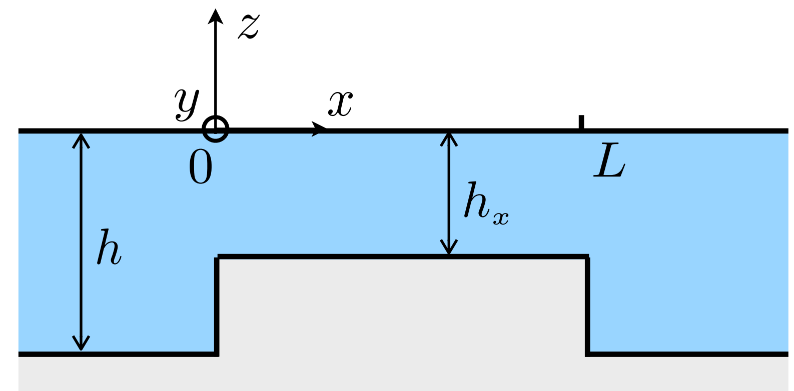
2D homogenized problem

$$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla \phi \right) - \frac{1}{g} \partial_{tt} \phi = 0$$

$$(h_x = \langle h^{-1} \rangle^{-1}, \quad h_y = \langle h \rangle)$$

Propagation of water waves over structured ridges





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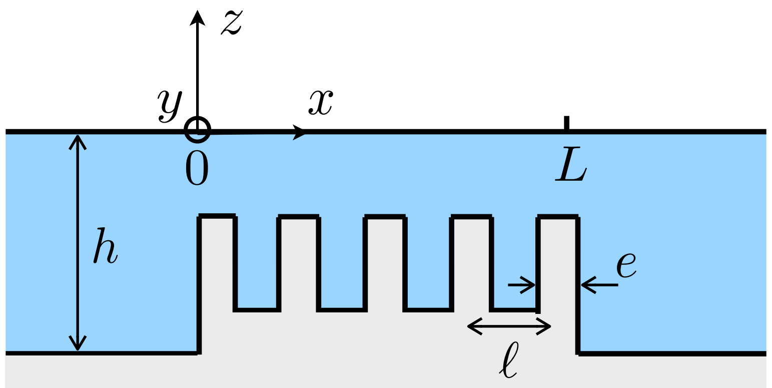
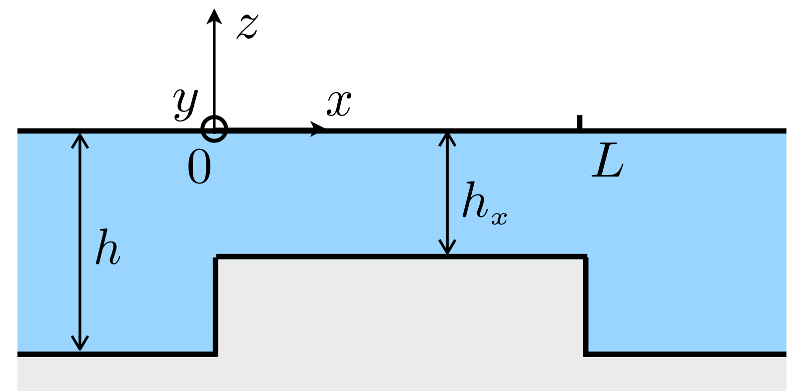
tempting because it offers a one-to-one correspondance with the electromagnetic case $\nabla \cdot \left(\frac{1}{\epsilon(x)} \nabla E \right) - \partial_{tt} E = 0$

Propagation of water waves over structured ridges





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transmission conditions at $x = 0, L$

BC: intuitive continuity of the potential and of the normal flux

$$[\![\phi]\!] = [\![D]\!] = 0$$

$$(D = h\partial_x\phi, \quad D = h_x\partial_x\phi)$$

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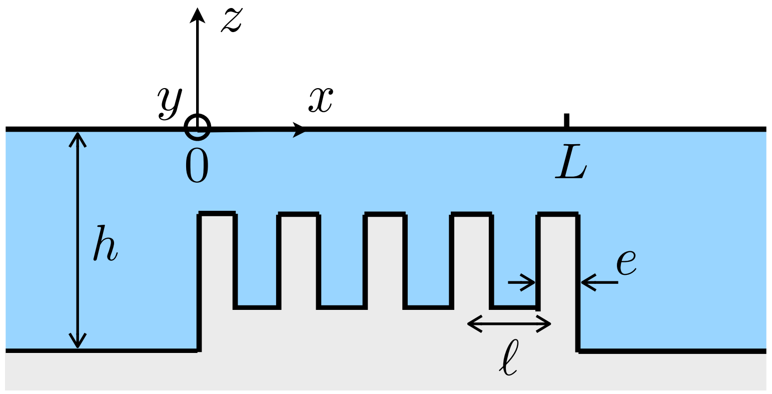

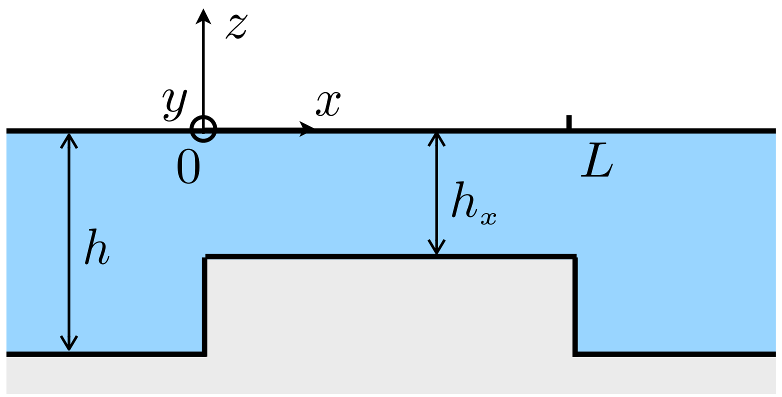

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Propagation of water waves over structured ridges

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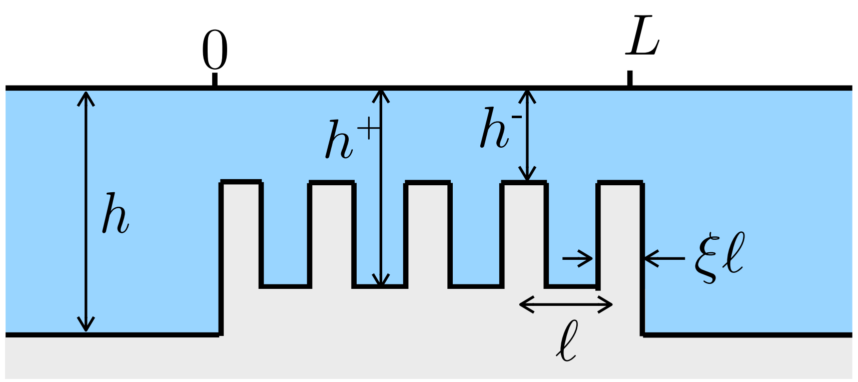

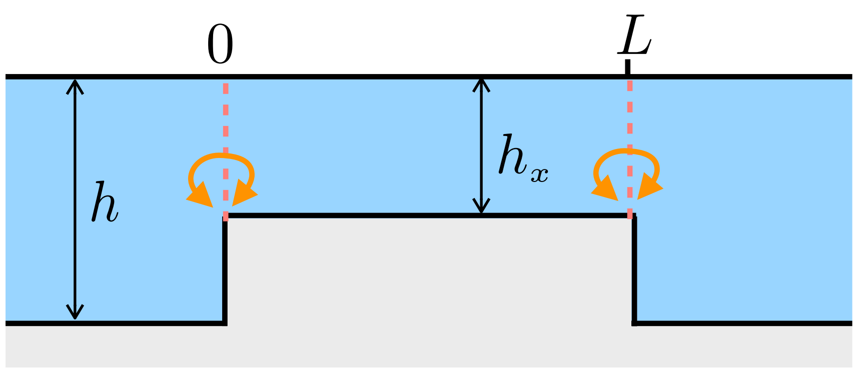

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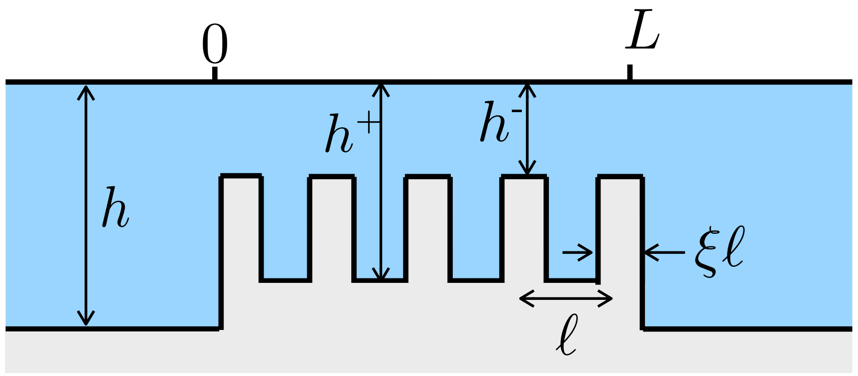
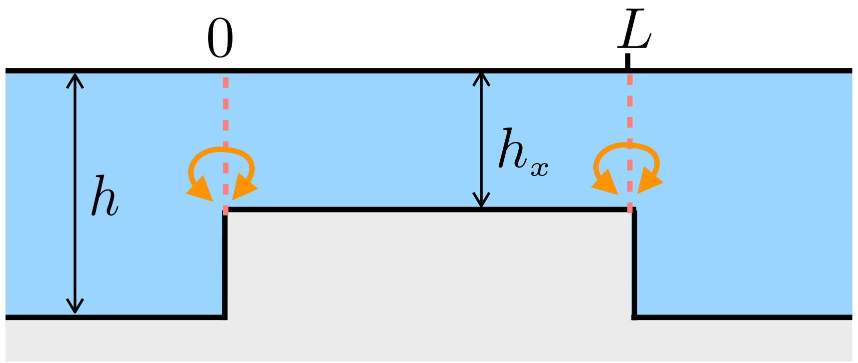
Propagation of water waves over structured ridges



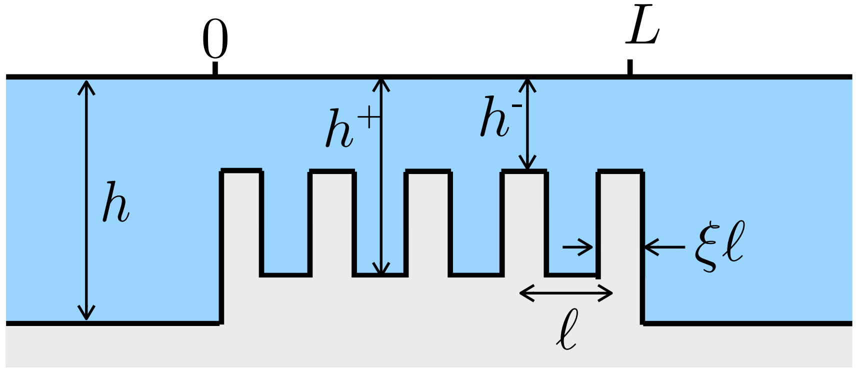


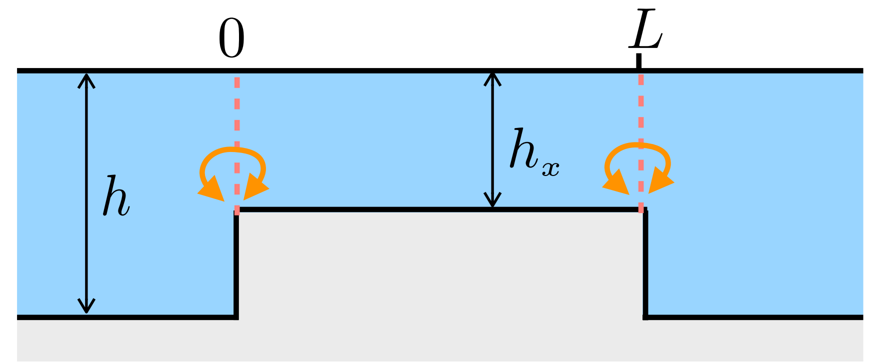
$$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla \phi \right) - \frac{1}{g} \partial_{tt} \phi = 0 \qquad h_x = \langle h^{-1} \rangle^{-1}, \quad h_y = \langle h \rangle$$

$$[\![\phi]\!] = [\![D]\!] = 0$$

$$(D = h \partial_x \phi, \quad D = h_x \partial_x \phi)$$

Propagation of water waves over structured ridges





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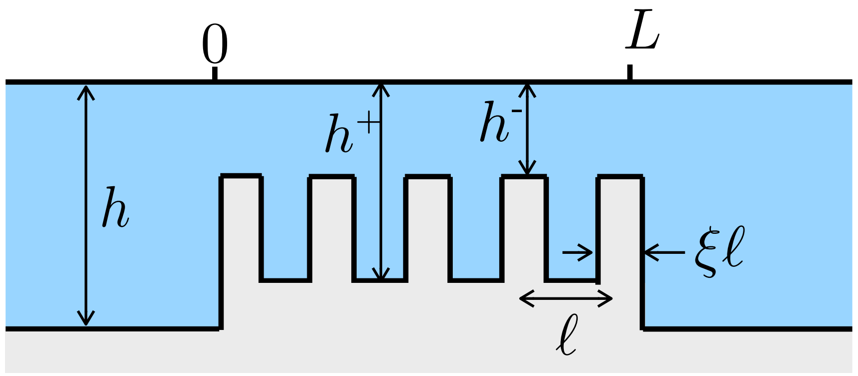
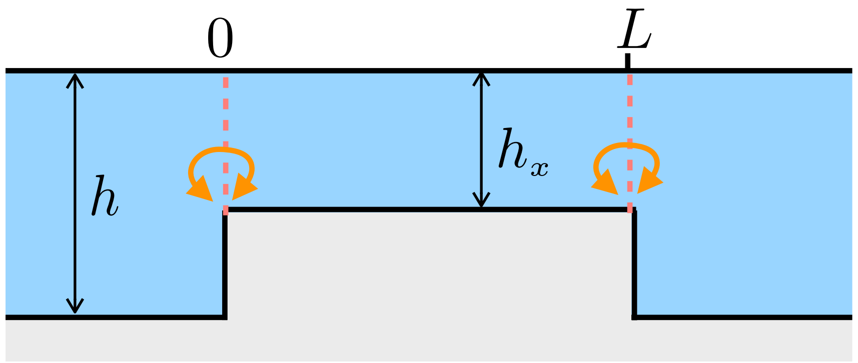
Propagation of water waves over structured ridges





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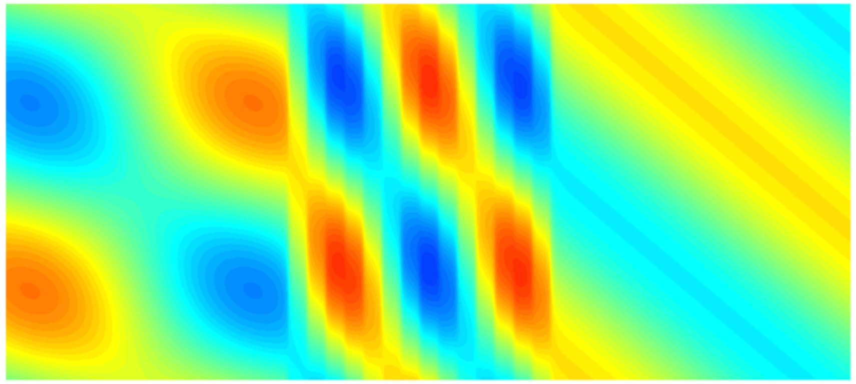
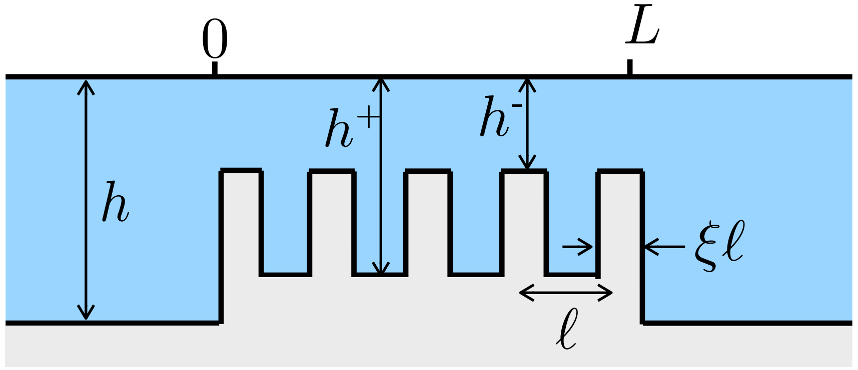
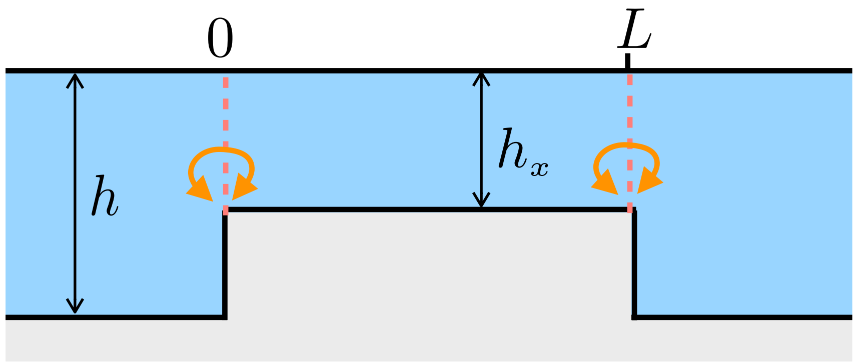


Propagation of water waves over structured ridges





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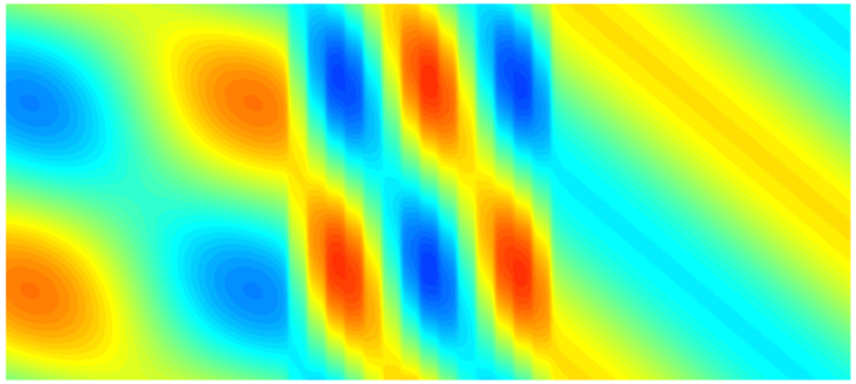
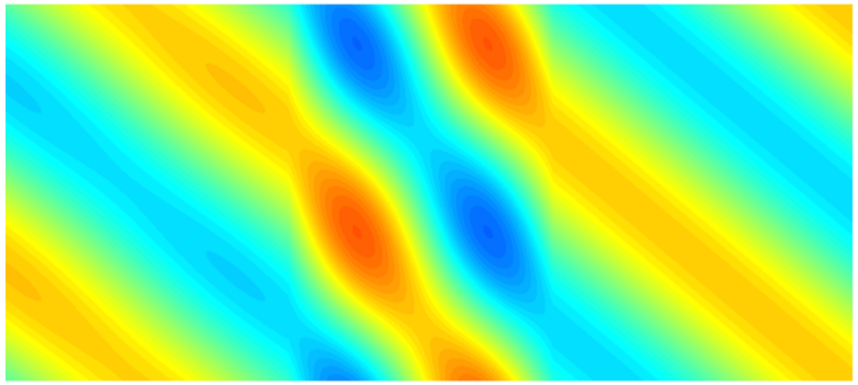
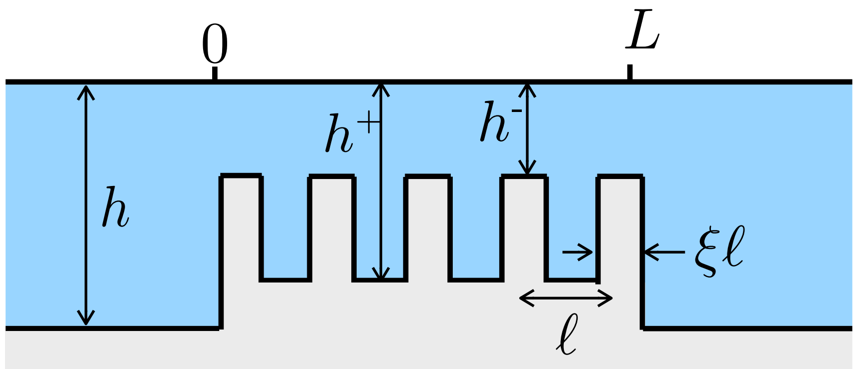
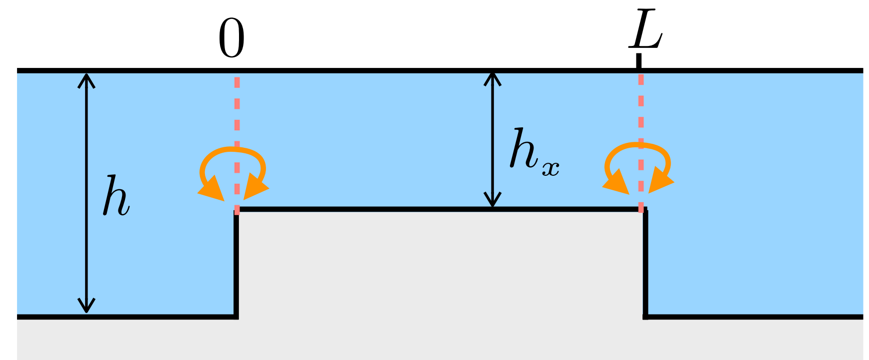




Propagation of water waves over structured ridges





$$\nabla\cdot\left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix}\nabla\phi\right) - \frac{1}{g}\partial_{tt}\phi = 0 \qquad h_x = \langle h^{-1}\rangle^{-1}, \quad h_y = \langle h\rangle$$

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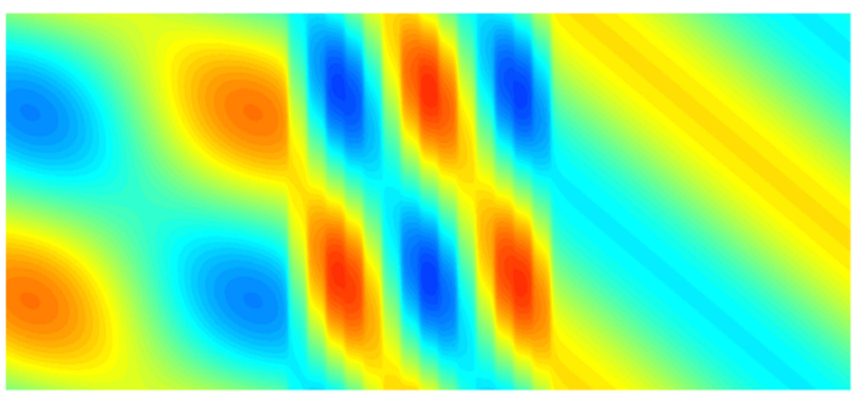
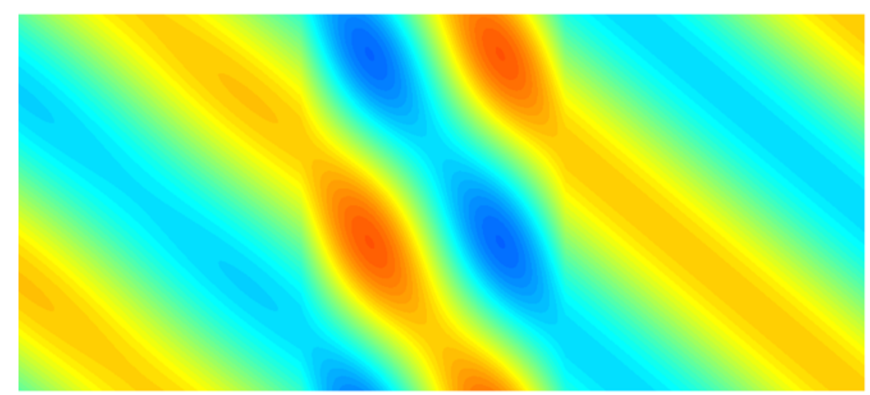
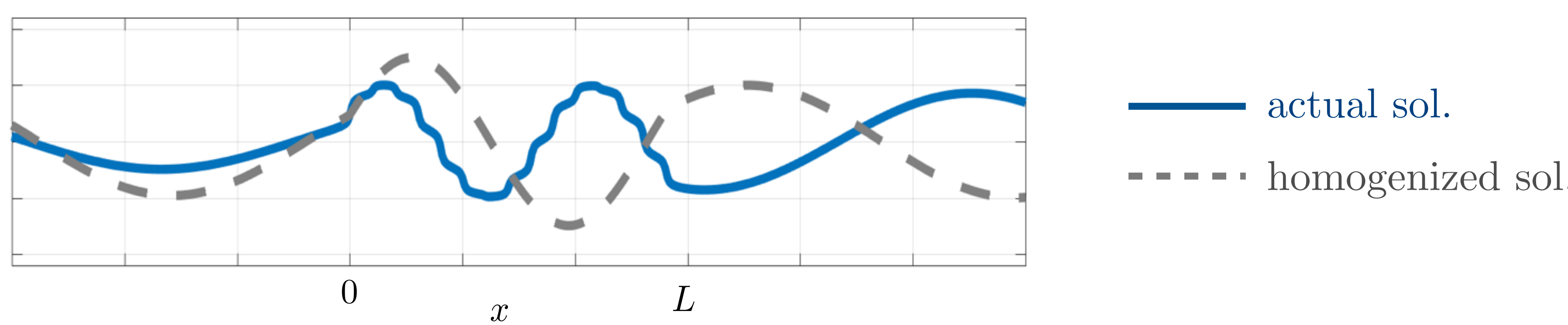
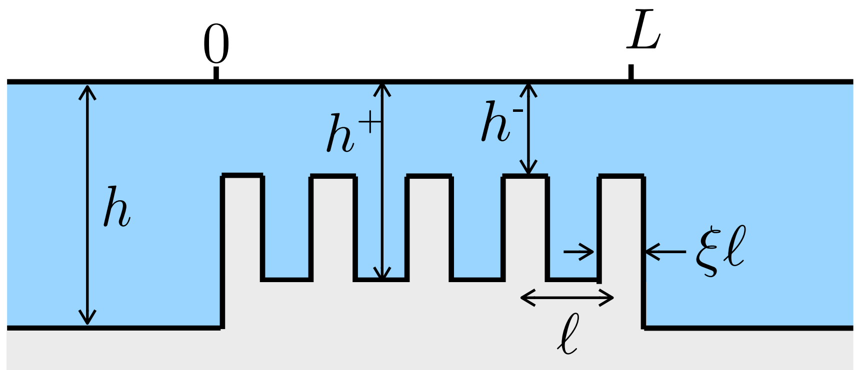
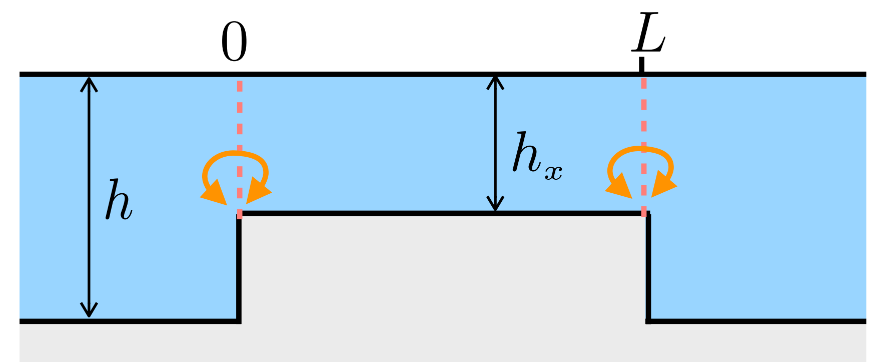






Propagation of water waves over structured ridges





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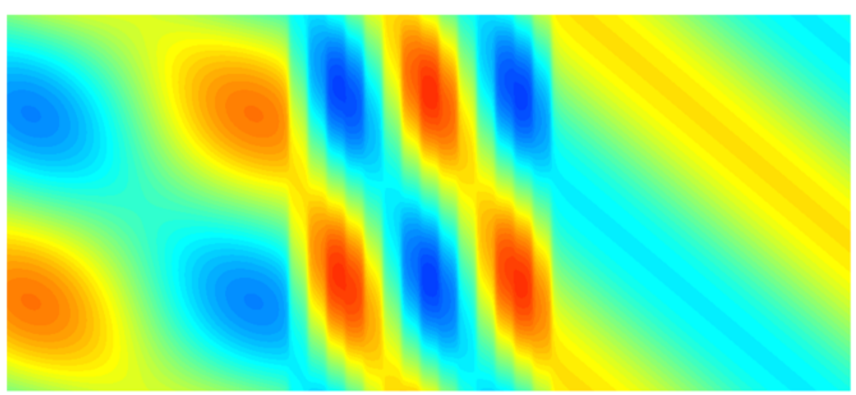
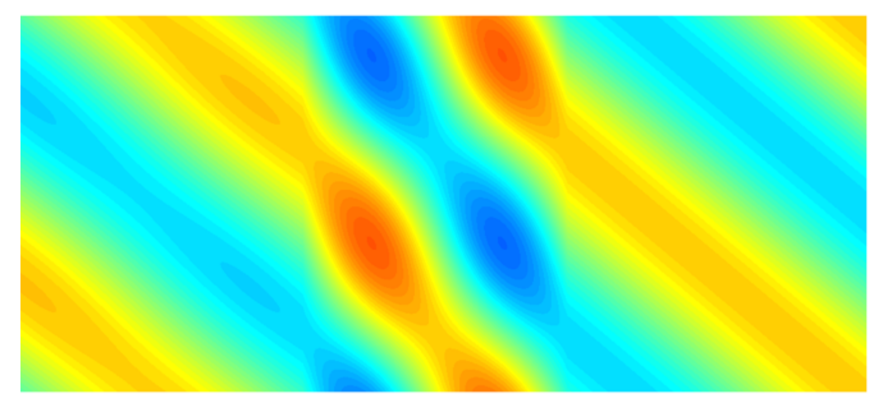
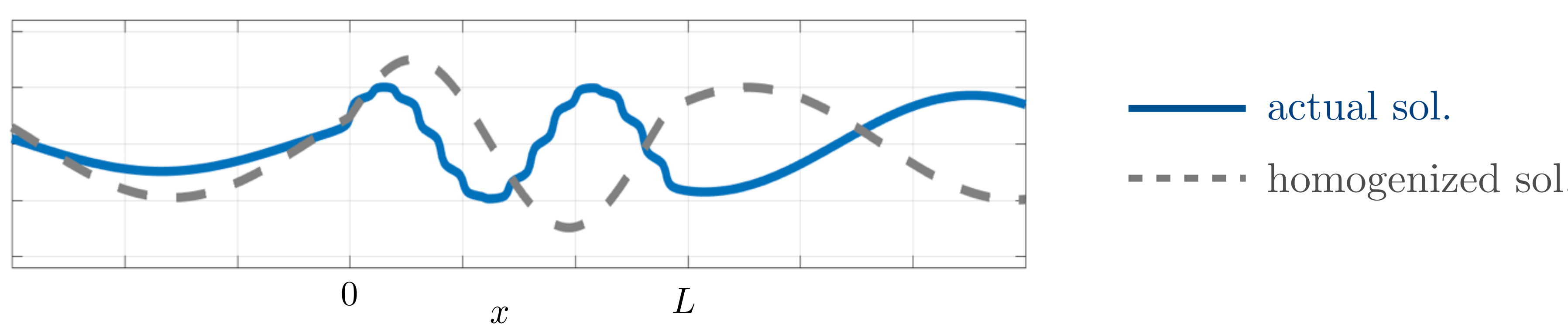
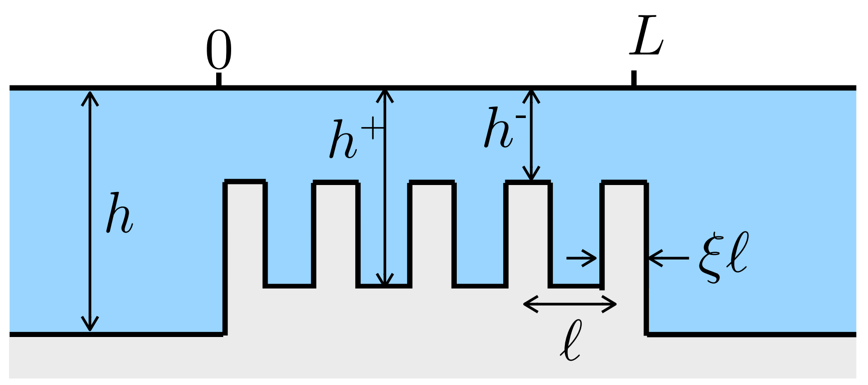
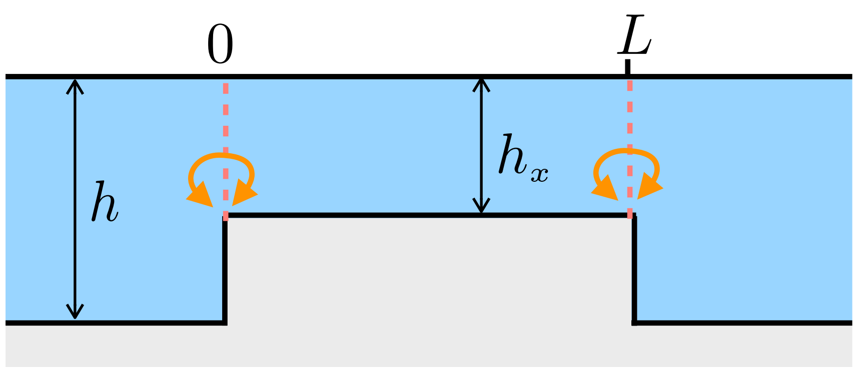






Propagation of water waves over structured ridges

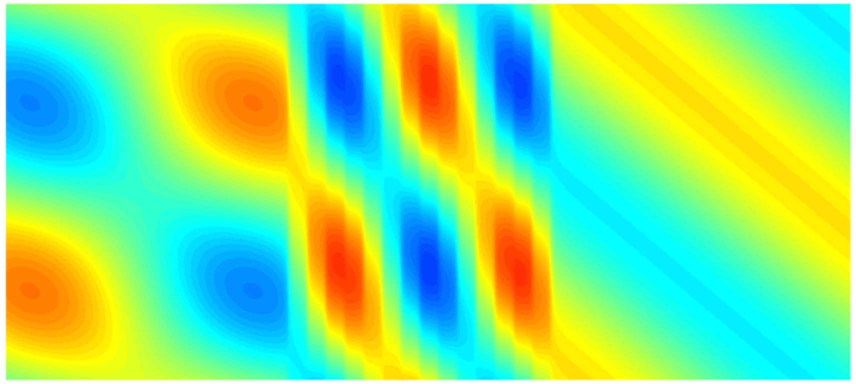


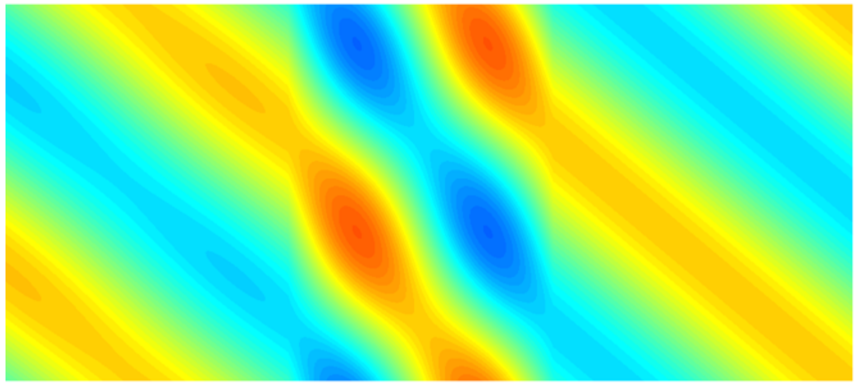


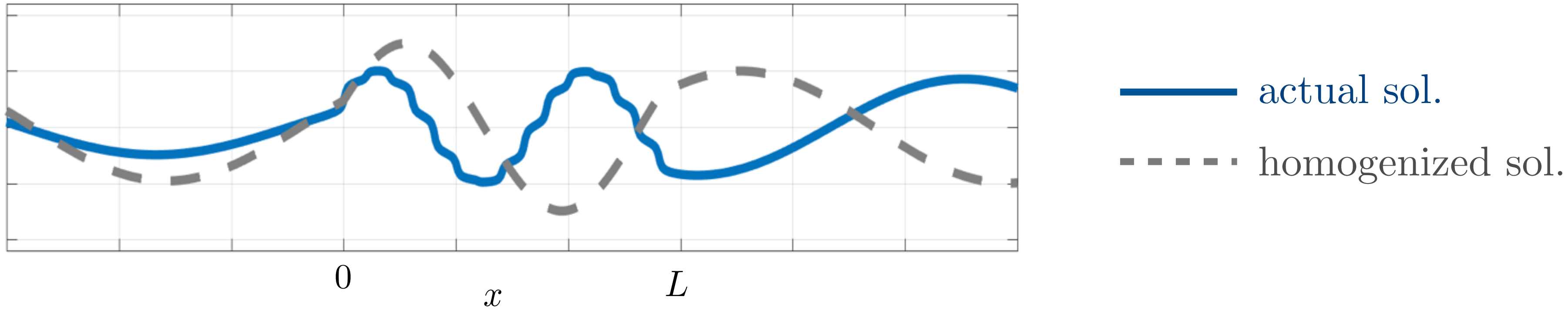
$$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla \phi \right) - \frac{1}{g} \partial_{tt} \phi = 0 \qquad \boxed{h_x = \langle h^{-1} \rangle^{-1},} \quad h_y = \langle h \rangle$$

$$h_x < \langle h^{-1} \rangle^{-1},$$

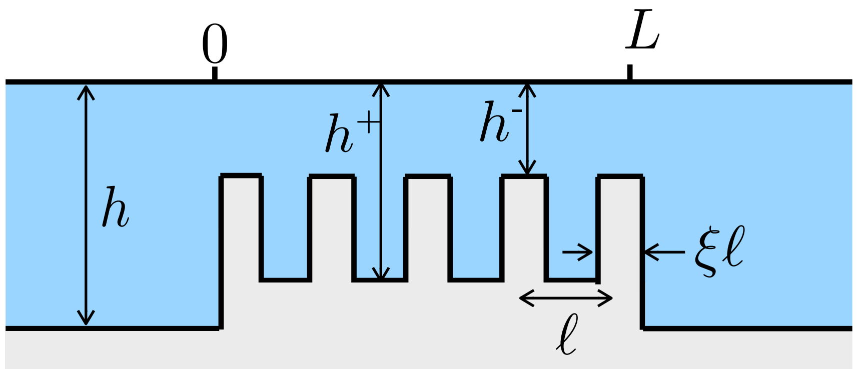
$$[\![\phi]\!] = 0, \quad [\![D]\!] = 0$$

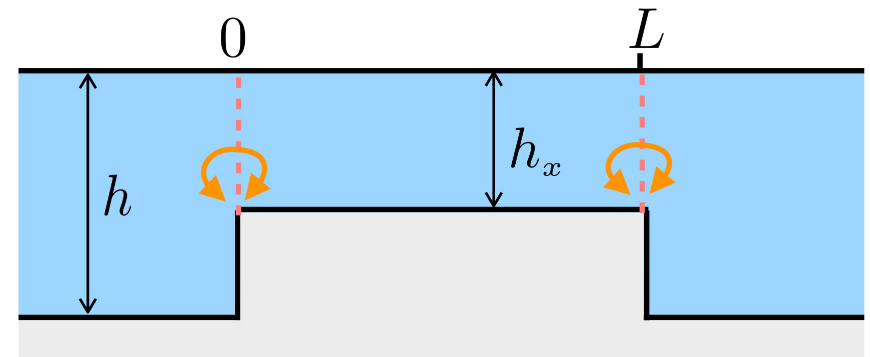






Propagation of water waves over structured ridges

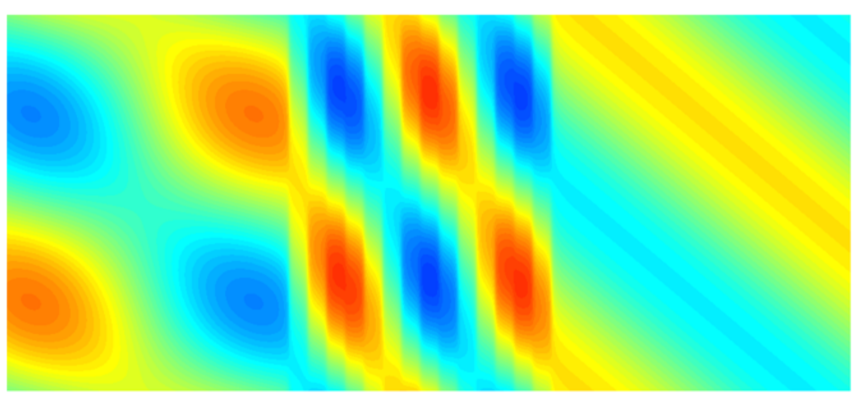


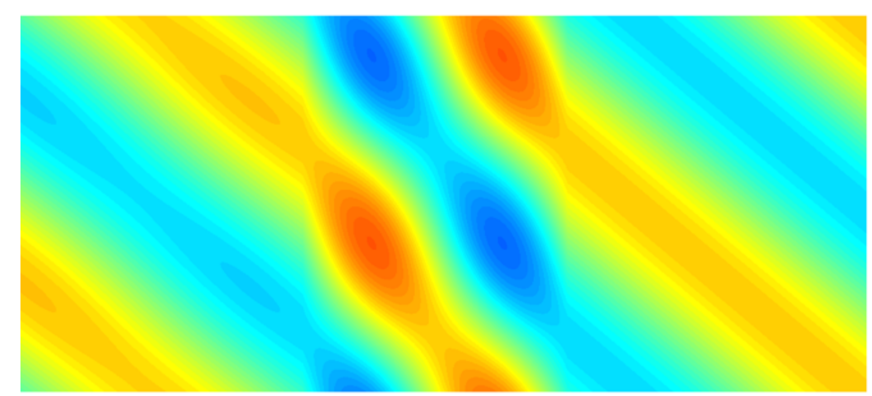


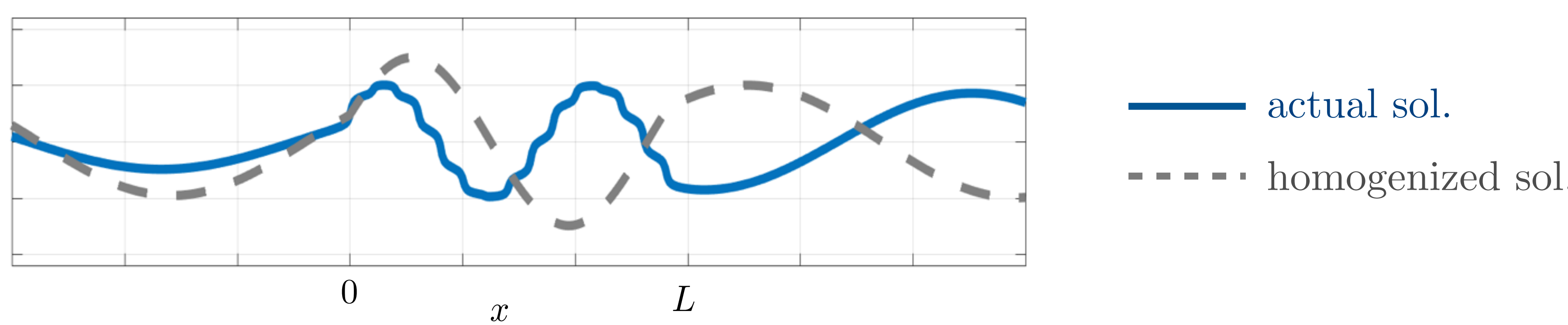
$$\nabla\cdot\left(\begin{pmatrix} h_x & 0\\ 0 & h_y\end{pmatrix}\nabla\phi\right) - \frac{1}{g}\partial_{tt}\phi = 0 \qquad h_x = \langle h^{-1}\rangle^{-1}, \quad h_y = \langle h\rangle$$

$$[\![\phi]\!] = 0, \quad [\![D]\!] = 0$$

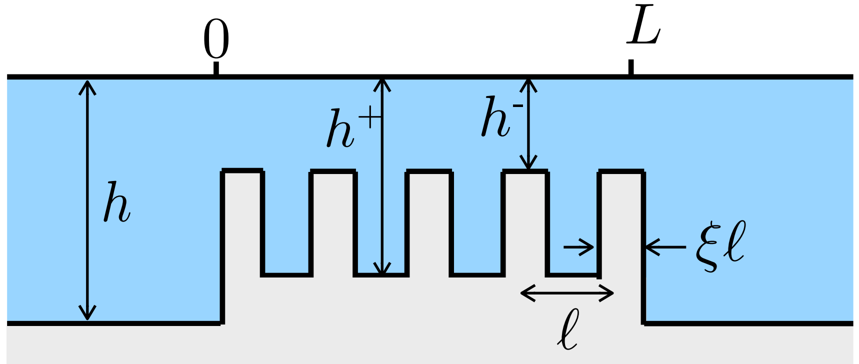
$h_x < \langle h^{-1}\rangle^{-1}$,
let us correct h_x

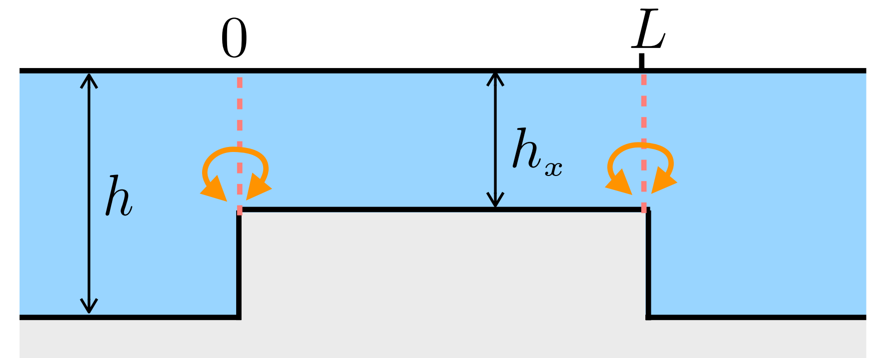






Propagation of water waves over structured ridges





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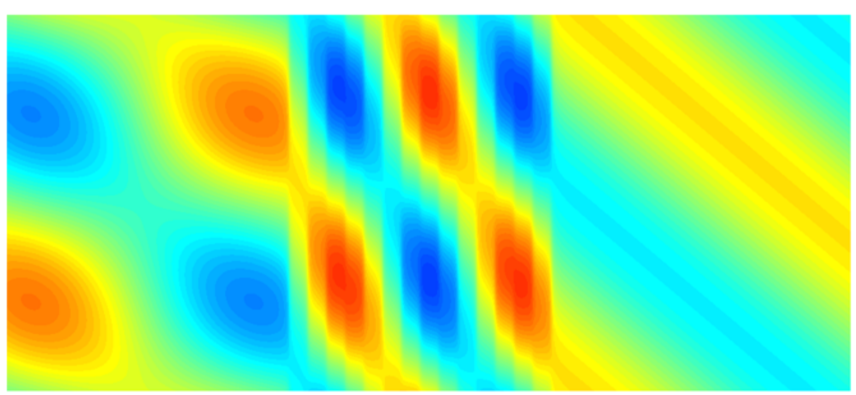
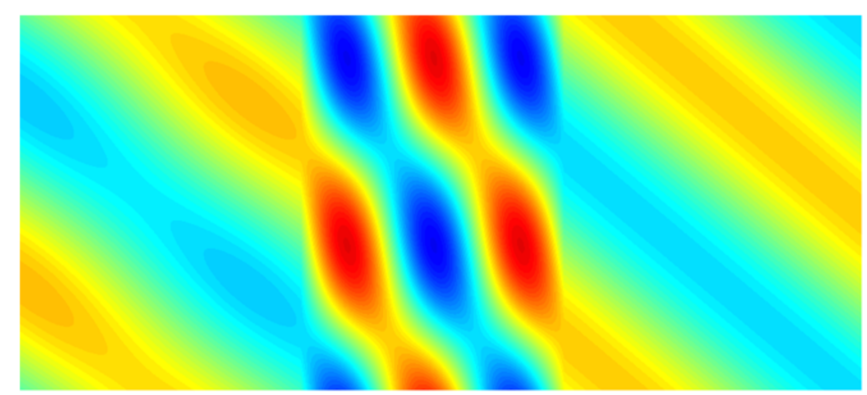
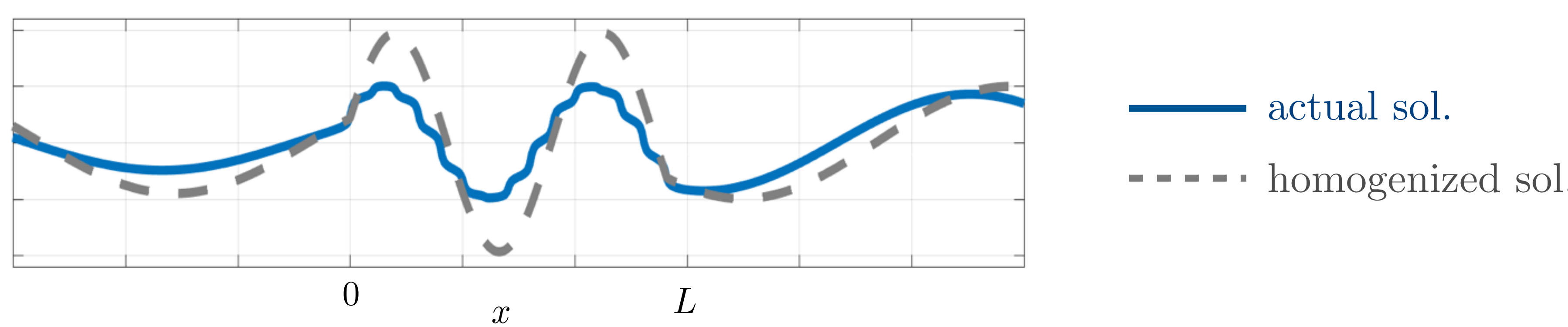
$$h_x < \langle h^{-1}\rangle^{-1},$$

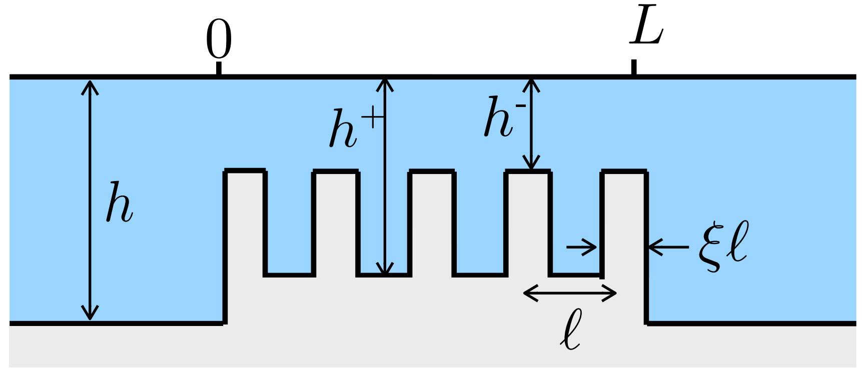
$$[\![\phi]\!] = 0, \quad [\![D]\!] = 0$$

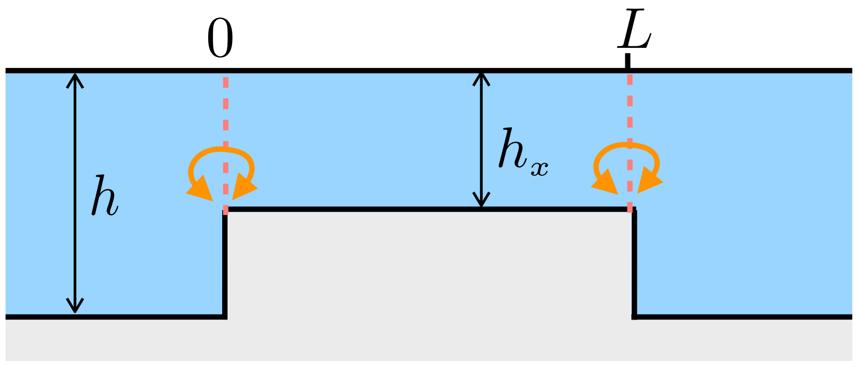






Propagation of water waves over structured ridges





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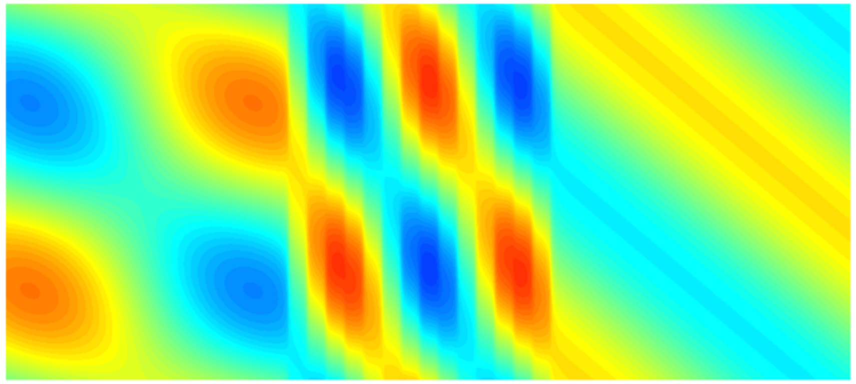
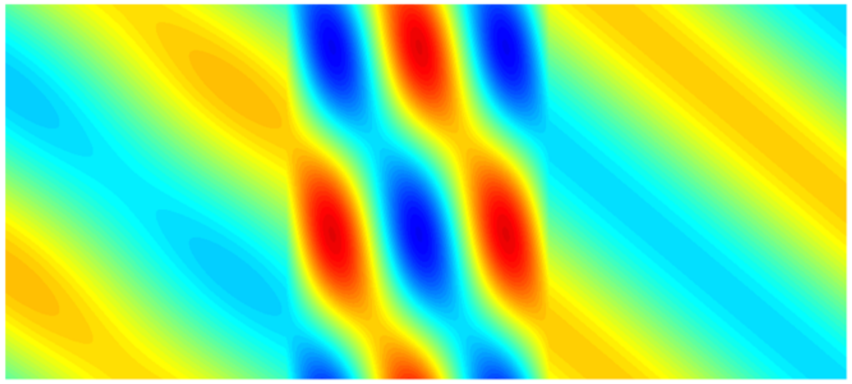
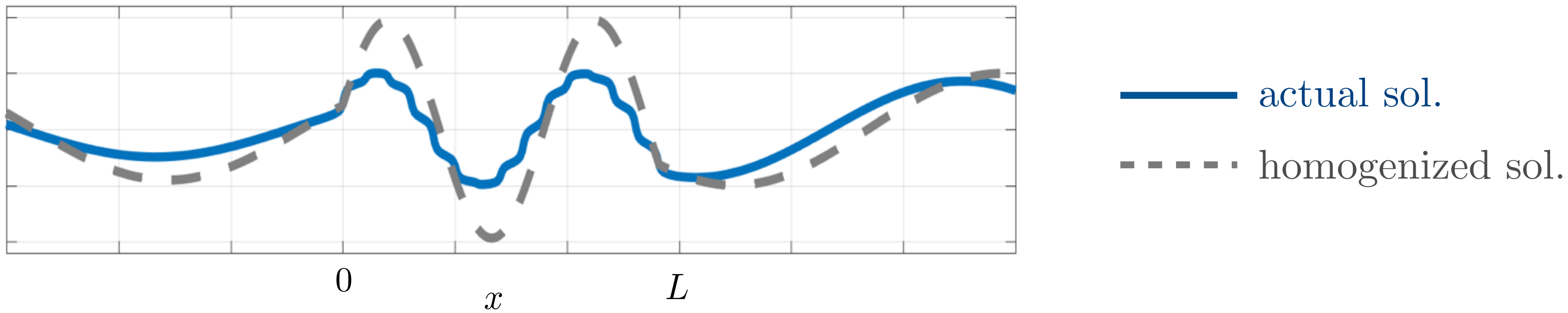
$h_x < \langle h^{-1} \rangle^{-1},$

$$[\![\phi]\!] = 0, \quad [\![D]\!] = 0$$

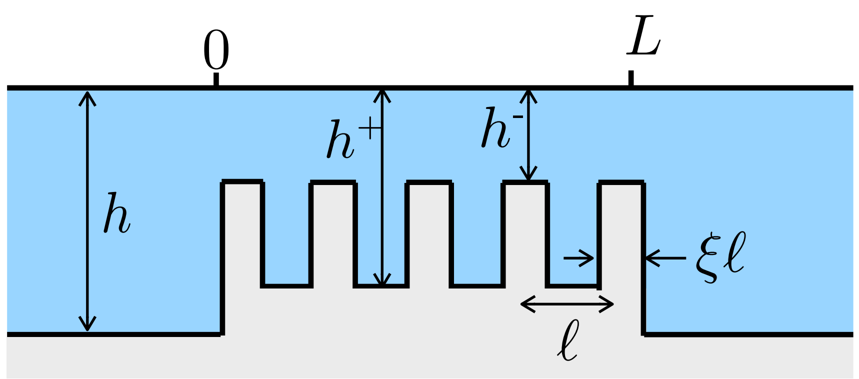
$[\![\phi]\!] \neq 0, \quad [\![D]\!] \neq 0$

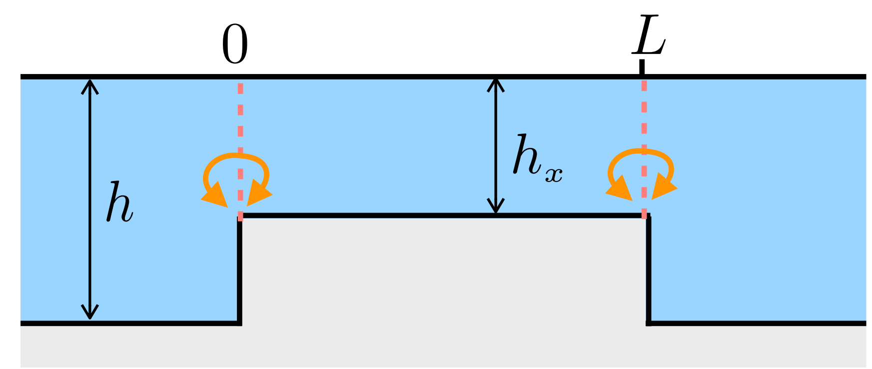






Propagation of water waves over structured ridges





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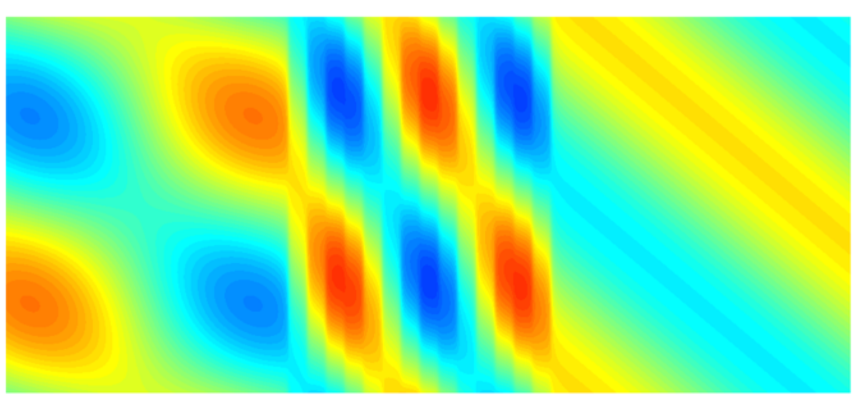
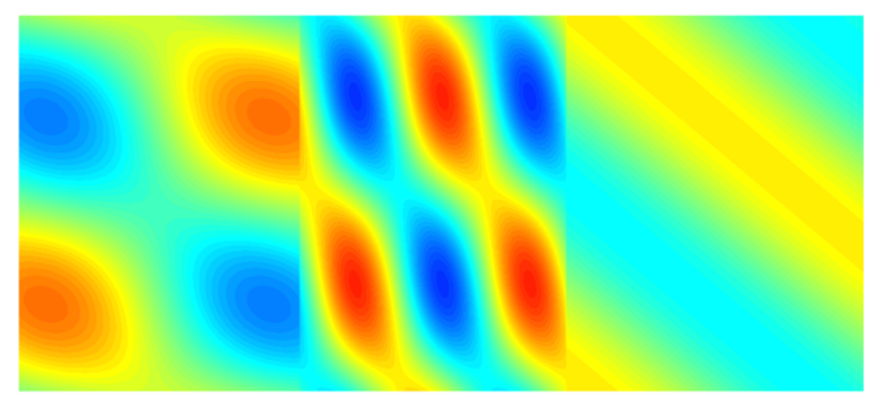
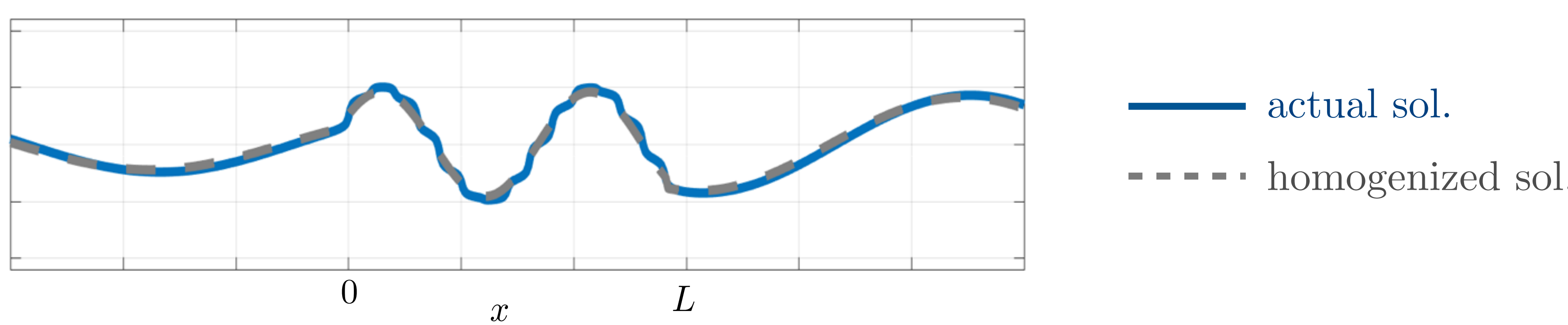
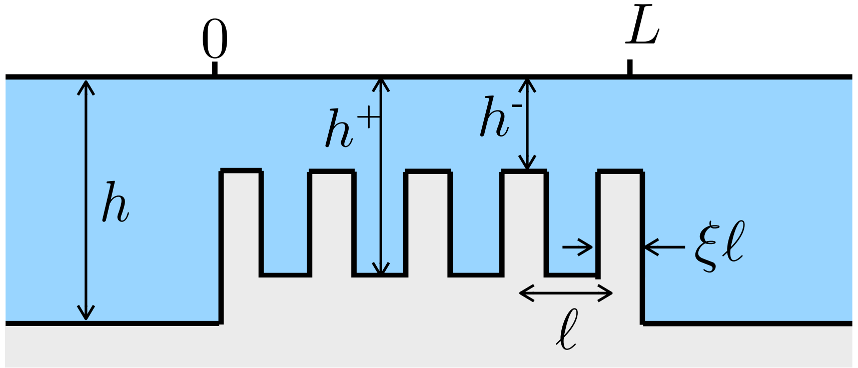
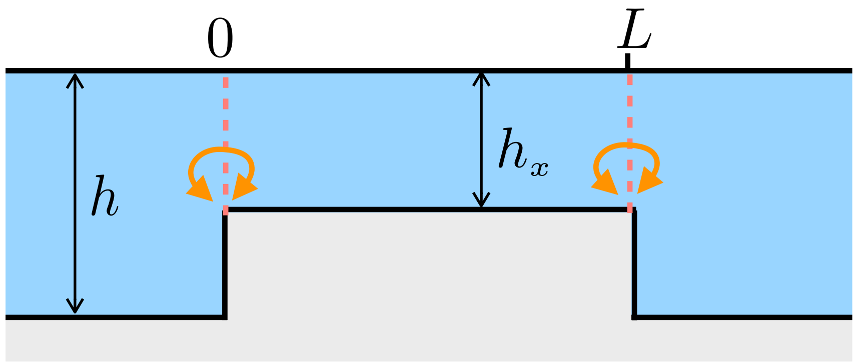






Propagation of water waves over structured ridges





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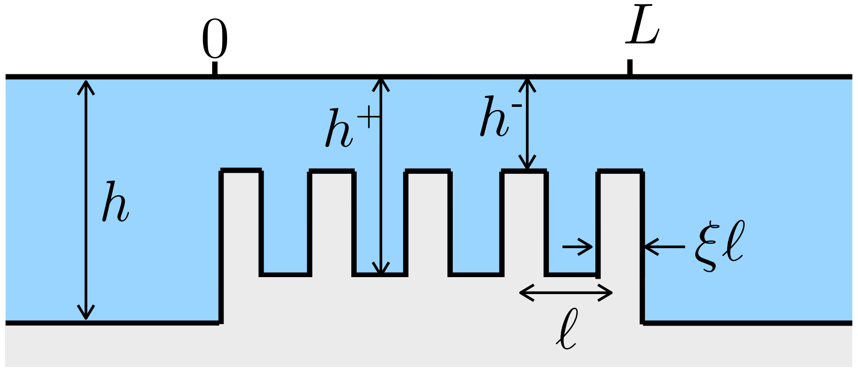
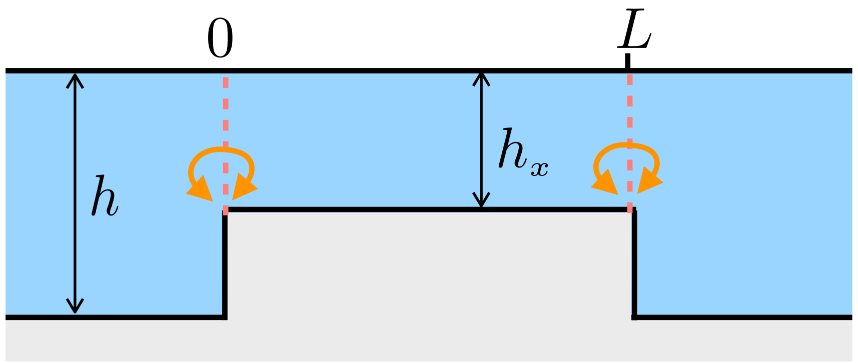
Propagation of water waves over structured ridges





$$\nabla\cdot\left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix}\nabla\phi\right) - \frac{1}{g}\partial_{tt}\phi = 0$$

$h_x = \langle h^{-1}\rangle^{-1}$, $\quad h_y = \langle h\rangle$

$h_x < \langle h^{-1}\rangle^{-1}$,

$[\![\phi]\!] = 0, \quad [\![D]\!] = 0$

$[\![\phi]\!] \neq 0, \quad [\![D]\!] \neq 0$

actual 3D problem

$\Phi(x, y, z)$

$\Delta\Phi = 0,$

$\nabla\Phi\cdot\mathbf{n} = 0 \qquad \partial_z\Phi = -\frac{1}{g}\partial_{tt}\Phi$

↓ 3D to 2D reduction

2D problem (shallow water app.)

$\phi(x, y)$

$$\nabla\cdot(h(x)\nabla\phi) - \frac{1}{g}\partial_{tt}\phi = 0$$

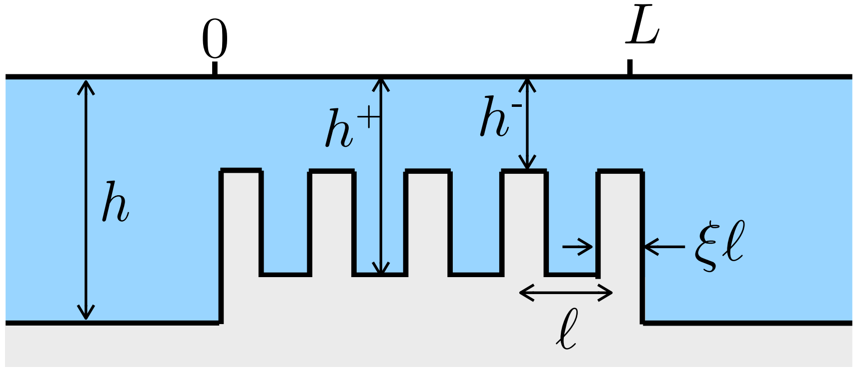
→ homogenization

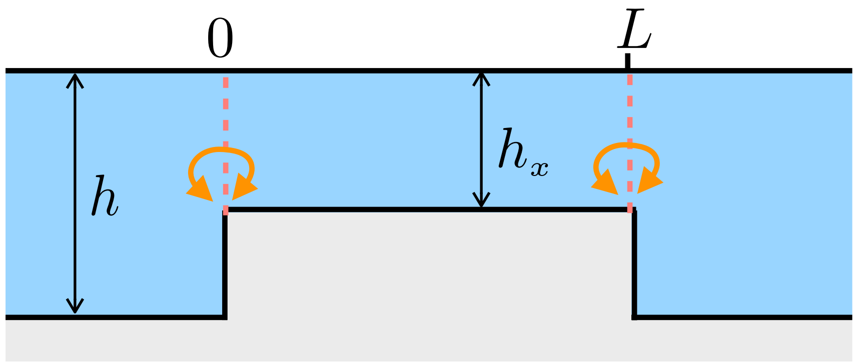
2D homogenized problem

$$\nabla\cdot\left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix}\nabla\phi\right) - \frac{1}{g}\partial_{tt}\phi = 0$$

$(h_x = \langle h^{-1}\rangle^{-1}, \quad h_y = \langle h\rangle)$

Propagation of water waves over structured ridges





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actual 3D problem

$\Phi(x,y,z)$

$\Delta\Phi = 0,$

$\nabla\Phi\cdot\mathbf{n} = 0 \qquad \partial_z\Phi = -\frac{1}{g}\partial_{tt}\Phi$

unfair step

↓ 3D to 2D reduction

2D problem (shallow water app.)

$\phi(x,y)$

$$\nabla\cdot(h(x)\nabla\phi) - \frac{1}{g}\partial_{tt}\phi = 0$$

→ homogenization

2D homogenized problem

$$\nabla\cdot\left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix}\nabla\phi\right) - \frac{1}{g}\partial_{tt}\phi = 0$$

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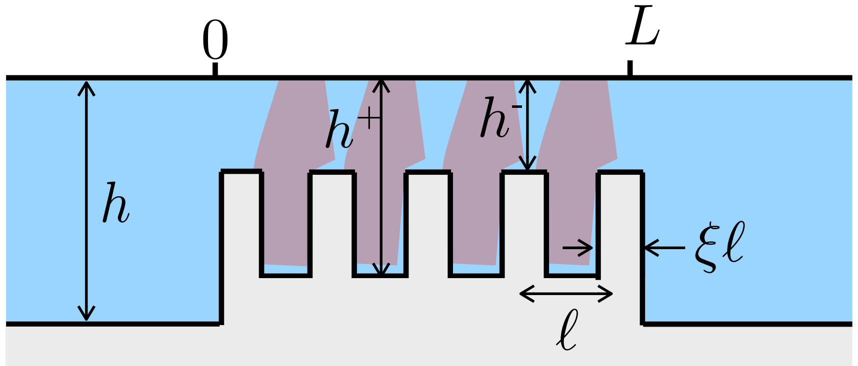
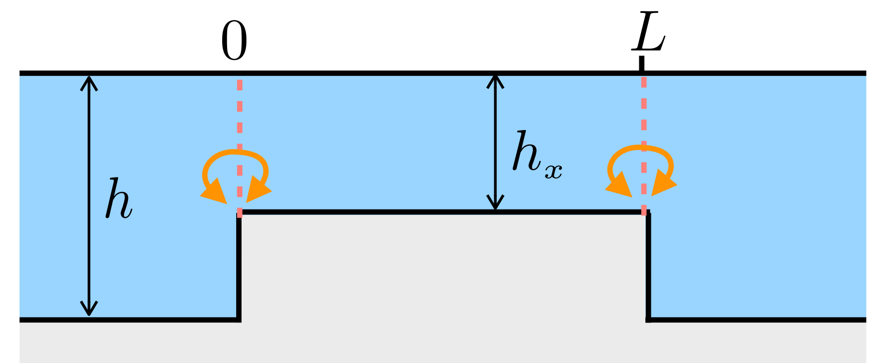
Propagation of water waves over structured ridges





$$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla \phi \right) - \frac{1}{g}\partial_{tt}\phi = 0 \qquad h_x = \langle h^{-1} \rangle^{-1}, \quad h_y = \langle h \rangle$$

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actual 3D problem

$\Phi(x, y, z)$

$\Delta \Phi = 0,$

$\nabla \Phi \cdot \mathbf{n} = 0 \qquad \partial_z \Phi = -\frac{1}{g}\partial_{tt}\Phi$

↓ 3D to 2D reduction ← unfair step

2D problem (shallow water app.)

$\phi(x, y)$

$\nabla \cdot (h(x)\nabla \phi) - \frac{1}{g}\partial_{tt}\phi = 0$

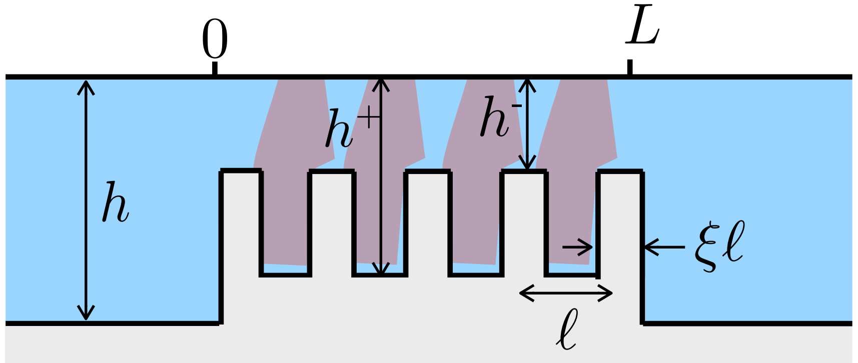
→ homogenization →

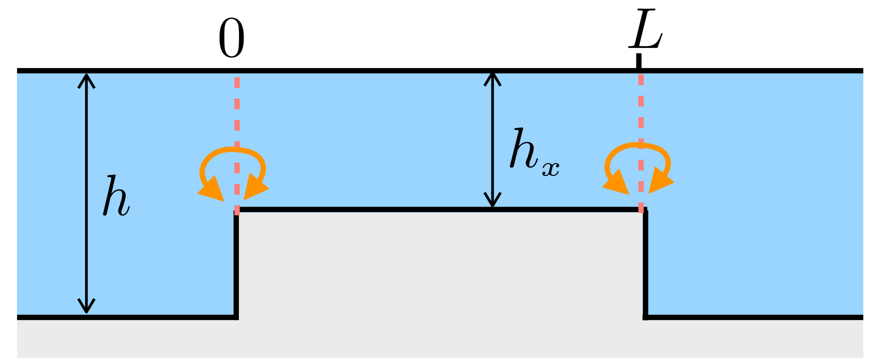
2D homogenized problem

$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla \phi \right) - \frac{1}{g}\partial_{tt}\phi = 0$

$(h_x = \langle h^{-1} \rangle^{-1}, \quad h_y = \langle h \rangle)$

Propagation of water waves over structured ridges





$$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla \phi \right) - \frac{1}{g} \partial_{tt} \phi = 0 \qquad h_x = \langle h^{-1} \rangle^{-1}, \quad h_y = \langle h \rangle$$

$h_x < \langle h^{-1} \rangle^{-1},$

$[\![\phi]\!] = 0, \quad [\![D]\!] = 0$

$[\![\phi]\!] \neq 0, \quad [\![D]\!] \neq 0$

actual 3D problem

$\Phi(x, y, z)$

$\Delta \Phi = 0,$

$\nabla \Phi \cdot \mathbf{n} = 0 \qquad \partial_z \Phi = -\frac{1}{g} \partial_{tt} \Phi$

unfair step → omits the evanescent field

3D to 2D reduction

2D problem (shallow water app.)

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$\nabla \cdot (h(x) \nabla \phi) - \frac{1}{g} \partial_{tt} \phi = 0$

homogenization

2D homogenized problem

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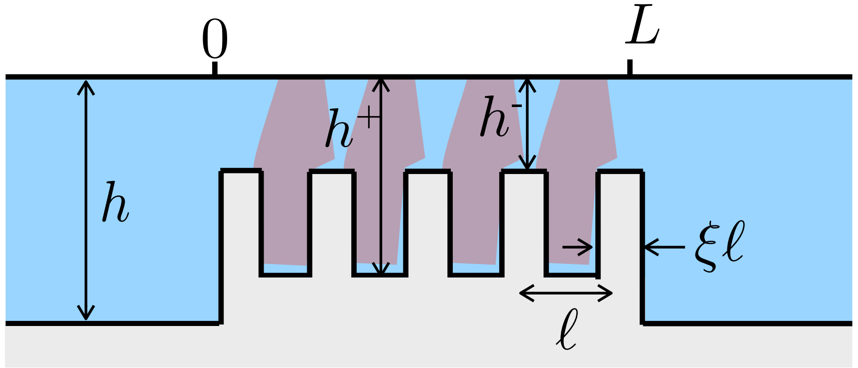
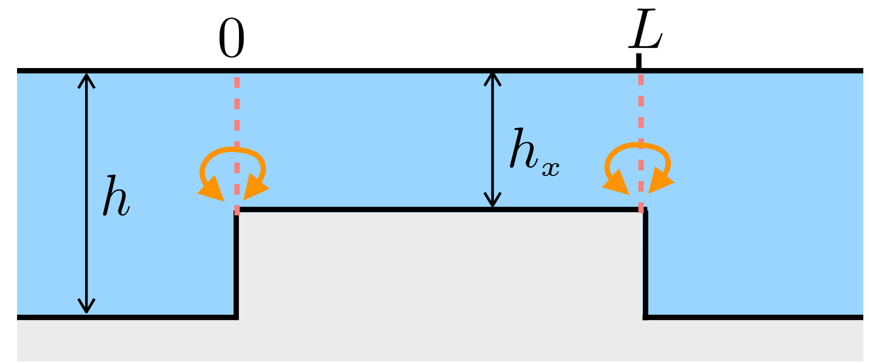
Propagation of water waves over structured ridges





$$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla\phi \right) - \frac{1}{g}\partial_{tt}\phi = 0 \quad h_x = \langle h^{-1}\rangle^{-1}, \quad h_y = \langle h\rangle$$

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↓ 3D to 2D reduction

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you have to conduct the homogenization procedure on the 3D pb

2D problem (shallow water app.)

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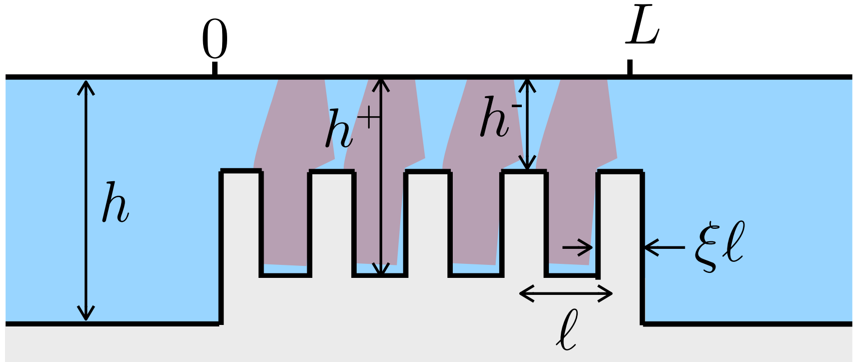
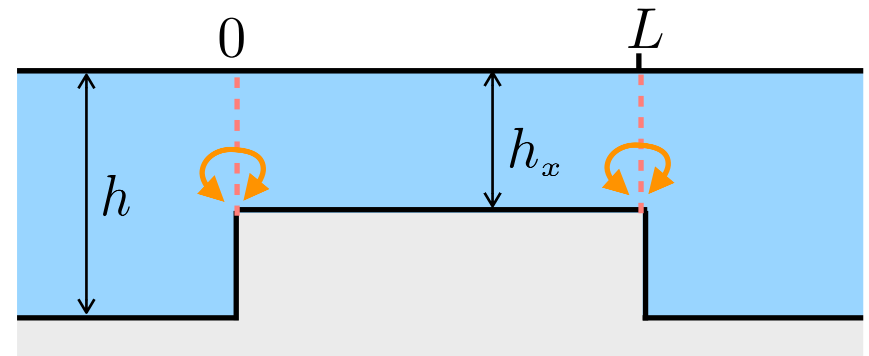
Propagation of water waves over structured ridges





$$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla\phi \right) - \frac{1}{g}\partial_{tt}\phi = 0 \qquad h_x = \langle h^{-1}\rangle^{-1}, \quad h_y = \langle h \rangle$$

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$[\![\phi]\!] \neq 0, \quad [\![D]\!] \neq 0$

more subtle (less known)

actual 3D problem

$\Phi(x, y, z)$

$\Delta\Phi = 0,$

$\nabla\Phi \cdot \mathbf{n} = 0 \qquad \partial_z\Phi = -\frac{1}{g}\partial_{tt}\Phi$

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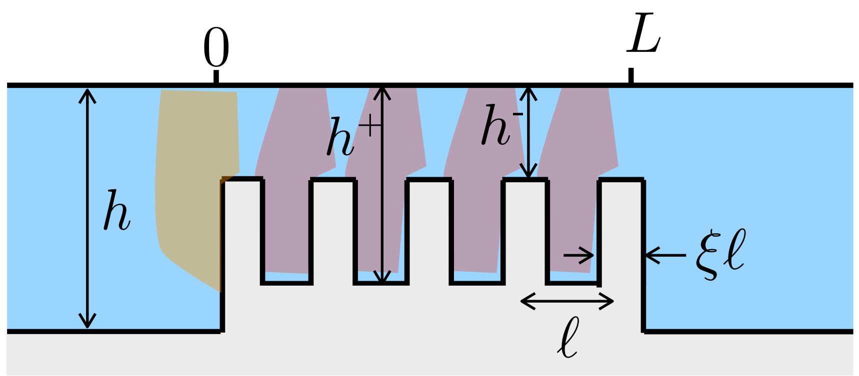
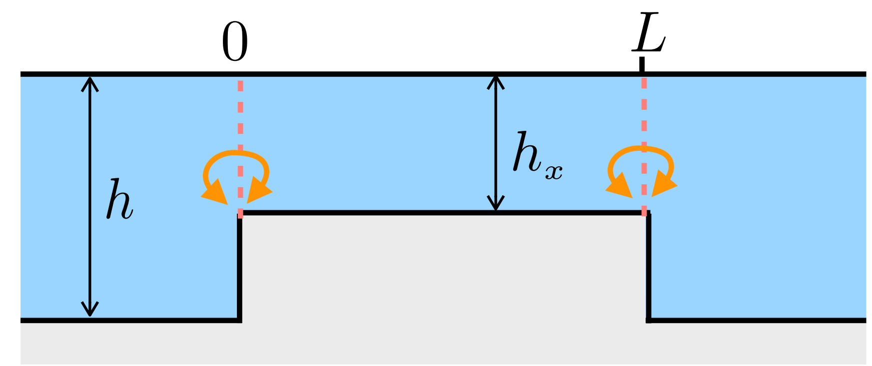
Propagation of water waves over structured ridges





$$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla \phi \right) - \frac{1}{g} \partial_{tt} \phi = 0 \quad h_x = \langle h^{-1} \rangle^{-1}, \quad h_y = \langle h \rangle$$

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actual 3D problem

$$\Phi(x, y, z)$$

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2D problem (shallow water app.)

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2D homogenized problem

$$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla \phi \right) - \frac{1}{g} \partial_{tt} \phi = 0$$

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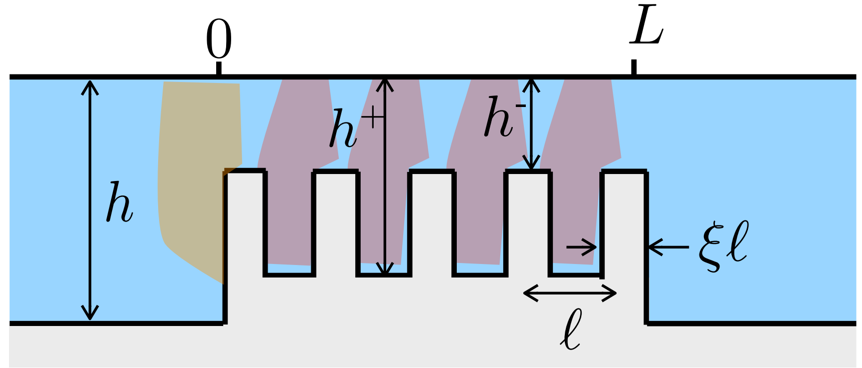
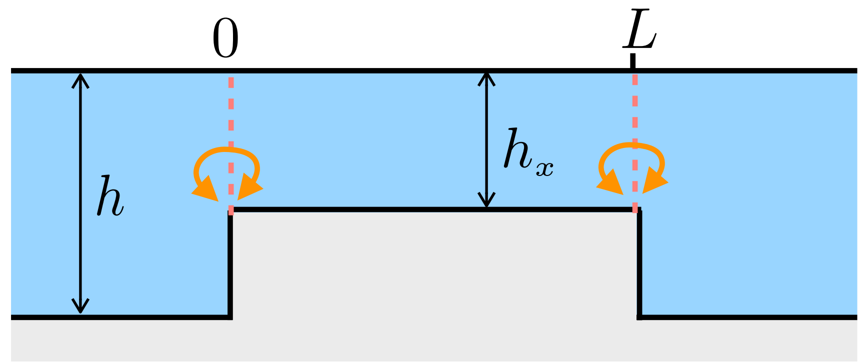
Propagation of water waves over structured ridges





$$\nabla\cdot\left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix}\nabla\phi\right) - \frac{1}{g}\partial_{tt}\phi = 0 \qquad h_x = \langle h^{-1}\rangle^{-1}, \quad h_y = \langle h\rangle$$

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$[\![\phi]\!] = 0, \quad [\![D]\!] = 0$

$[\![\phi]\!] \neq 0, \quad [\![D]\!] \neq 0$

more subtle (less known)

these b. layer effects require a specific treatment

actual 3D problem

$\Phi(x, y, z)$

$\Delta\Phi = 0,$

$\nabla\Phi\cdot\mathbf{n} = 0 \qquad \partial_z\Phi = -\frac{1}{g}\partial_{tt}\Phi$

↓ 3D to 2D reduction

unfair step → omits the evanescent field

you have to conduct the homogenization procedure on the 3D pb

2D problem (shallow water app.)

$\phi(x, y)$

$\nabla\cdot(h(x)\nabla\phi) - \frac{1}{g}\partial_{tt}\phi = 0$

→ homogenization

2D homogenized problem

$$\nabla\cdot\left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix}\nabla\phi\right) - \frac{1}{g}\partial_{tt}\phi = 0$$

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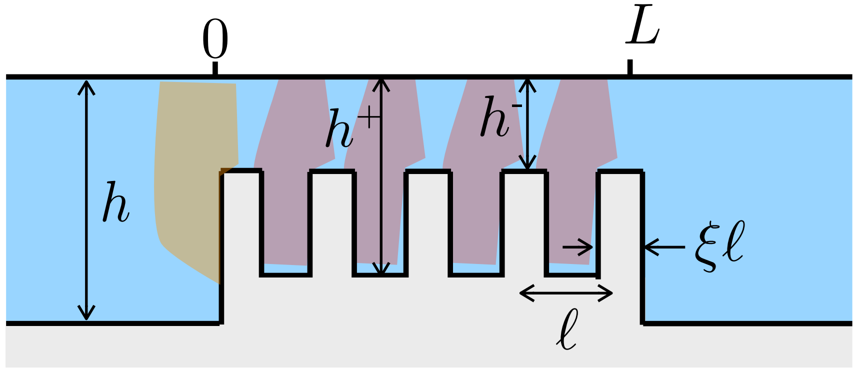
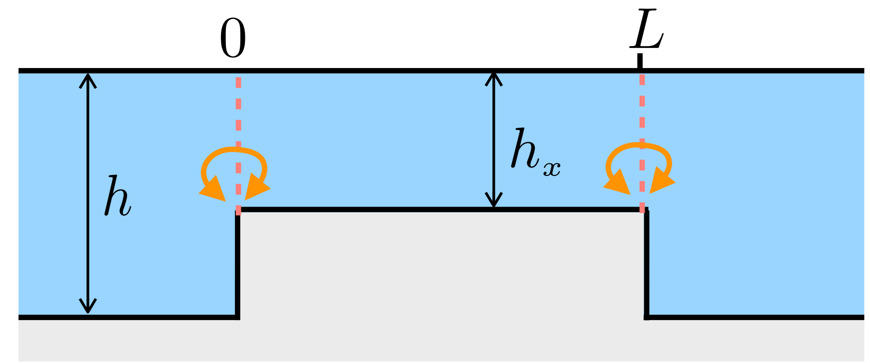
Propagation of water waves over structured ridges





$$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla \phi \right) - \frac{1}{g} \partial_{tt} \phi = 0$$

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more subtle (less known)

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actual 3D problem

$\Phi(x, y, z)$

$\Delta \Phi = 0,$

$\nabla \Phi \cdot \mathbf{n} = 0 \qquad \partial_z \Phi = -\frac{1}{g} \partial_{tt} \Phi$

↓ 3D to 2D reduction

1 in the bulk of the structure

unfair step → omits the evanescent field

you have to conduct the homogenization procedure on the 3D pb

2D problem (shallow water app.)

$\phi(x, y)$

$$\nabla \cdot (h(x) \nabla \phi) - \frac{1}{g} \partial_{tt} \phi = 0$$

→ homogenization →

2D homogenized problem

$$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla \phi \right) - \frac{1}{g} \partial_{tt} \phi = 0$$

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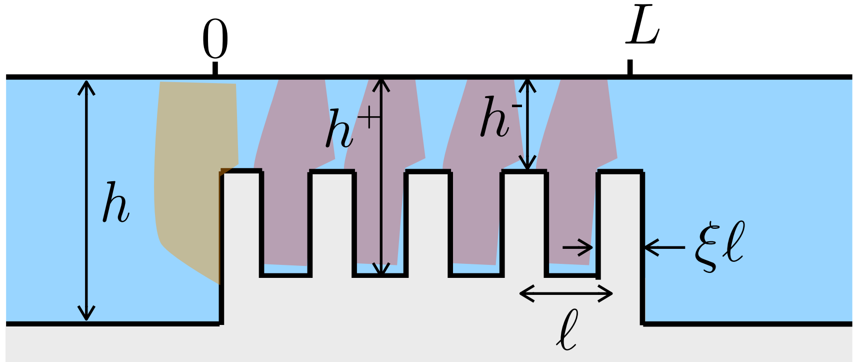
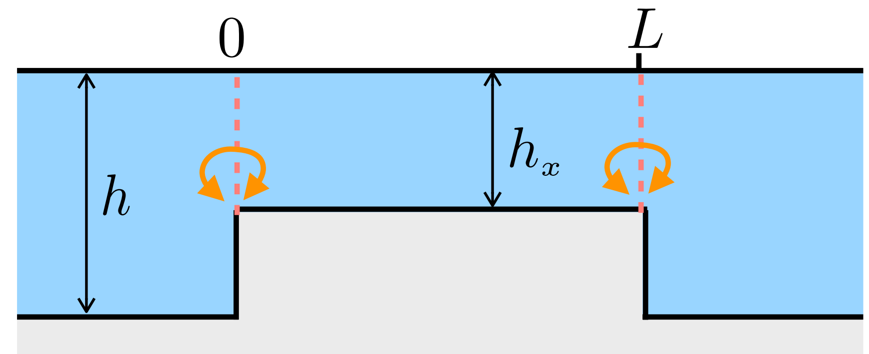
Propagation of water waves over structured ridges





$$\nabla\cdot\left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix}\nabla\phi\right) - \frac{1}{g}\partial_{tt}\phi = 0$$

$h_x = \langle h^{-1}\rangle^{-1}$, $h_y = \langle h\rangle$

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$[\![\phi]\!] \neq 0, \quad [\![D]\!] \neq 0$

more subtle (less known)

these b. layer effects require a specific treatment

2 at the end boundaries of the structure

actual 3D problem

$\Phi(x, y, z)$

$\Delta\Phi = 0,$

$\nabla\Phi\cdot\mathbf{n} = 0 \qquad \partial_z\Phi = -\frac{1}{g}\partial_{tt}\Phi$

3D to 2D reduction

1 in the bulk of the structure

unfair step → omits the evanescent field

you have to conduct the homogenization procedure on the 3D pb

2D problem (shallow water app.)

$\phi(x, y)$

$$\nabla\cdot(h(x)\nabla\phi) - \frac{1}{g}\partial_{tt}\phi = 0$$

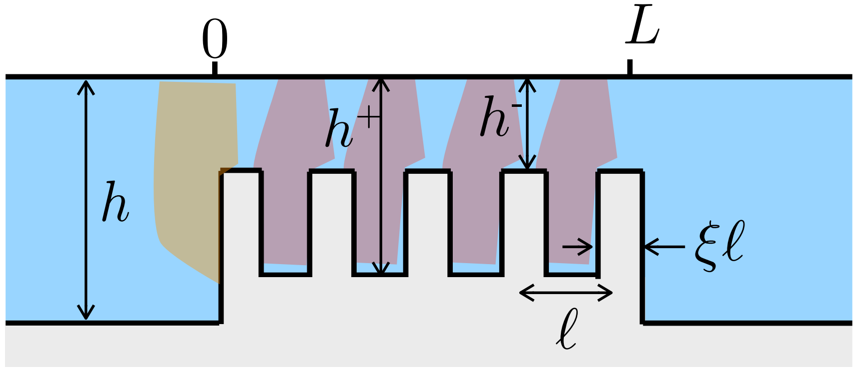
homogenization

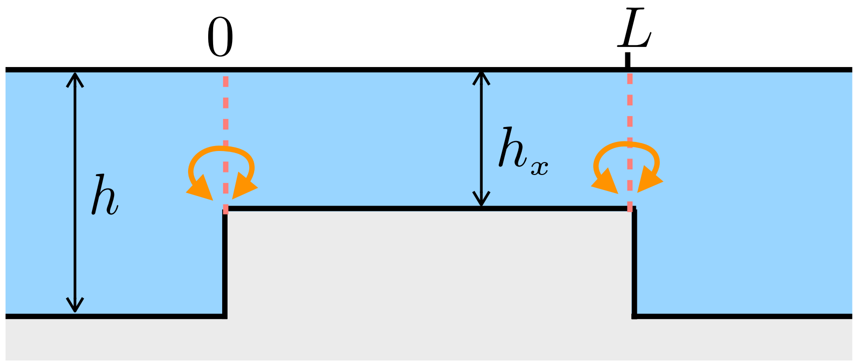
2D homogenized problem

$$\nabla\cdot\left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix}\nabla\phi\right) - \frac{1}{g}\partial_{tt}\phi = 0$$

$(h_x = \langle h^{-1}\rangle^{-1}, \quad h_y = \langle h\rangle)$

Propagation of water waves over structured ridges





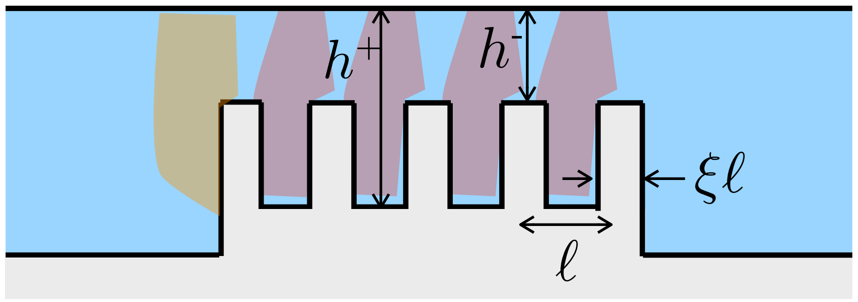
$$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla \phi \right) - \frac{1}{g} \partial_{tt} \phi = 0 \quad h_x < \langle h^{-1} \rangle^{-1}, \; h_y = \langle h \rangle$$

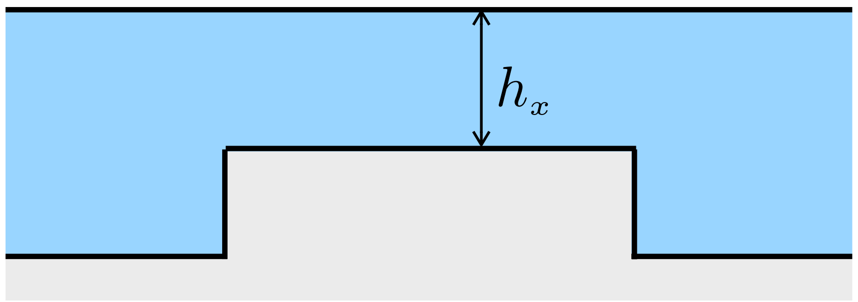
1 in the bulk of the structure

unfair step → omits the evanescent field

you have to conduct the homogenization procedure on the 3D pb

Propagation of water waves over structured ridges





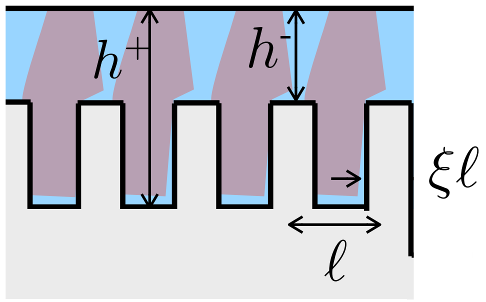
$$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla\phi \right) - \frac{1}{g}\partial_{tt}\phi = 0 \quad h_x < \langle h^{-1}\rangle^{-1}, \; h_y = \langle h \rangle$$

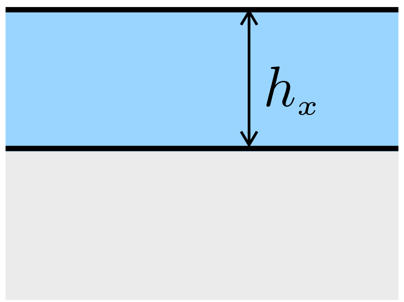
1 in the bulk of the structure

unfair step $\rightarrow$ omits the evanescent field

you have to conduct the homogenization procedure on the 3D pb

Propagation of water waves over structured ridges





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Propagation of water waves over structured ridges

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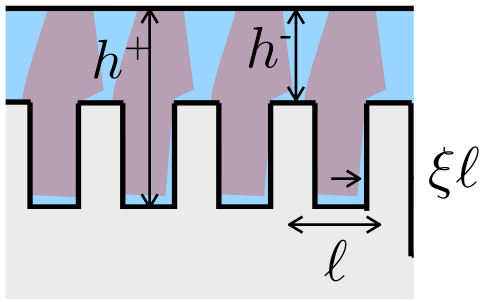

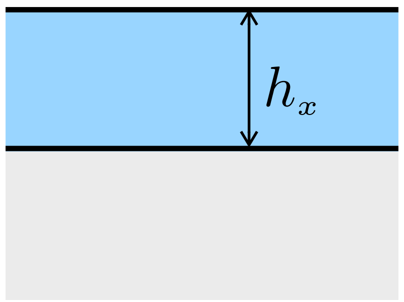

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Propagation of water waves over structured ridges

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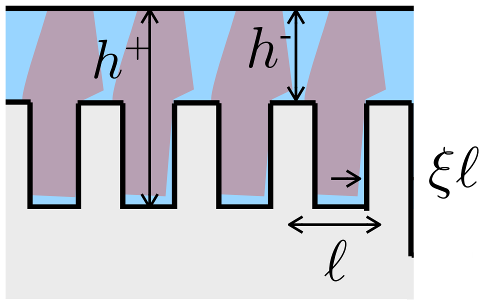

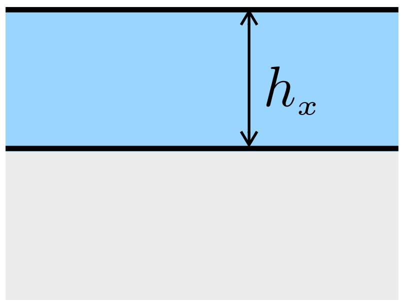

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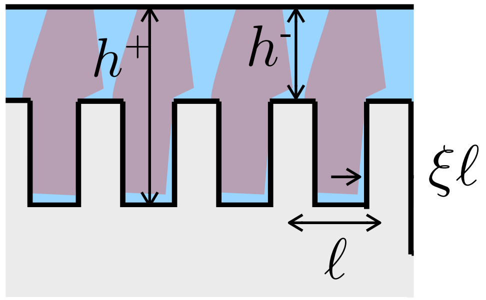
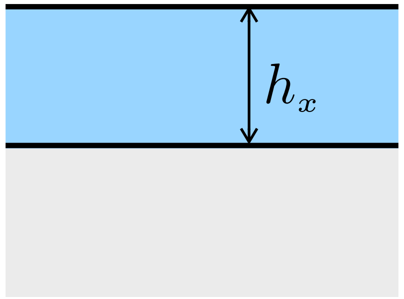
Main ingredients are: • rescaling : $x \to x, x' = x/\ell$,

Propagation of water waves over structured ridges

1 in the bulk of the structure





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Propagation of water waves over structured ridges

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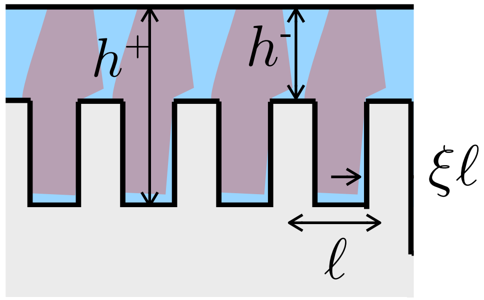

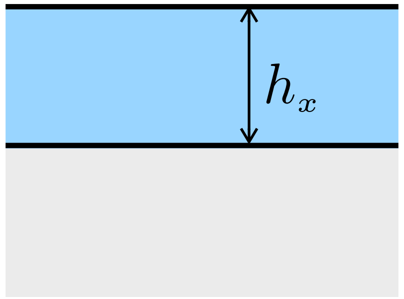

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Propagation of water waves over structured ridges

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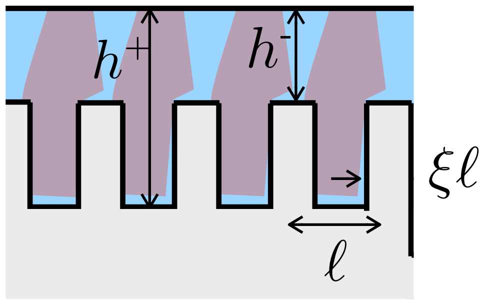

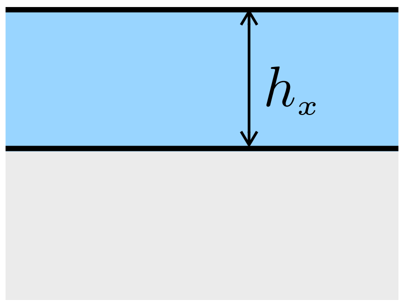

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Main ingredients are: • rescaling : $x \to x, x' = x/\ell, \quad y \to y, \quad z \to z' = z/h,$

(x', z') micro scales

Propagation of water waves over structured ridges

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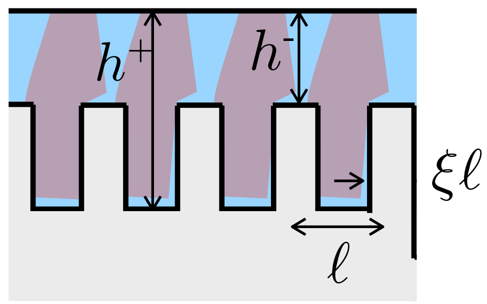

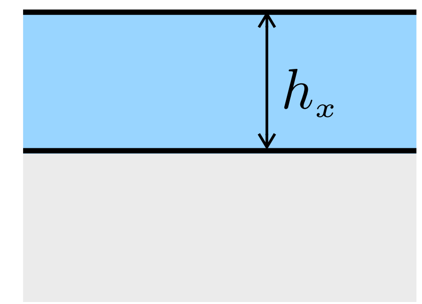

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Main ingredients are: • rescaling : $x \to x, x' = x/\ell, \quad y \to y, \quad z \to z' = z/h,$

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Propagation of water waves over structured ridges

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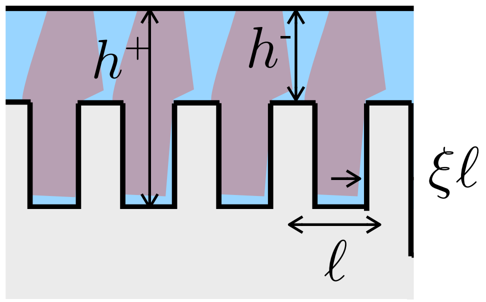

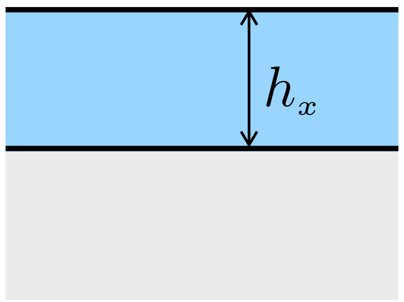

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Main ingredients are: • rescaling : $x \to x, x' = x/\ell, \quad y \to y, \quad z \to z' = z/h,$

(x', z') micro scales aim to disappear $\phi(x,y) = \int_\Omega \Phi(x,x',y,z')dx'dz'$

Propagation of water waves over structured ridges

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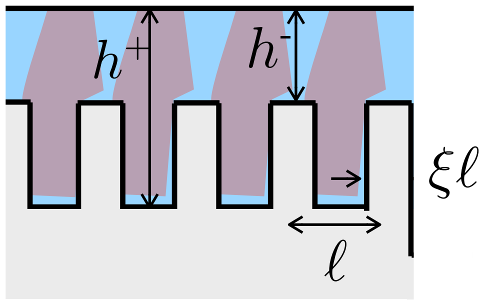

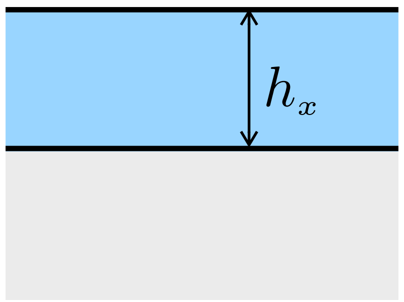

$$\nabla \cdot \left(\begin{pmatrix} h_x & 0 \\ 0 & h_y \end{pmatrix} \nabla \phi \right) - \frac{1}{g} \partial_{tt} \phi = 0 \quad h_x < \langle h^{-1} \rangle^{-1}, \; h_y = \langle h \rangle$$

Main ingredients are:

- rescaling : $x \to x, x' = x/\ell, \quad y \to y, \quad z \to z' = z/h,$

  (x', z') micro scales aim to disappear $\phi(x, y) = \int_\Omega \Phi(x, x', y, z') dx' dz'$

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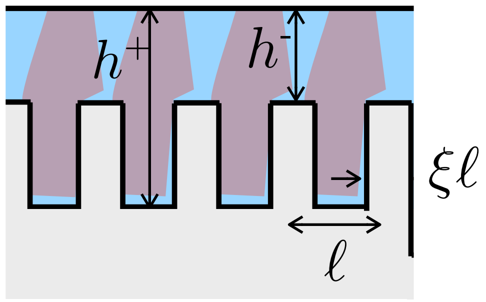
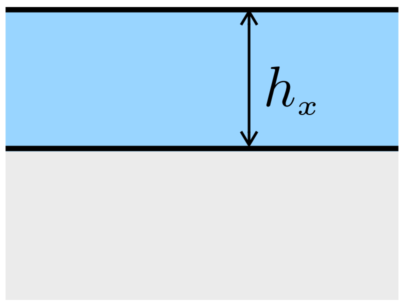
Propagation of water waves over structured ridges

1 in the bulk of the structure





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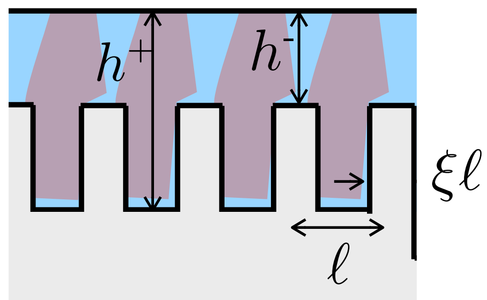
potential flow pb.

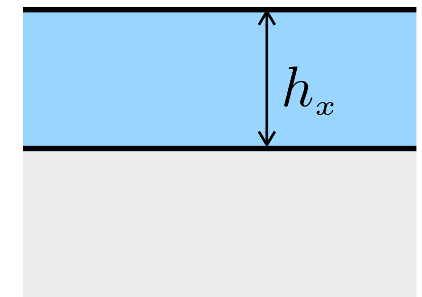
$\Delta \Phi^{\text{cell}} = 0$

set in a "unit cell"

Propagation of water waves over structured ridges

1 in the bulk of the structure





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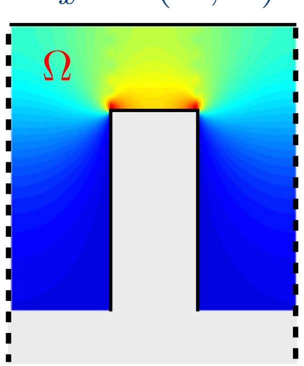

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Propagation of water waves over structured ridges

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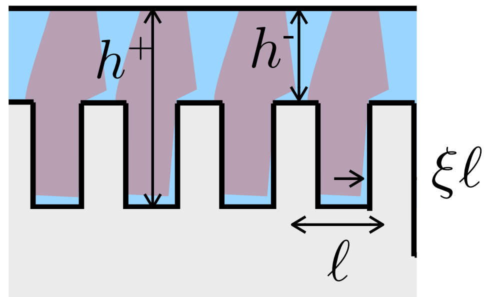

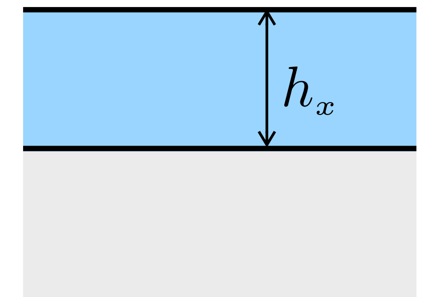

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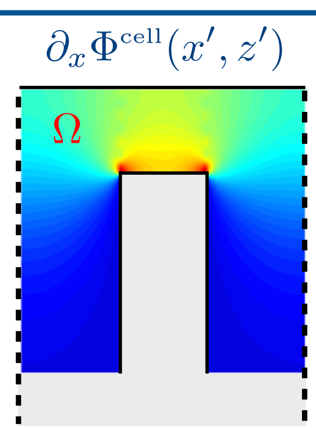

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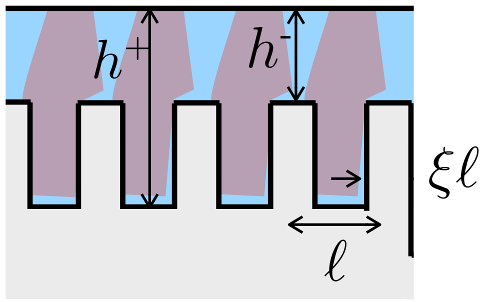

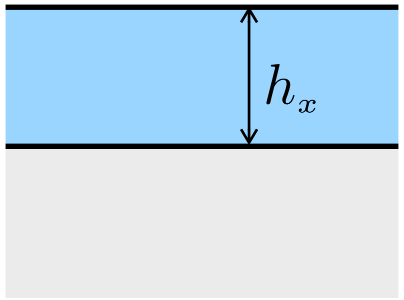

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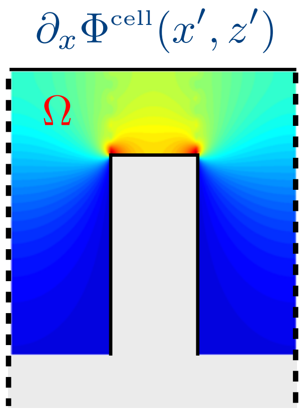

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Propagation of water waves over structured ridges

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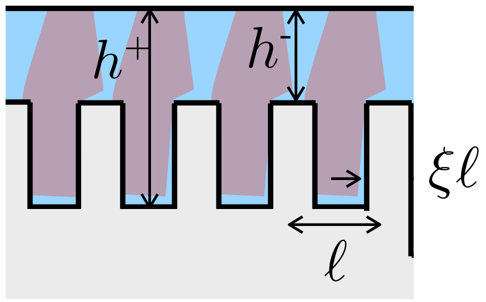

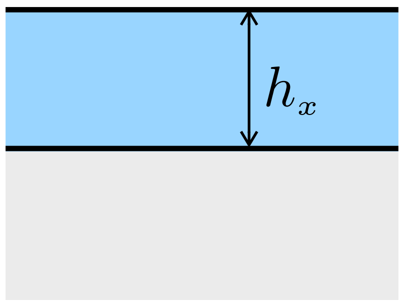

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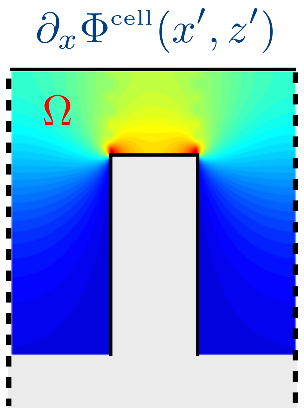

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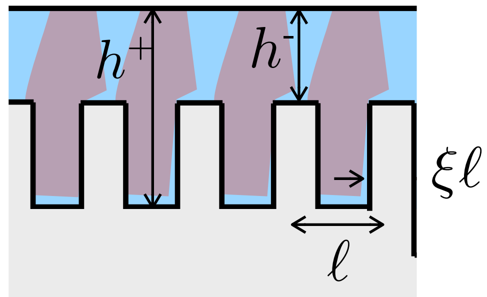

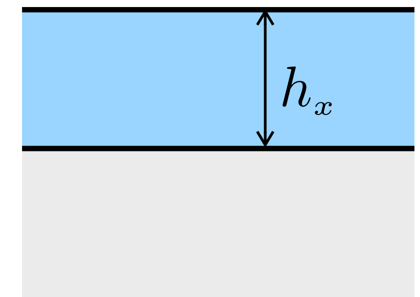

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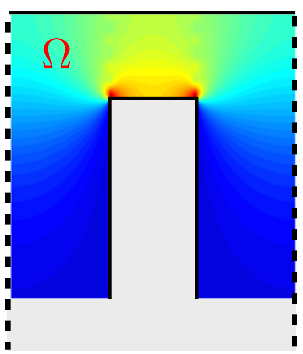

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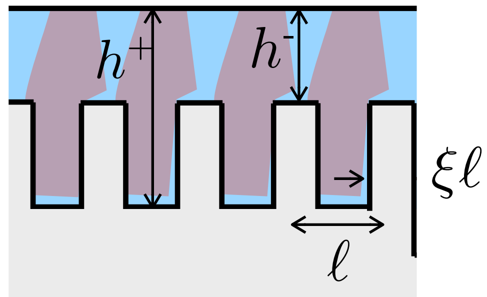

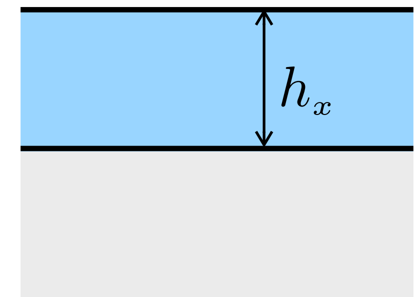

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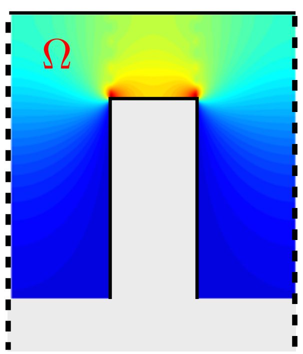

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Gravity Waves in a Channel with a Rough Bottom*

By Rodolfo R. Rosales and George C. Papanicolaou

Studies in Applied Mathematics, *68*(2), 89-102 1983

We derive effective equations for the surface elevation of gravity waves in a shallow channel with a rough bottom.

Propagation of water waves over structured ridges

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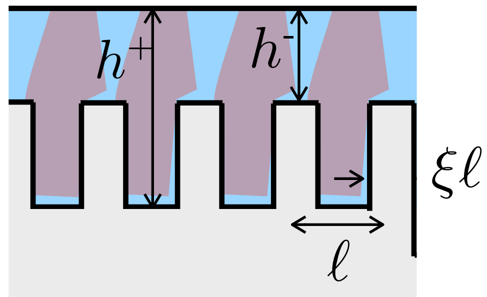

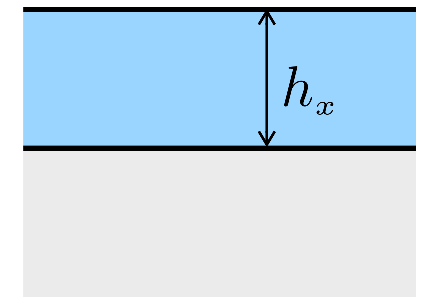

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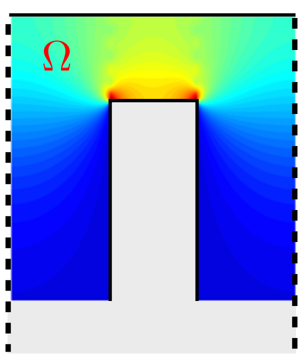

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PHYSICAL REVIEW B **96**, 134310 (2017)

Revisiting the anisotropy of metamaterials for water waves

A. Maurel, J.-J. Marigo, P. Cobelli, P. Petitjeans, and V. Pagneux
I. Langevin/ESPCI, 1 rue Jussieu, 75005 Paris, France;
LMS/Ecole Polytechnique, route de Saclay, 91120 Palaiseau, France;
Dpt Fisica/Univ. Buenos Aires and IFIBA, Cuidad Universitaria, Buenos Aires 1428, Argentina;
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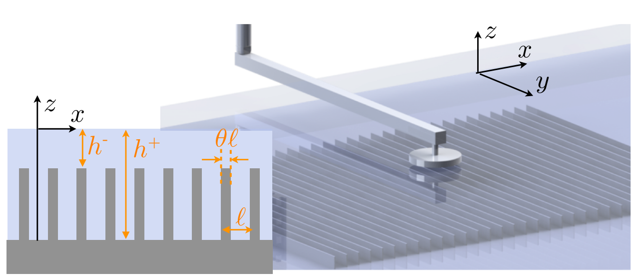

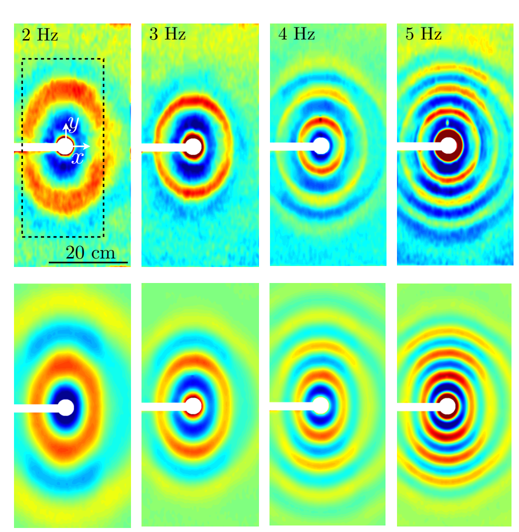

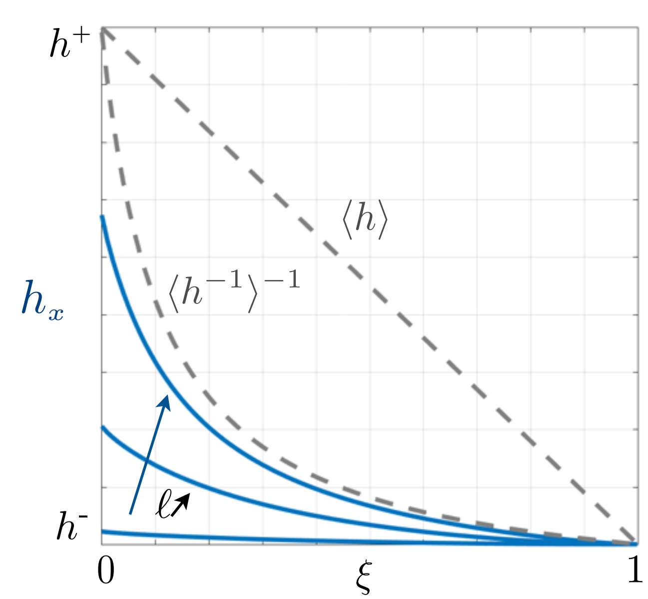

Propagation of water waves over structured ridges

1 in the bulk of the structure

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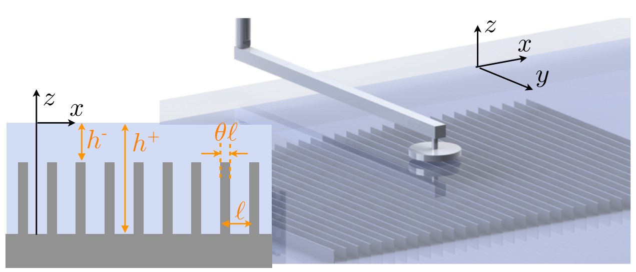

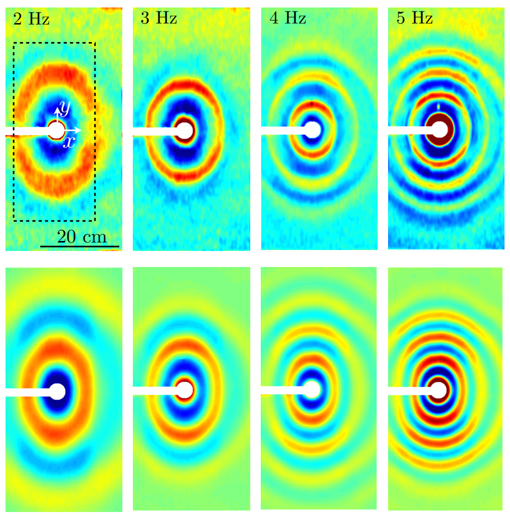

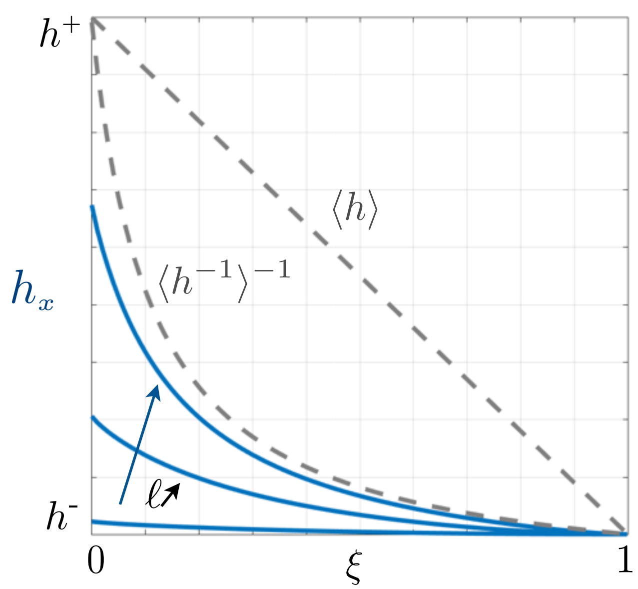

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Propagation of water waves over structured ridges

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2 at the end boundaries of the structure

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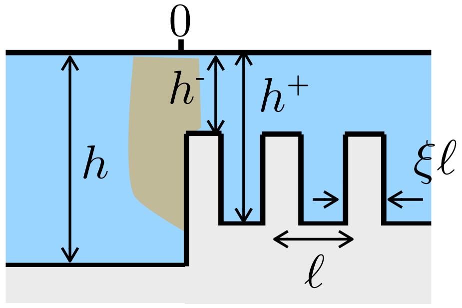

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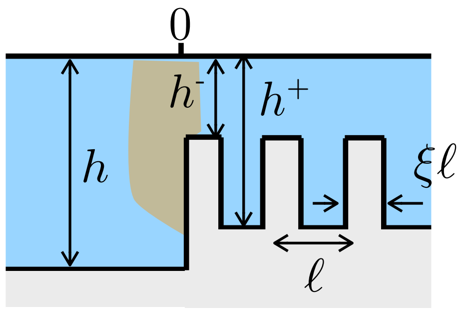

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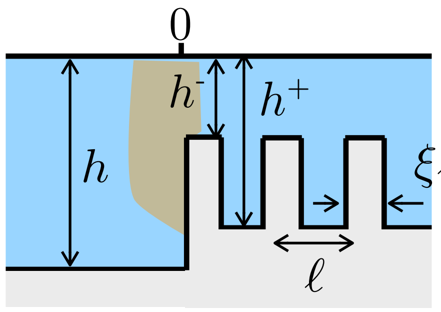

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$\mathcal{B} \to$ potential flow pb

$\Delta \Psi = 0$

set in an "elementary strip"

$\mathcal{C}$ is a geometrical parameter

Propagation of water waves over structured ridges

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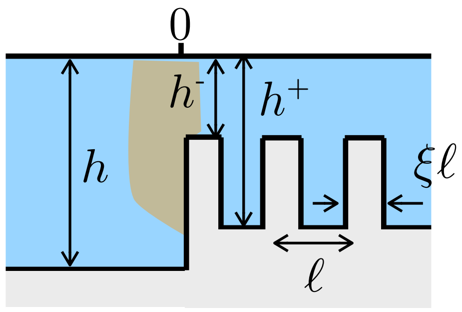

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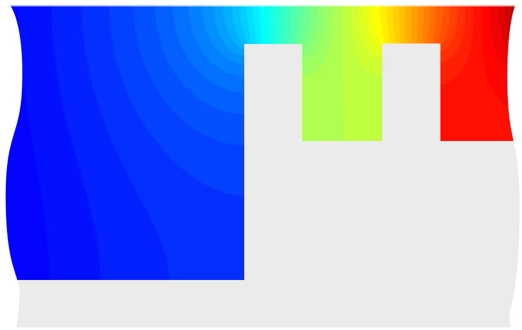
$\Psi \to \dfrac{x'}{h}$ $\quad x' \to -\infty$



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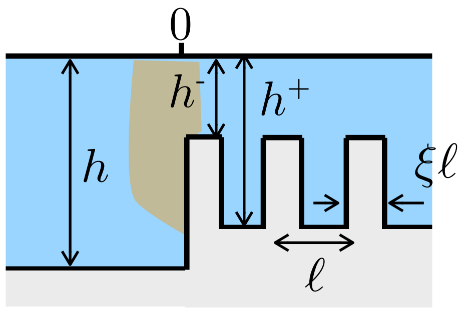

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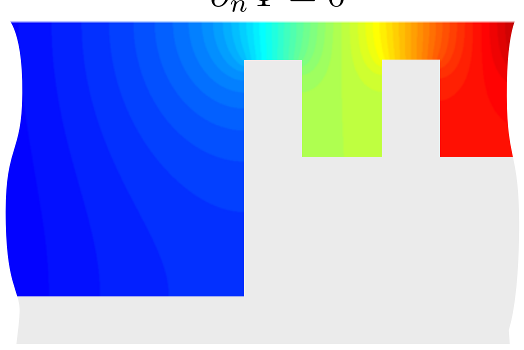

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$$[\![\phi]\!] = \mathcal{B}\, \overline{D}, \qquad [\![D]\!] = \mathcal{C}\, \frac{\partial^2 \overline{\phi}}{\partial y^2} \qquad\qquad (D = h\partial_x \phi, \quad D = h_x \partial_x \phi)$$

Propagation of water waves over structured ridges

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Propagation of water waves over structured ridges

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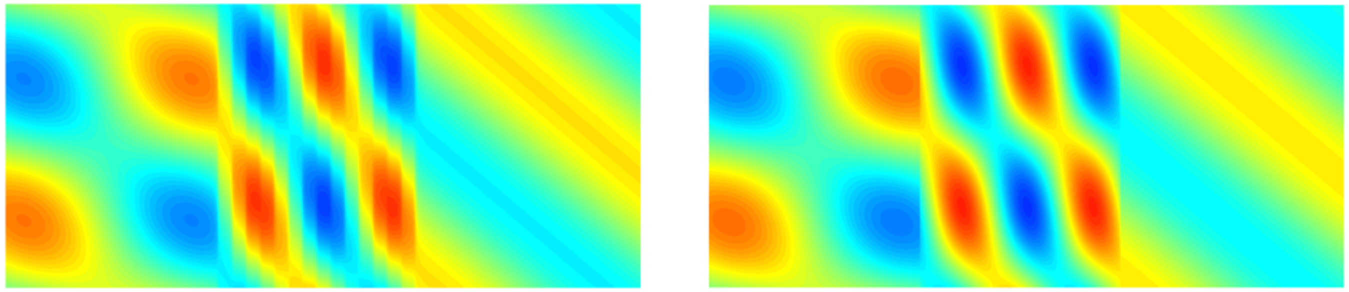

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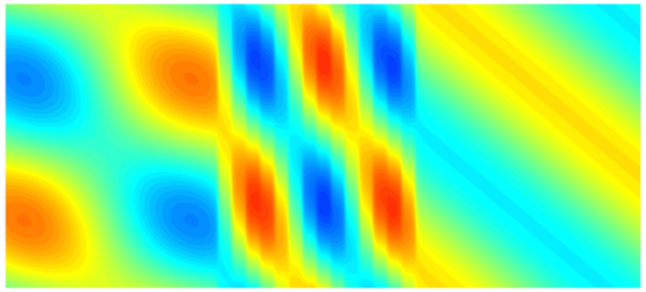

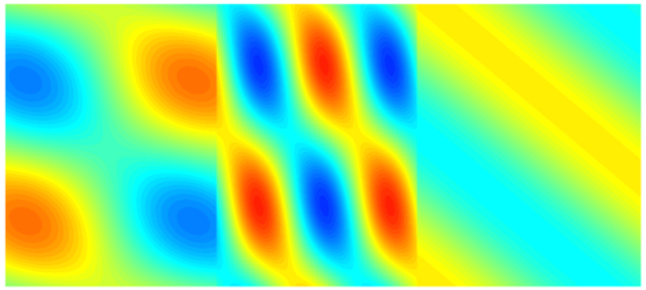

Concluding remarks

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- they involve effective parameters (approximate explicit expressions can be guessed)

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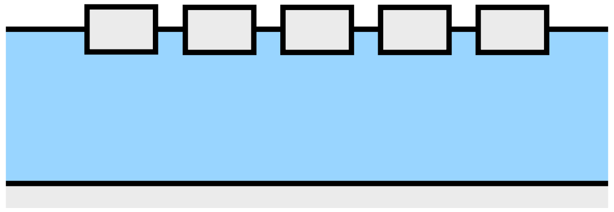
Ongoing work

Structured dock



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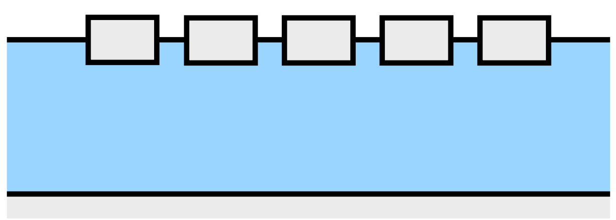

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floating ice

Ongoing work

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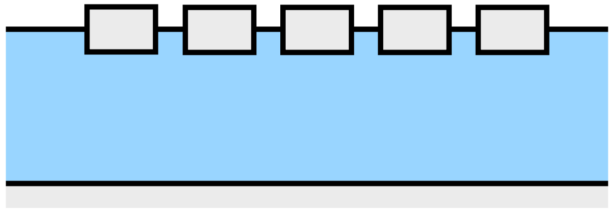

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wave breakers