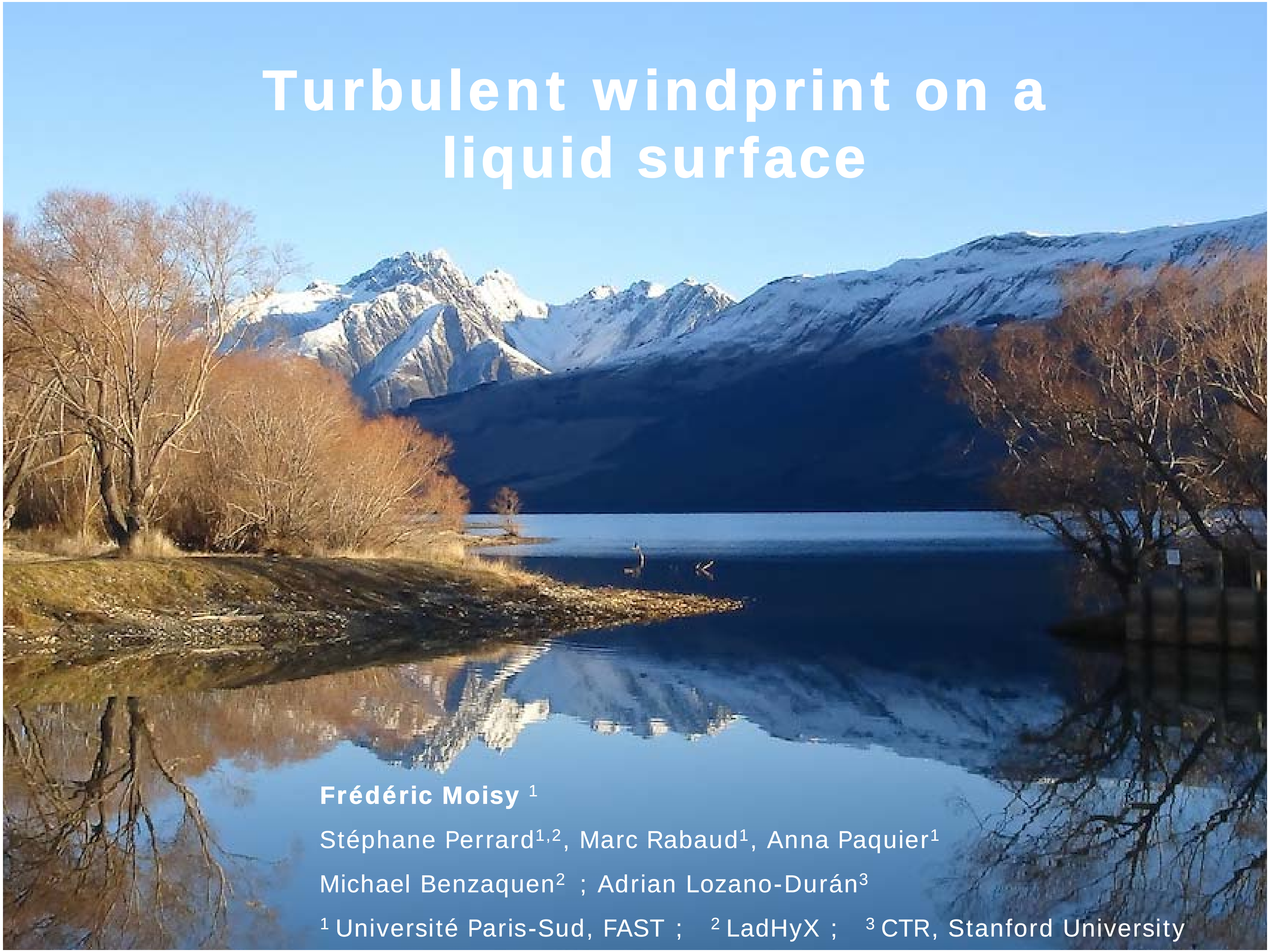
Turbulent windprint on a liquid surface



Frédéric Moisy [1]

Stéphane Perrard[1,2], Marc Rabaud[1], Anna Paquier[1]

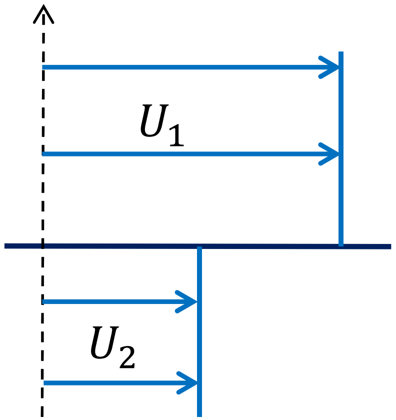
Michael Benzaquen[2] ; Adrian Lozano-Durán[3]

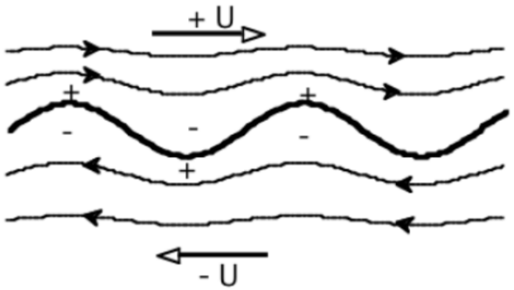
[1] Université Paris-Sud, FAST ; [2] LadHyX ; [3] CTR, Stanford University

Simplest model: Kelvin-Helmholtz (1868)

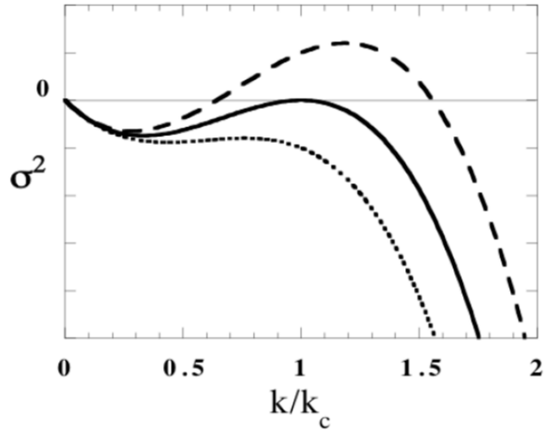
Linear stability analysis

- piecewise velocity profile
- surface tension
- potential flow (no viscosity)





Growth rate



Threshold: $$|U_1 - U_2|_{min} = \left[2\frac{\rho_1 + \rho_2}{\rho_1 \rho_2} (\Delta \rho g \gamma)^{1/2} \right]^{1/2}$$

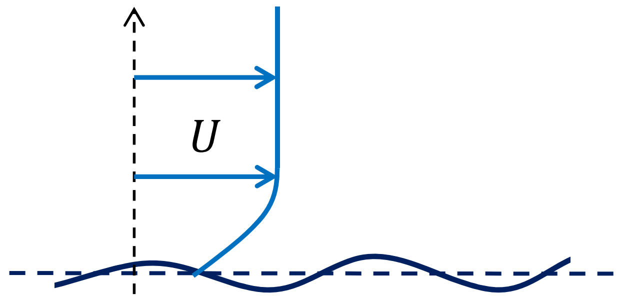
For the air-water interface, $\lambda_c = 2\pi / k_c \simeq 1.7\ cm$, $|U_1 - U_2| \min \simeq 6.6\ m/s$

too small!! too large!!

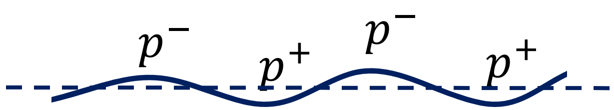
Key models

- **Miles (1957)**

Linear stability for inviscid Orr-Sommerfeld problem
using the (« laminar ») averaged turbulent velocity profile

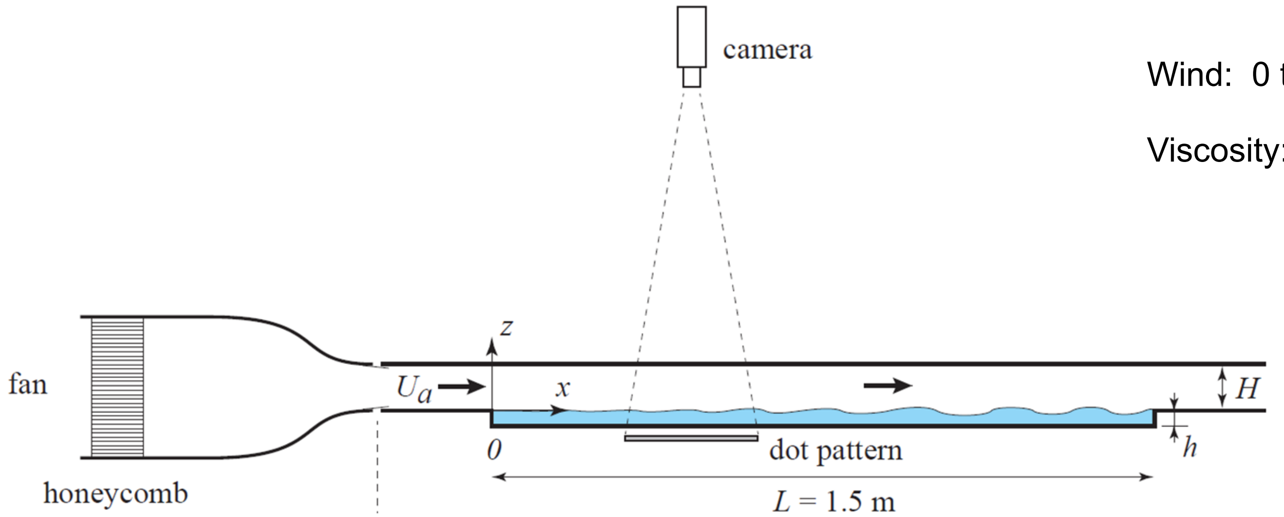


- **Phillips (1957)** : resonance between waves and travelling pressure fluctuations in the turbulent boundary layer



$$\overline{\xi^2} \sim \frac{\overline{p^2}\, t}{2\sqrt{2}\rho^2 U_c\, g}$$

Experimental setup

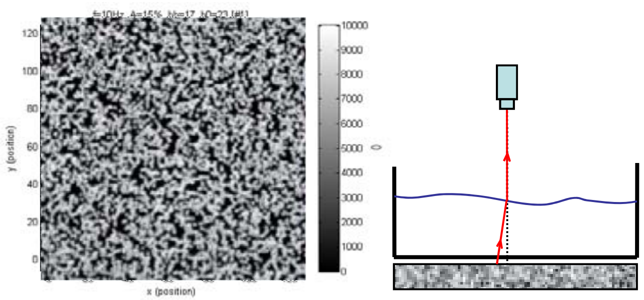


Wind: 0 to 10 m/s

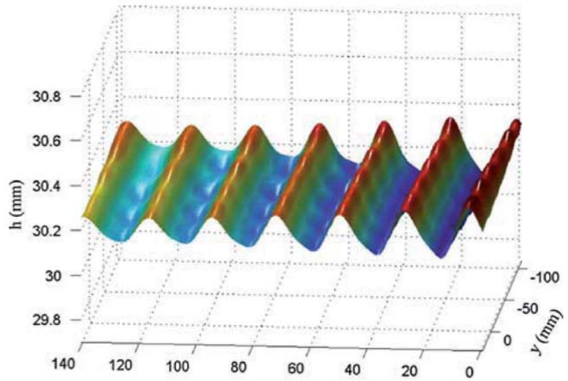
Viscosity: $1 - 1000\ \nu_{water}$

Paquier *et al.* Phys Rev Fluids (2015, 2016)

Free-surface synthetic Schlieren (FS-SS)

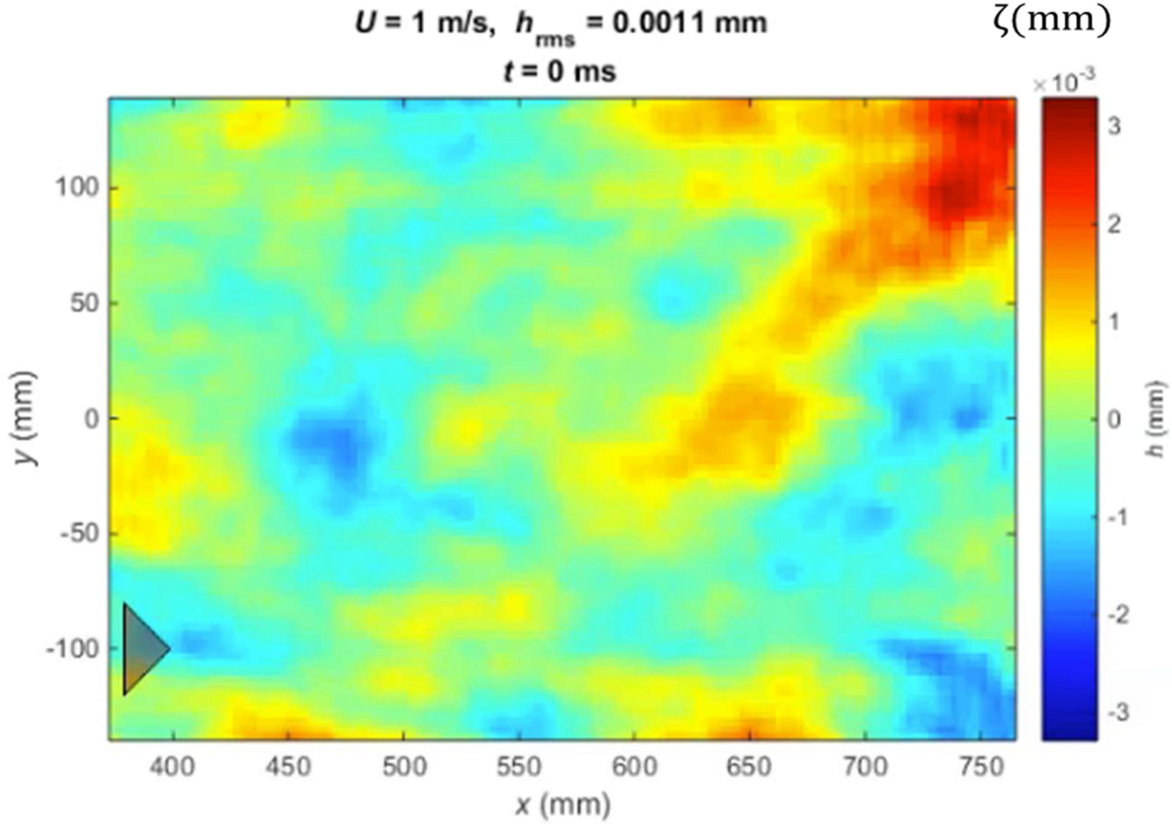


Digital Image Correlation + ∇^{-1} using a least-square inversion



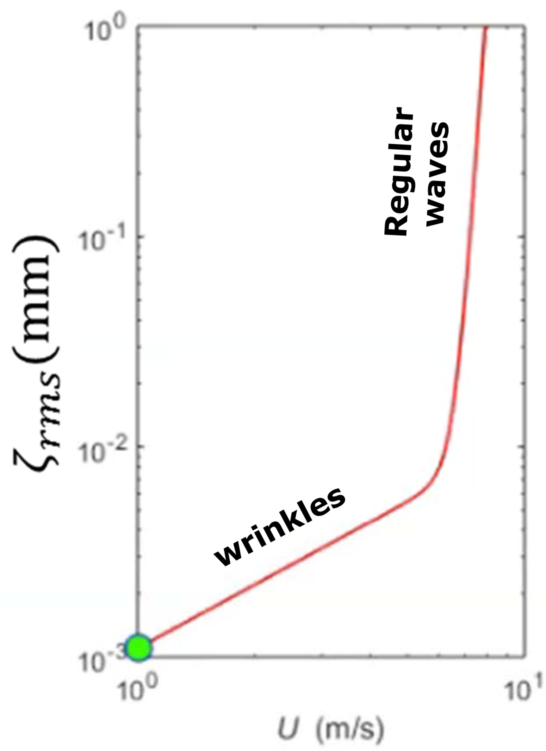
Moisy, Rabaud & Salsac, Exp. in Fluids (2009)

Transtion from « wrinkles » to regular waves



Amplitude :

$$\zeta_{rms} = \langle \zeta^2(x,y,t) \rangle^{1/2}$$



Paquier, Moisy, Rabaud (2015, 2016)

Growth of deformation amplitude with wind

Wave amplitude rms: $\boxed{\zeta_{rms} = \langle \zeta^2(x,y,t) \rangle^{1/2}}$

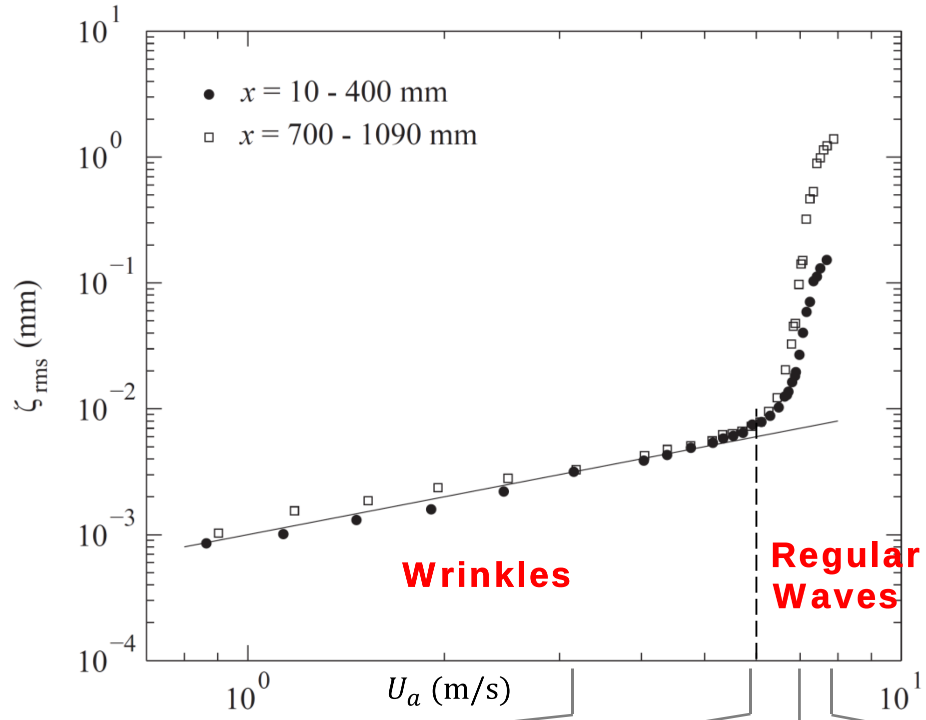
Wrinkles

~ linear growth

with U_a

~ independent

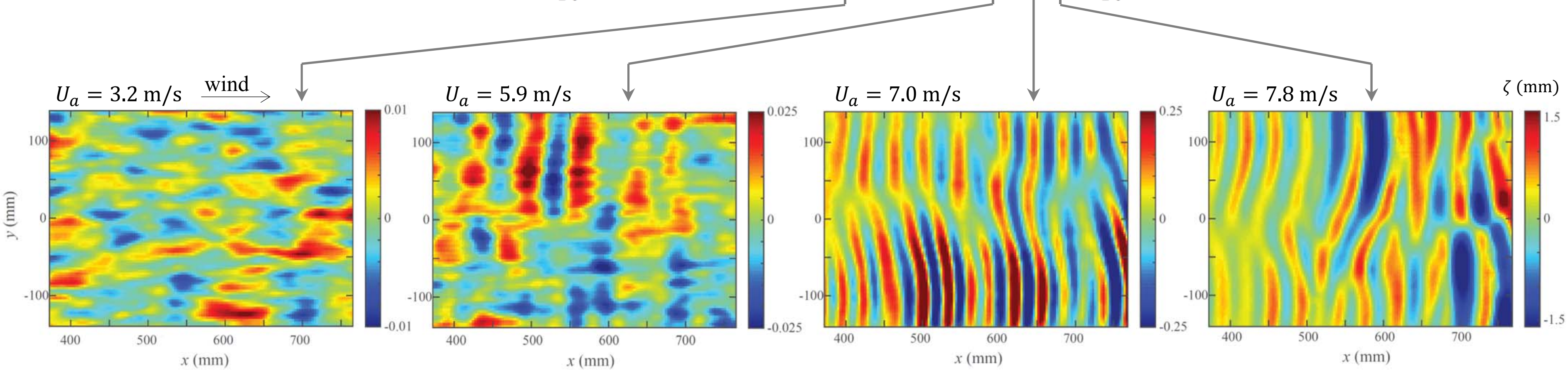
of fetch x



Regular waves

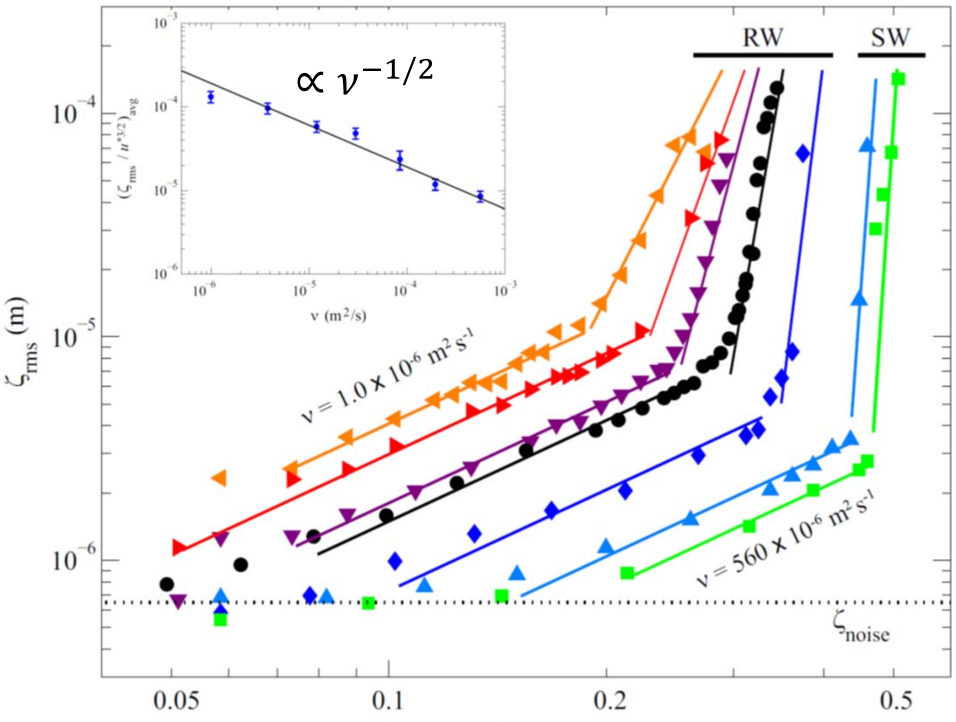
- rapid growth

with U_a **and** x



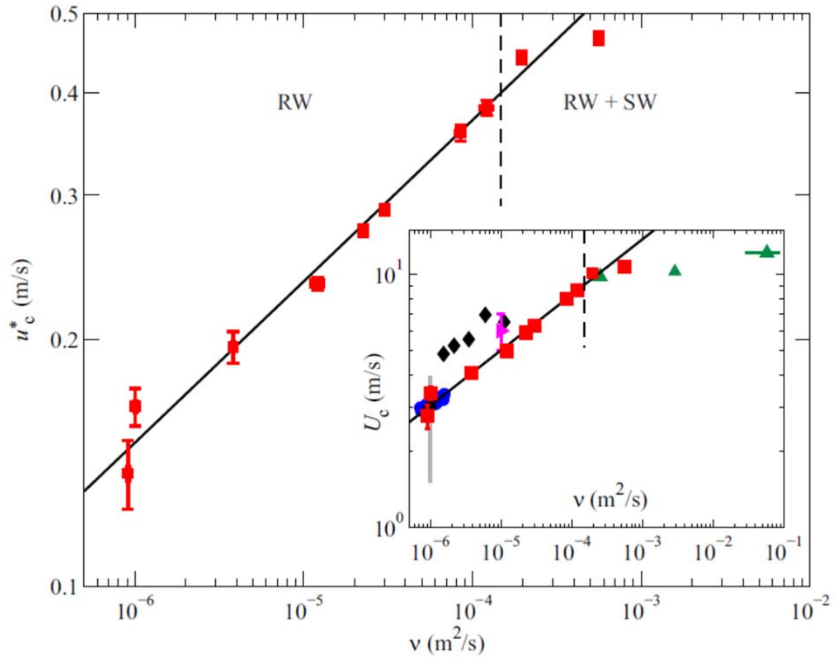
Influence of viscosity on wind-wave generation

Evolution of the surface deformation



Friction velocity u^* (here $u^* \simeq 0.05\, U_a$)

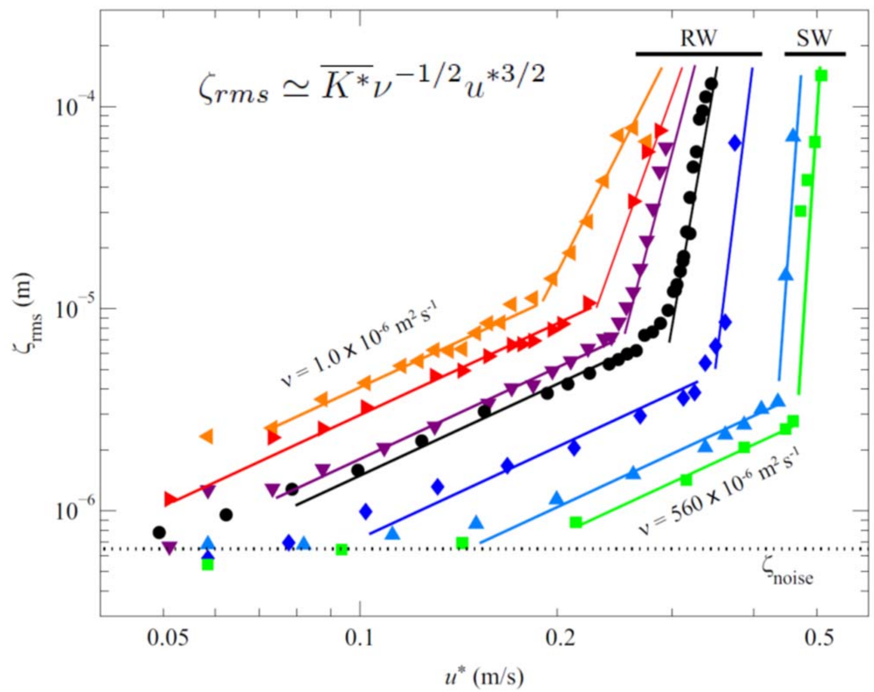
Evolution of the threshold velocity

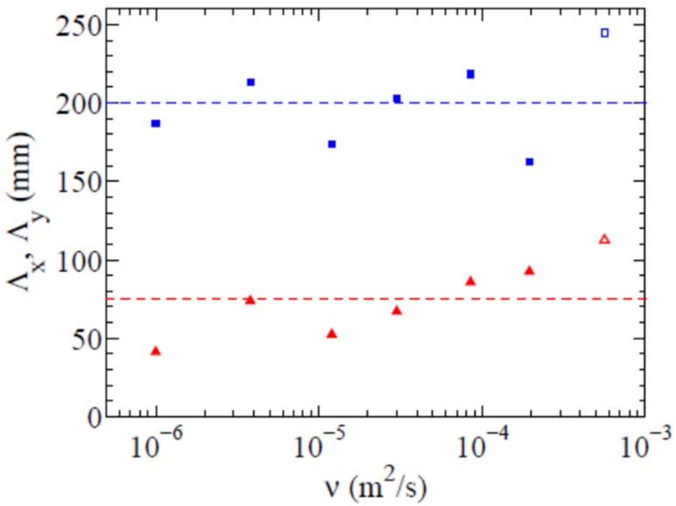


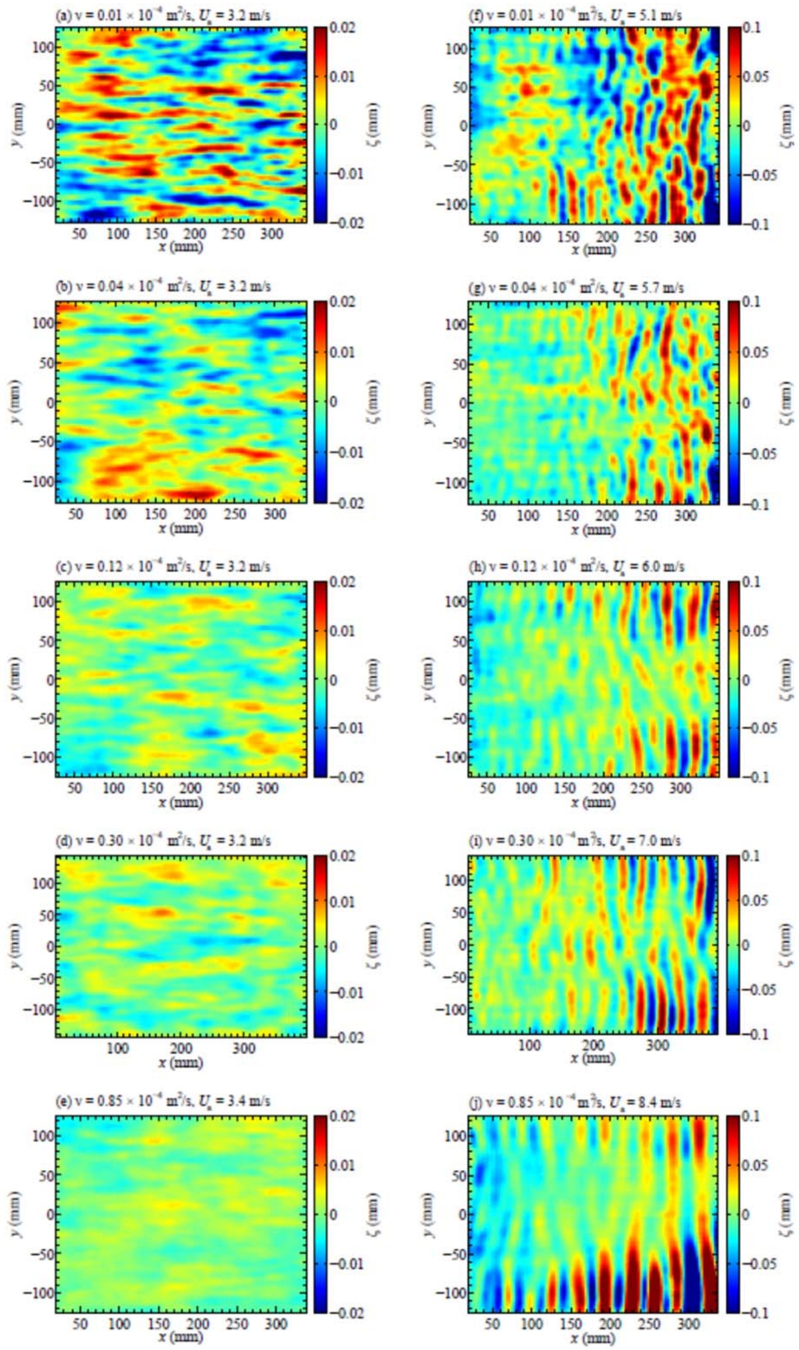
In the wrinkles regime: $\boxed{\zeta_{rms} \propto \nu^{-1/2}\, u^{*3/2}}$

$\boxed{u^*_c \propto \nu^{0.20}}$

Influence of the liquid viscosity



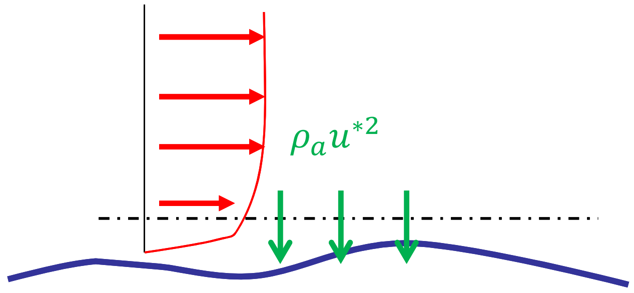




Scaling of the wrinkle amplitude: phenomenology

Balance between turbulent energy flux

and viscous dissipation in the liquid



$$\frac{dE_w}{dt} = P_{inj} - \frac{E_w}{\tau} = 0$$

Wave energy: $E_w \simeq \rho_\ell \, g \, \zeta^2$,

viscous time scale: $\tau \simeq \frac{\delta^2}{\nu}$

Work of turbulent stress per unit time: $P_{inj} \simeq p_{rms} \; \dot{\zeta} \simeq \rho_a u^{*2} \dot{\zeta}$

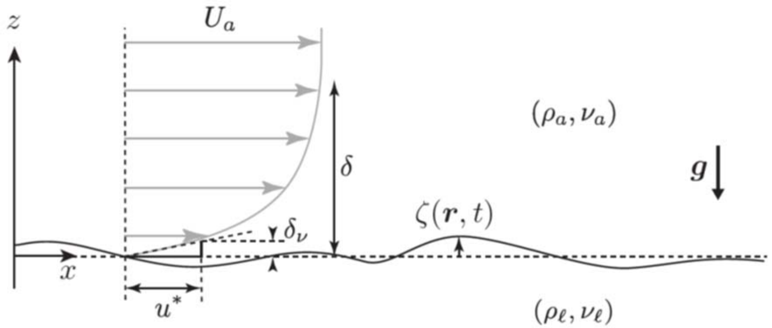
Momentum conservation: $\rho_\ell \dot{\zeta} \simeq \rho_a u^*$

$$\frac{\zeta_{\rm rms}}{\delta} \simeq C \frac{\rho_a}{\rho_\ell} \frac{u^{*3/2}}{(g\nu_\ell)^{1/2}}$$

A spectral theory for the wrinkles

Assumptions

- Viscous flow in the liquid
- Waves of small amplitude: no feedback
  on the turbulent forcing
- No drift current



Stokes Equation + linearized b.c.

$$\partial_t \boldsymbol{v} = -\frac{1}{\rho_\ell} \boldsymbol{\nabla} p_\ell + \mathbf{g} + \nu_\ell \boldsymbol{\Delta} \boldsymbol{v}$$

Space-time Fourier transfom

$$\hat{\zeta}(\boldsymbol{k}, \omega) = \mathcal{F}\{\zeta(\boldsymbol{r}, t)\} = \int \mathrm{d}^2\boldsymbol{r}\, \mathrm{d}t\, \zeta(\boldsymbol{r}, t) e^{-i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)}$$

$$\zeta(\boldsymbol{r}, t) = \mathcal{F}^{-1}\{\hat{\zeta}(\boldsymbol{k}, \omega)\} = (2\pi)^{-3} \int \mathrm{d}^2\boldsymbol{k}\, \mathrm{d}\omega\, \hat{\zeta}(\boldsymbol{k}, \omega) e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)}$$

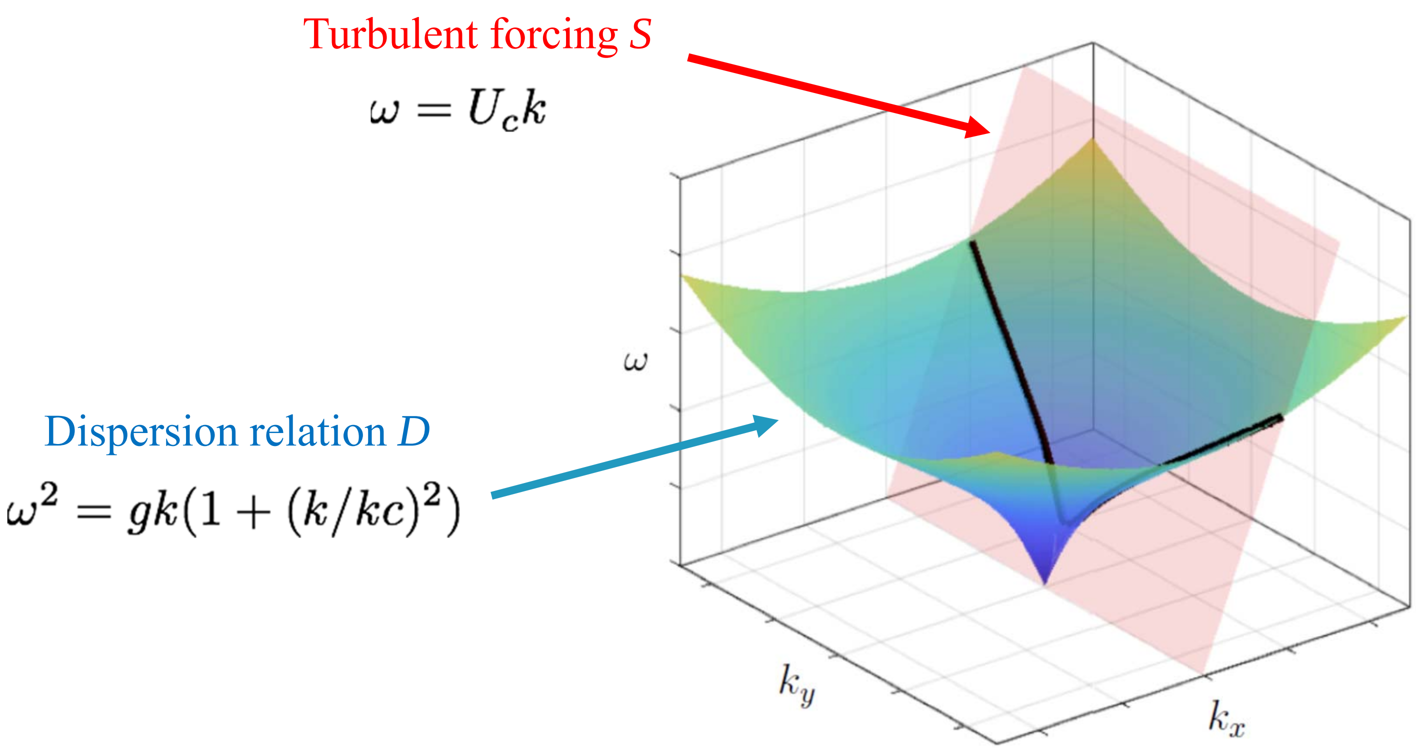
Low-viscosity limit $\left(k^{-1} \gg \ell_\nu = \left(\nu_\ell^2/g\right)^{1/3}\right)$

$$\boxed{\hat{\zeta}(\boldsymbol{k}, \omega) = \frac{1}{\rho_\ell} \frac{k\hat{N} + i\boldsymbol{k}\cdot\hat{\boldsymbol{T}}}{\omega^2 - g'k + 4i\nu_\ell \omega k^2}}$$

S. Perrard, A. Lozano-Duran, M. Rabaud, M. Benzaquen, subm. to JFM (2019)

Resonance in (2+1) dimensions

$$\hat{\zeta}(\boldsymbol{k},\omega) = \frac{1}{\rho_\ell}\frac{k\hat{N} + i\boldsymbol{k}\cdot\hat{\boldsymbol{T}}}{\omega^2 - g'k + 4i\nu_\ell\omega k^2} = \frac{\hat{S}(\boldsymbol{k},\omega)}{\hat{D}(\boldsymbol{k},\omega)}$$



Similar to Phillips (1957), but including viscosity & shear stress

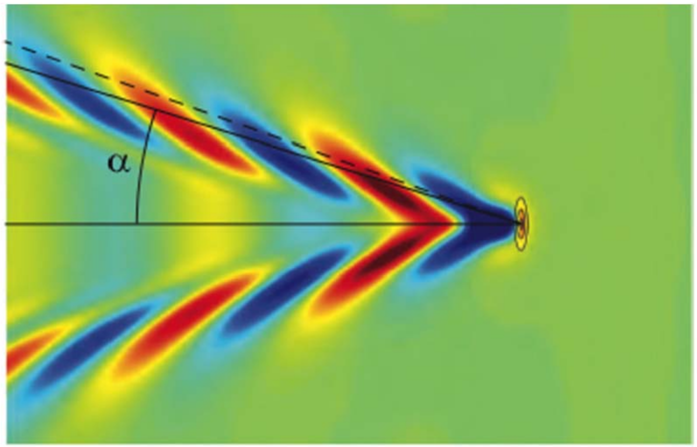
Connection to the ship wake problem (Havelock 1908)

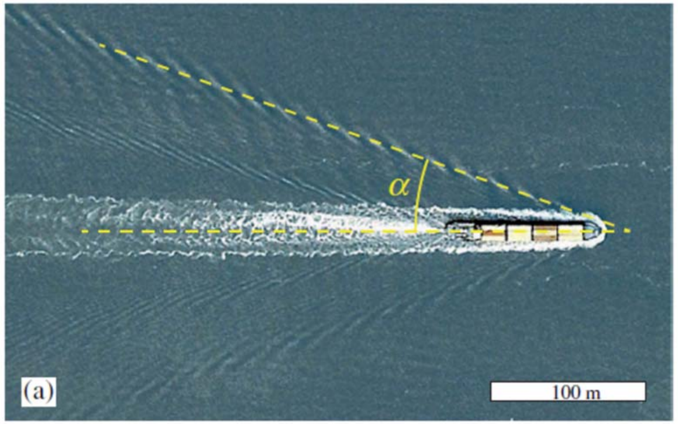
$$\hat{\zeta}(\boldsymbol{k},\omega) = \frac{1}{\rho_\ell}\frac{k\hat{N} + i\boldsymbol{k}\cdot\hat{\boldsymbol{T}}}{\omega^2 - g'k + 4i\nu_\ell\omega k^2}$$

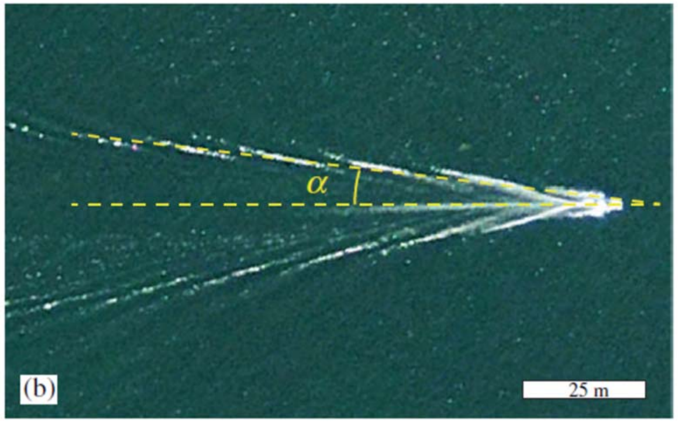
For a rigid traveling pressure patch, $\widehat{N}(\boldsymbol{k},\omega) = \delta(\omega - U_c k_x)\hat{\mathcal{N}}(\boldsymbol{k})$

we recover the classical Havelock's ship wake integral

$$\zeta(\mathbf{x}) = -\lim_{\epsilon\to 0}\frac{1}{(2\pi)^2}\iint\frac{k\hat{P}(\mathbf{k})/\rho}{\omega(\mathbf{k})^2 - (\mathbf{k}\cdot\mathbf{U} - i\epsilon)^2}e^{i\mathbf{k}\cdot\mathbf{x}}d^2\mathbf{k}$$



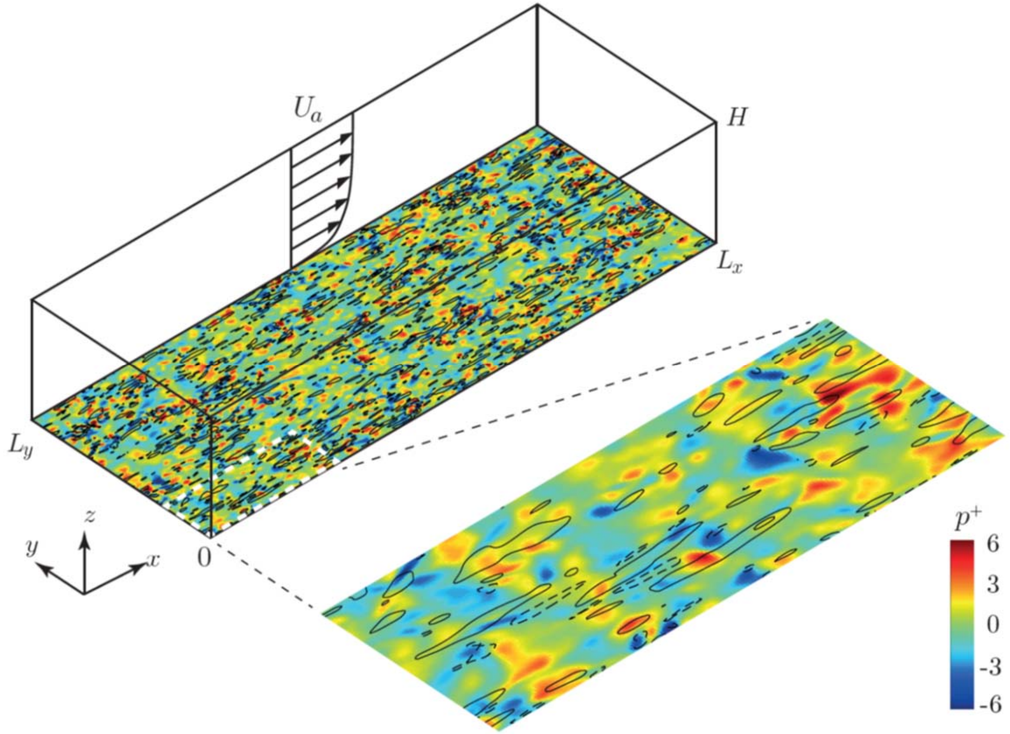




Rabaud & Moisy, Phys Rev Lett 2013
Darmon et al, JFM 2014

Evaluation of source term $S(k, \omega)$ from DNS

Data from A. Lozano Duran (CTR, Stanford)



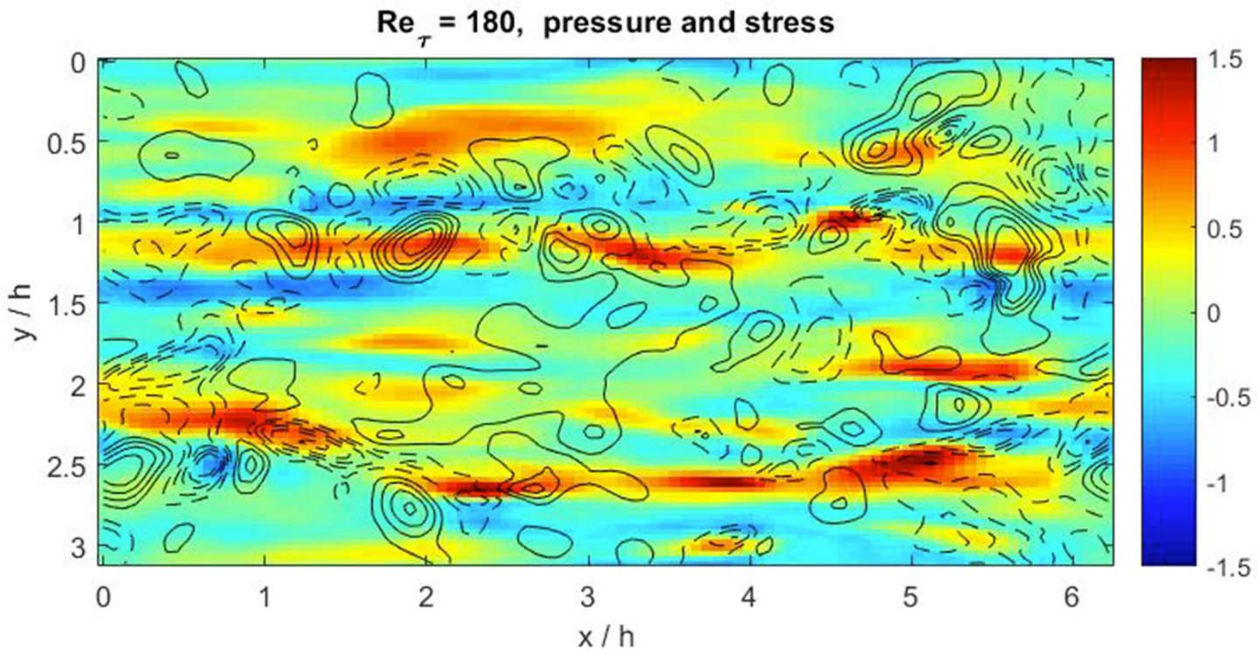
- Turbulent chanel flow

$$L_x \times L_y \times L_z = (8\pi, 3\pi, 2)H$$

- Periodic box in *x* and *y*
- Rigid wall
- No slip boundary condition

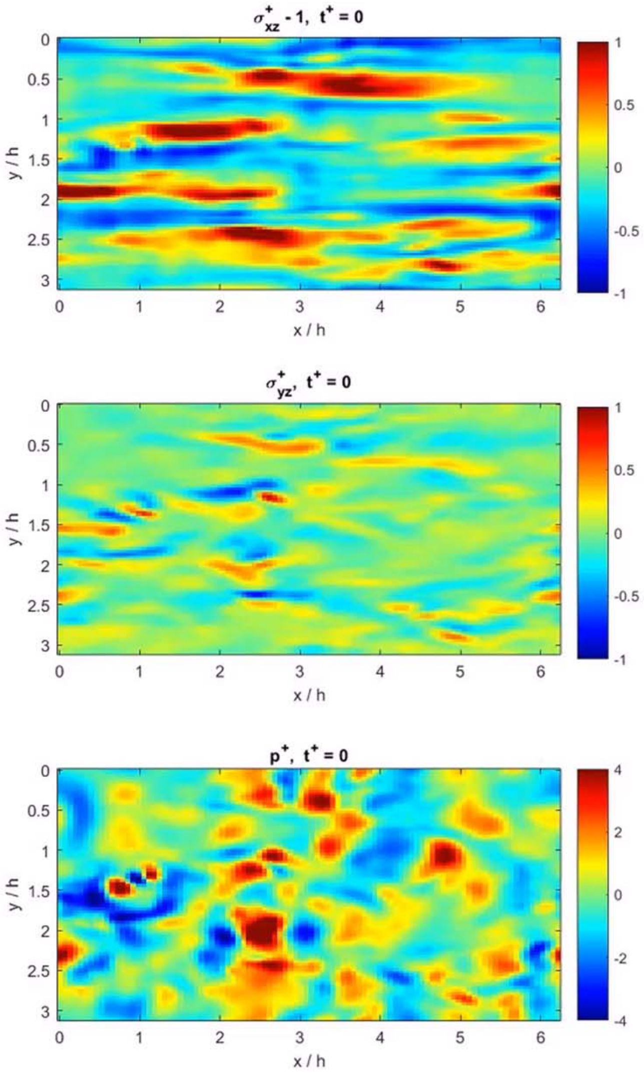
Re_δ	Δx^+	Δy^+	Δz^+_{min}	Δz^+_{max}	Δt^+	H/u^*	U_a (m s^{-1})
100	10.1	5.7	0.06	3.4	0.63	12.5	1
180	9.1	5.3	0.02	3.0	0.64	14.1	1.8
250	12.1	6.8	0.03	4.0	0.61	10.1	2.5
360	13.1	6.5	0.04	5.8	3.80	21.8	3.6
550	13.4	7.5	0.04	6.7	0.45	6.7	5.5

Normal & tangential stresses from DNS

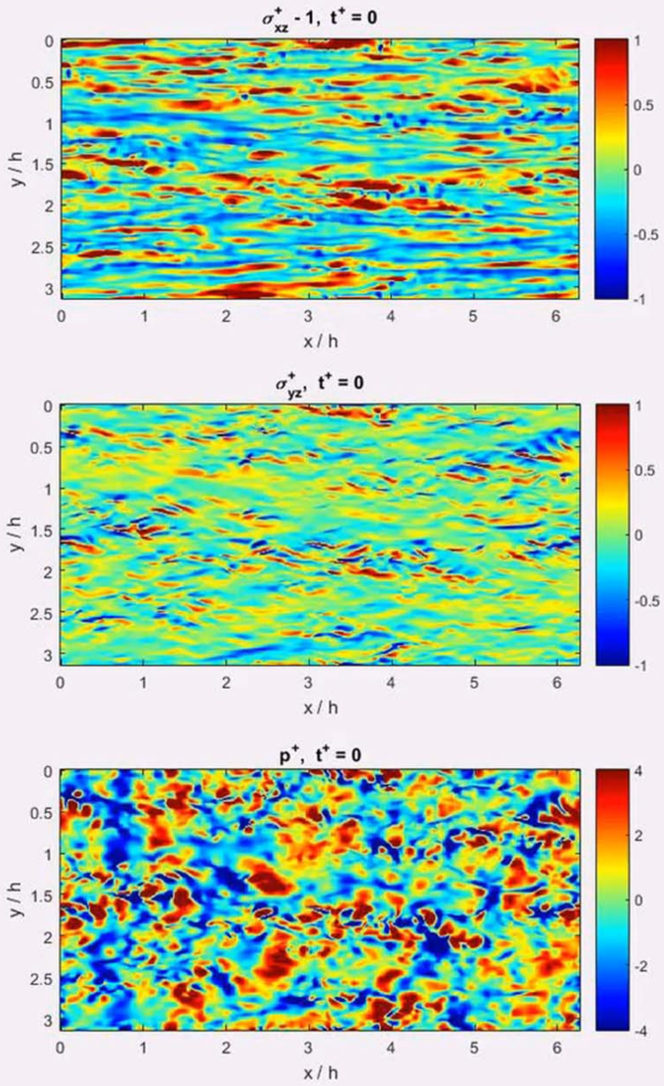


Normal & tangential stresses from DNS

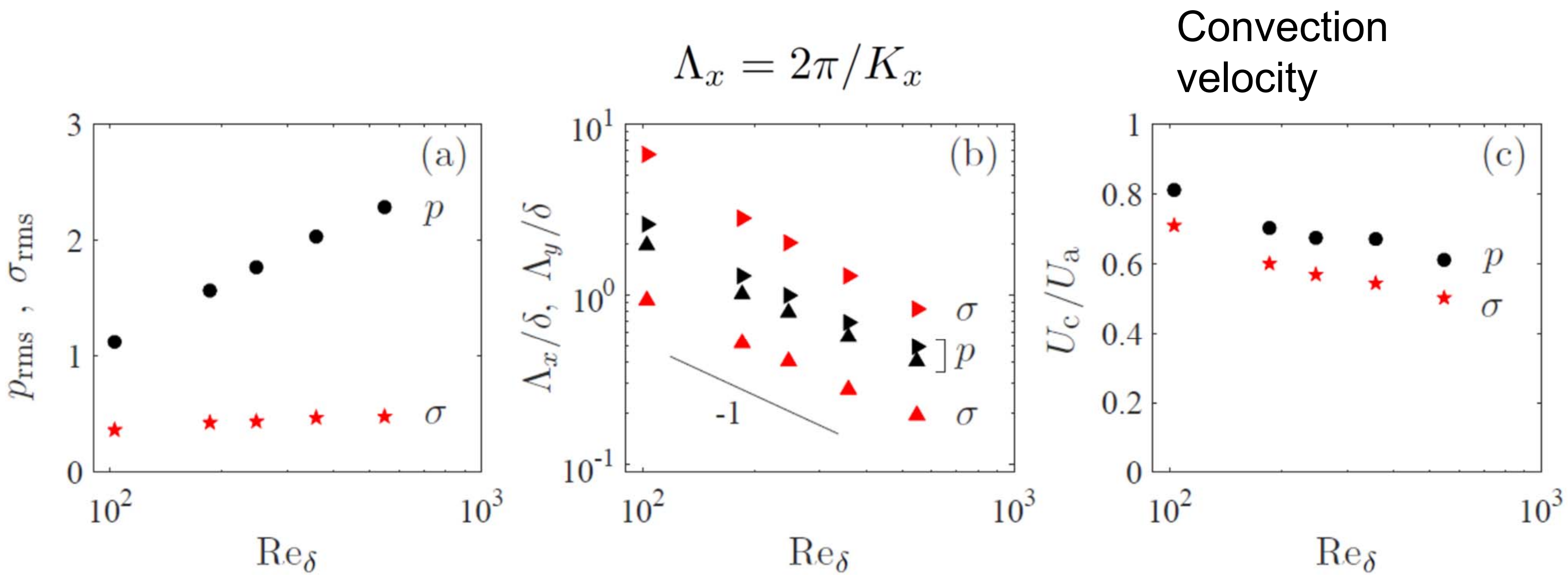
$Re_\tau = 180 \ \ (\coloneqq 1.8\, m/s)$



$Re_\tau = 550 \ \ (\coloneqq 5.5\, m/s)$



Statistical properties of turbulent stress fluctuations



Spectral barycenter $\boldsymbol{K} = K_x \mathbf{e_x} + K_y \mathbf{e_y} = \dfrac{\int_{\mathcal{D}} \mathrm{d}^2\boldsymbol{k}\mathrm{d}\omega \; \boldsymbol{k}|\hat{f}|}{\int_{\mathcal{D}} \mathrm{d}^2\boldsymbol{k}\mathrm{d}\omega \; |\hat{f}|}$

Characteristic size of pressure patches: $\Lambda_{x,y} \simeq 250\delta_\nu$

($\delta_\nu = \nu_a/u^*$: viscous sublayer thickness)

Wall pressure + wall stress
from DNS

$$P_0(\vec{r},t), \qquad \boldsymbol{\sigma}_0(\vec{r},t)$$

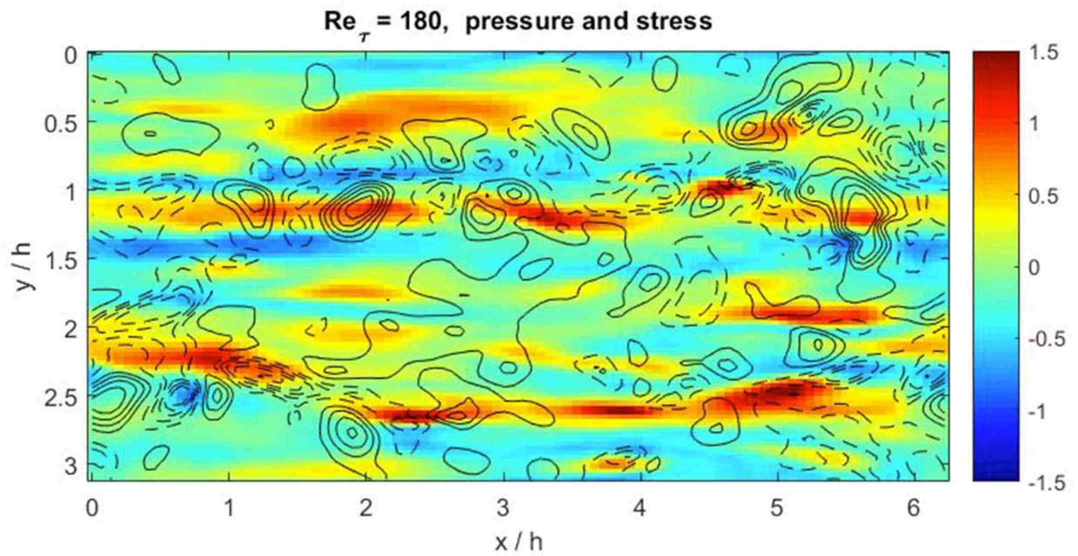
FFT

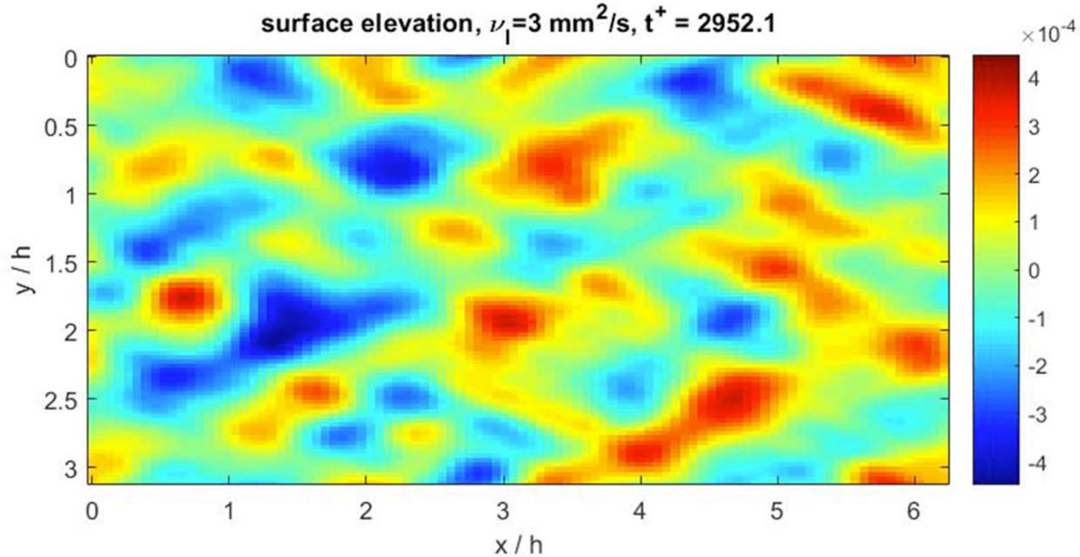
$$\hat{S} = k\hat{P}_0 + i\mathbf{k}.\hat{\sigma}_0$$

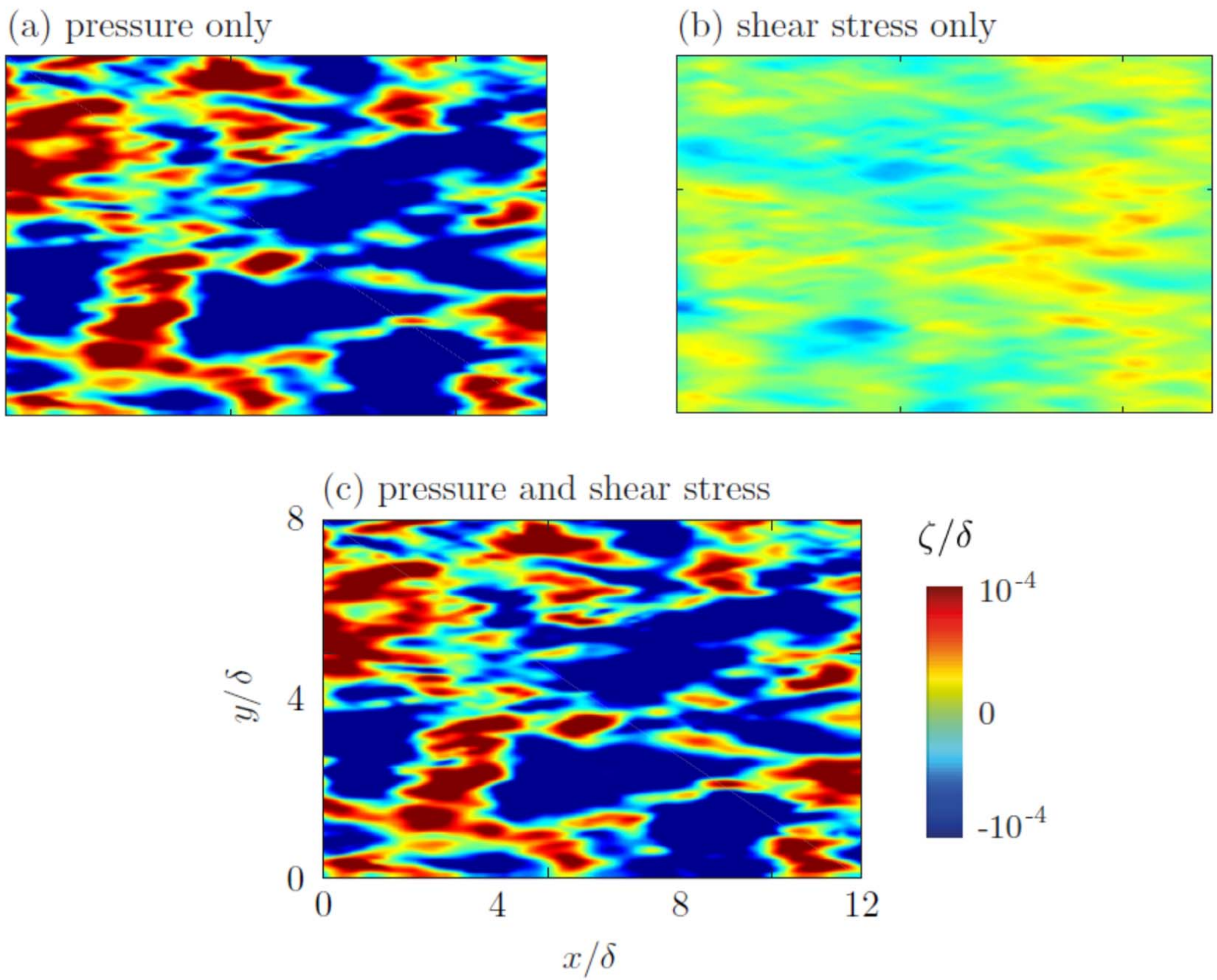
$$\hat{\zeta} = \frac{1}{\rho_\ell}\frac{\hat{S}}{\omega^2 - g'k + 4\nu i\omega k^2}$$

FFT $^{-1}$

$$\zeta(\vec{r},t)$$

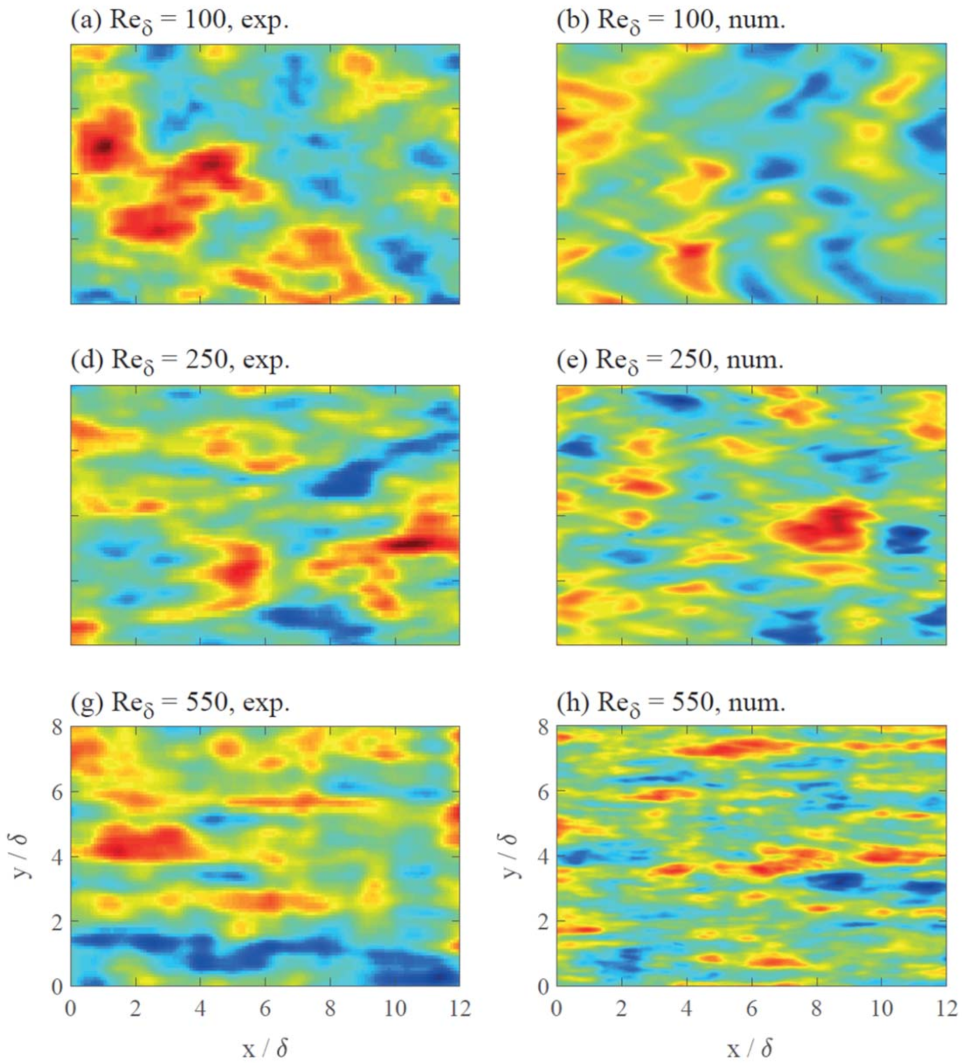


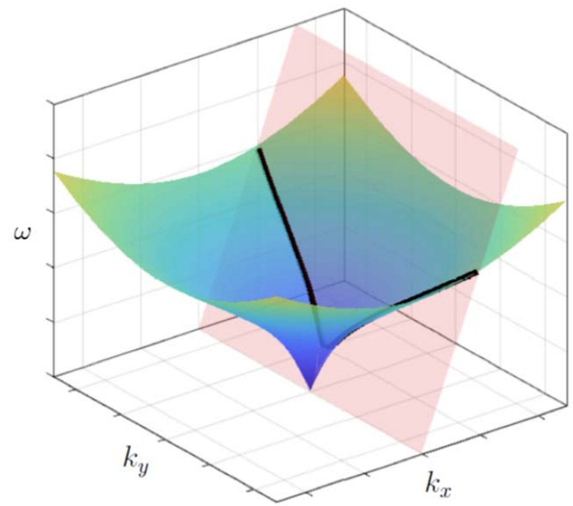




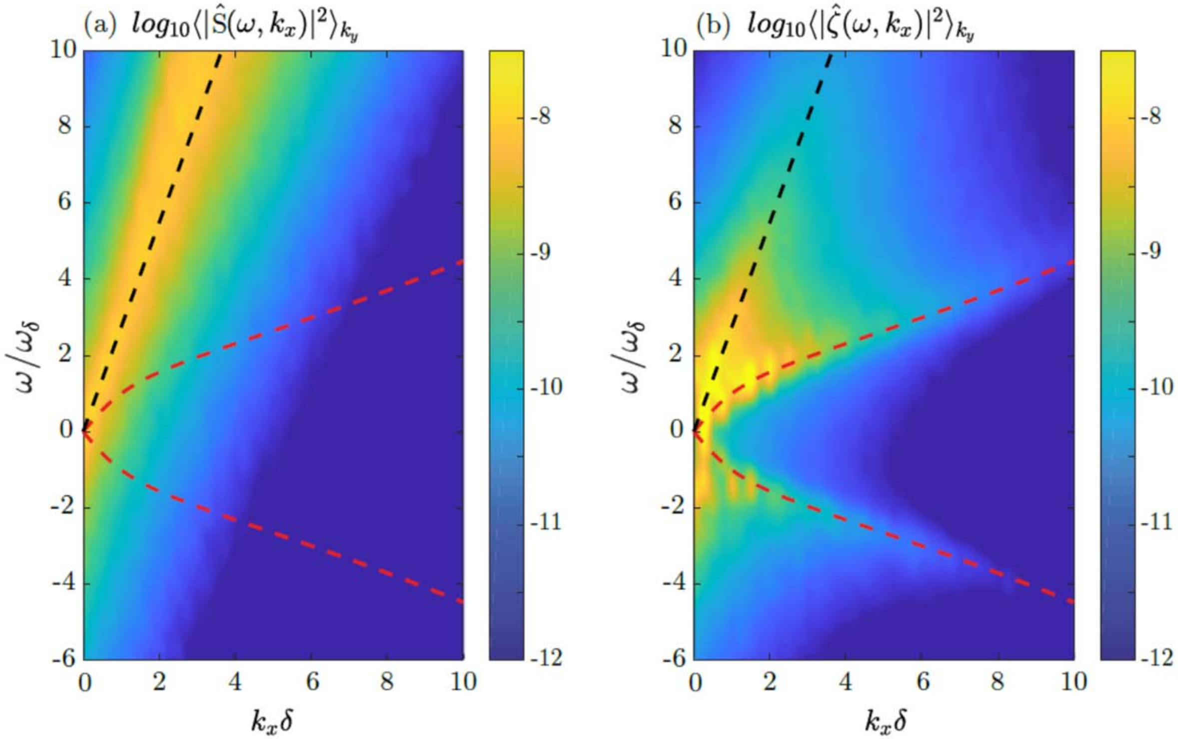
(a) pressure only
(b) shear stress only
(c) pressure and shear stress
8
4
0
0
4
8
12
y/δ
x/δ
ζ/δ
10^{-4}
0
-10^{-4}

True wrinkles vs. Synthetic wrinkles

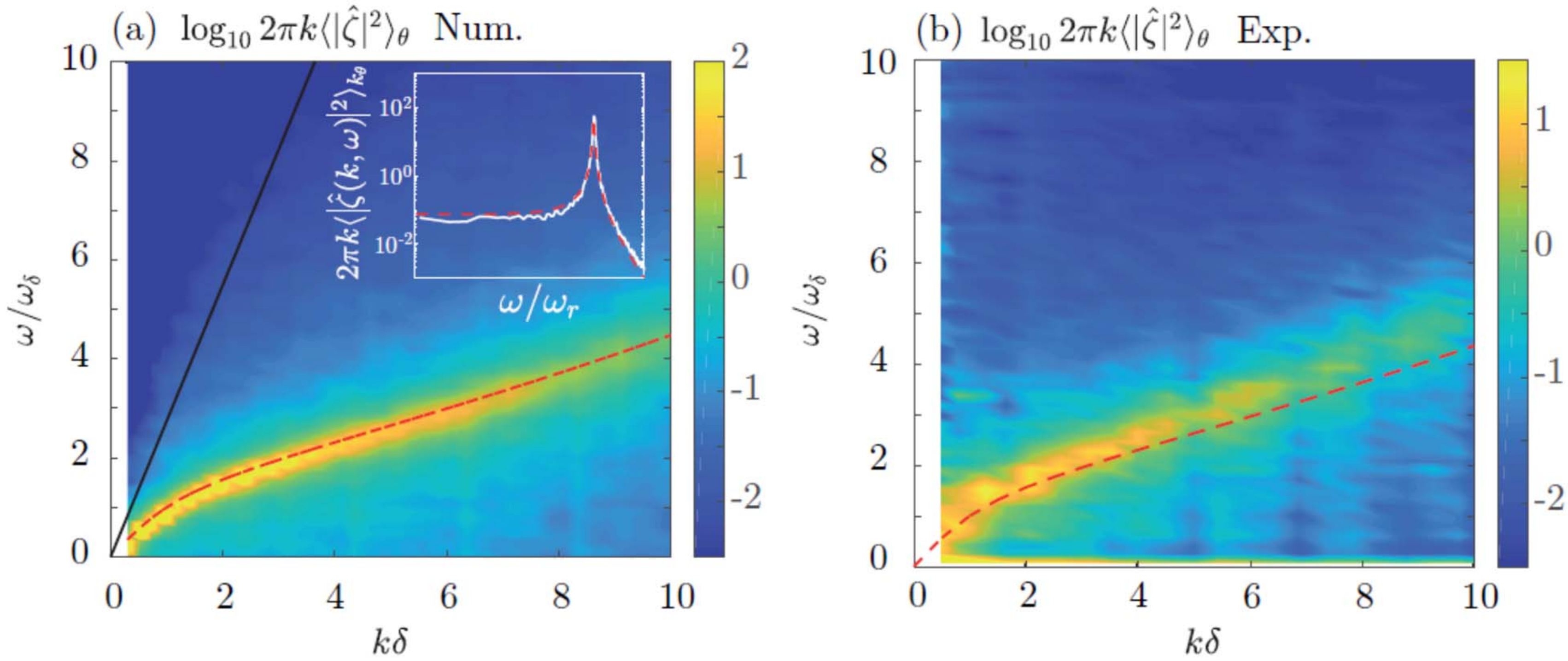




Source **Surface response**

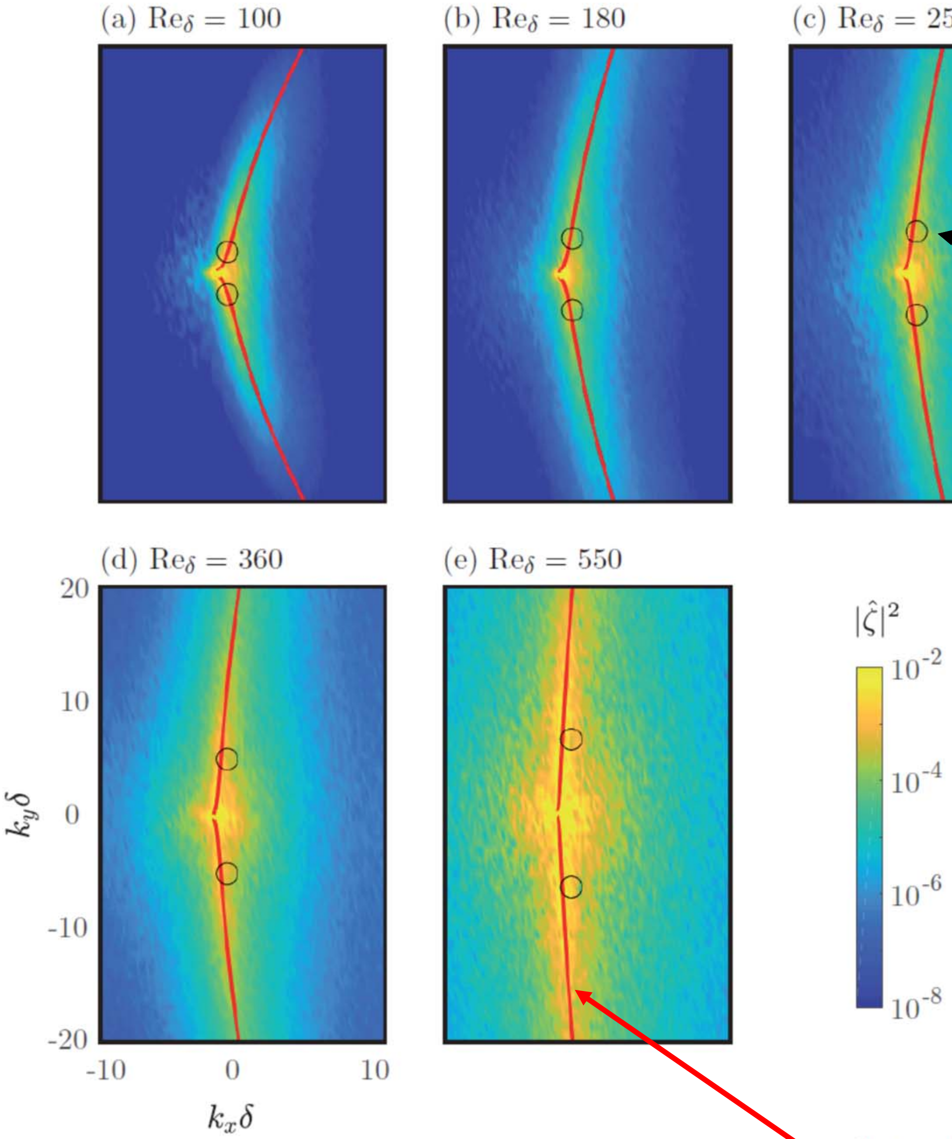


Surface response, in $(|k|, \omega)$ plane



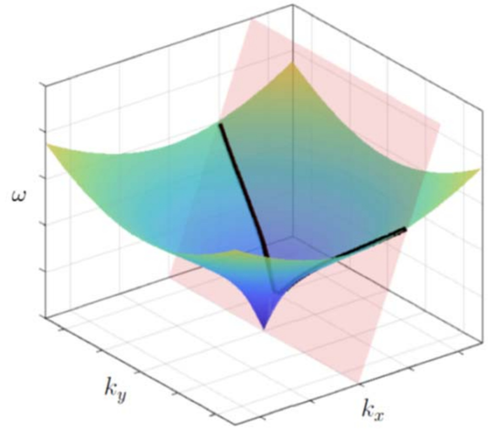
Most energy near the dispersion relation: resonant response

Surface response, in the (k_x, k_y) plane



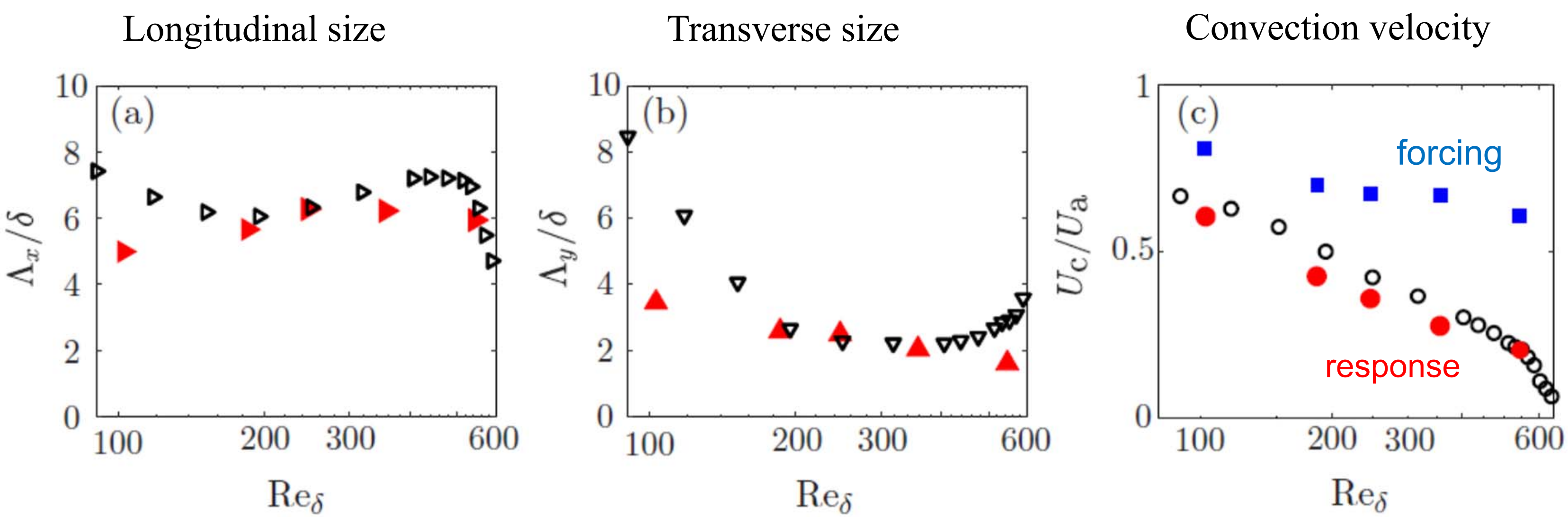
Spectral barycenter

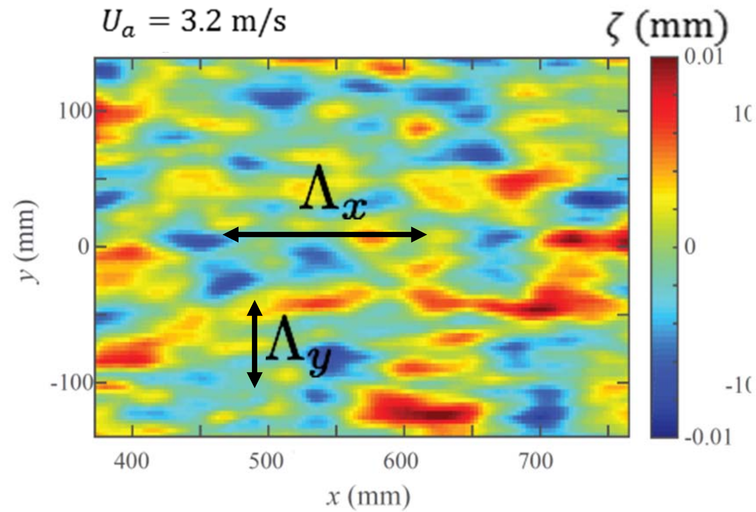
$$\boldsymbol{K} = K_x \mathbf{e_x} + K_y \mathbf{e_y} = \frac{\int_{\mathcal{D}} \mathrm{d}^2\boldsymbol{k}\mathrm{d}\omega\ \boldsymbol{k}|\hat{f}|}{\int_{\mathcal{D}} \mathrm{d}^2\boldsymbol{k}\mathrm{d}\omega\ |\hat{f}|}$$



$$gk\tanh(kh)(1 + (k\ell_c)^2) = (k_x U_c)^2$$

Comparison Experiment / Numerics





Scaling of the wrinkle amplitude

Space-time Parseval theorem (energy conservation)

$$\frac{\overline{\zeta^2}}{\delta^2} = \left(\frac{\rho_a}{\rho_\ell}\right)^2 \frac{u^{*3}}{\delta} \int \mathrm{d}^2\boldsymbol{k}\mathrm{d}\omega \, \frac{|\hat{S}^\dagger|^2}{(\omega^2 - \omega_r^2)^2 + \omega_\nu^2\omega^2}$$

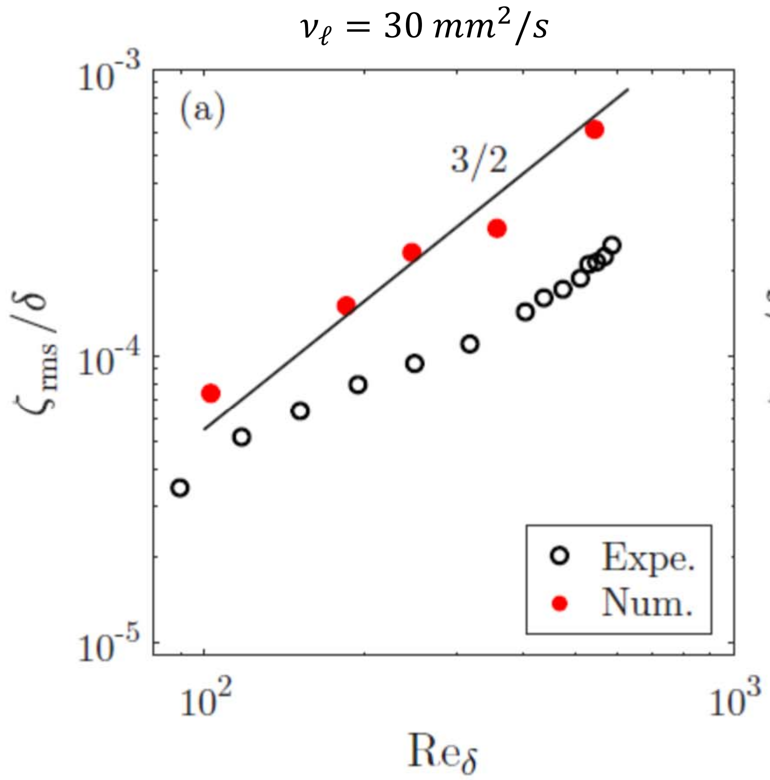
+ Assumption: wrinkle energy near the dispersion relation

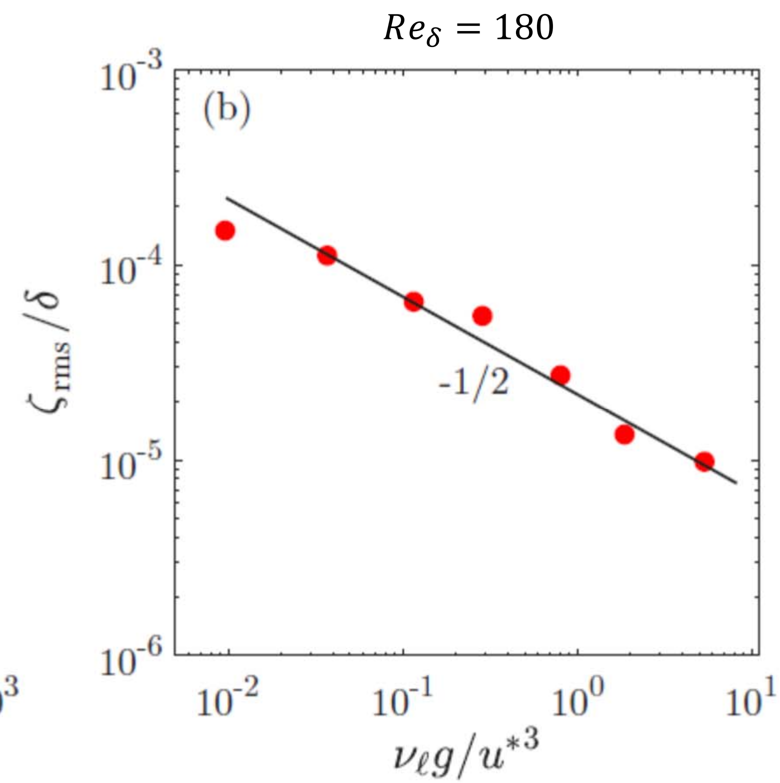
$$\langle|\hat{\zeta}^2|\rangle_\omega = \int \mathrm{d}\omega|\hat{\zeta}(\boldsymbol{k},\omega)|^2 = \left(\frac{\rho_a}{\rho_\ell}\right)^2 \frac{u^{*3}}{16 g \nu_\ell} W(\tilde{\boldsymbol{k}}) \, |\hat{S}^\dagger(\tilde{\boldsymbol{k}},\tilde{\omega}_r)|^2$$

Weighting factor $$W(\tilde{\boldsymbol{k}}) = \frac{1}{\tilde{k}^3(1+\mathrm{Bo}^{-2}\tilde{k}^2)\tanh(\tilde{k}h/\delta)}$$

For $Bo \gg 1$ and $h/\delta >\simeq 1$, $W(k) \simeq k^{-3}$

Shift of the surface response towards the upper bound of the forcing $[\delta_\nu, \delta]$





Good agreement with $$\frac{\zeta_{\rm rms}}{\delta} = \frac{\rho_{\rm a}}{\rho_\ell}\left(\frac{u^{*3}}{g\nu_\ell}\right)^{1/2} f_4\left(\mathrm{Re}_\delta, \mathrm{Bo}, \frac{h}{\delta}\right)$$

f_4 does not vary with Re_δ

(variations in Bo and h/δ not tested)

$$\frac{\zeta_{\rm rms}}{\delta} \simeq C\frac{\rho_a}{\rho_\ell}\frac{{u^*}^{3/2}}{(g\nu_\ell)^{1/2}}$$

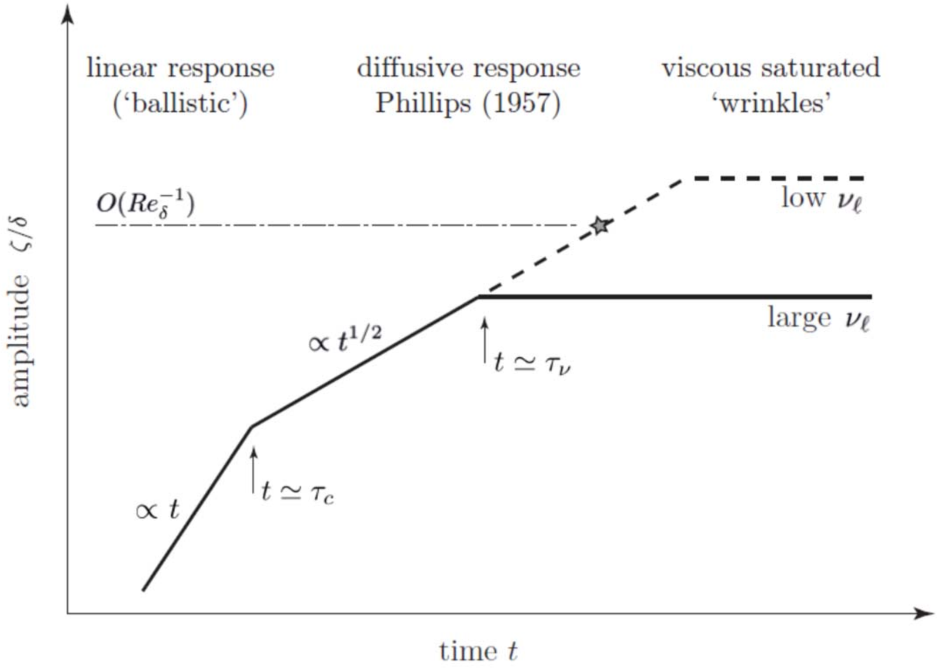
$C \simeq 0.022$

Transient before the wrinkle regime

Space Fourier transform (*not time*!) $$\breve{\zeta}(\boldsymbol{k},t) = \int \mathrm{d}^2\boldsymbol{r}\,\zeta(\boldsymbol{r},t)e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}$$

Damped oscillator, forced by noise (Langevin equation)

$$\partial_{tt}\breve{\zeta}(\boldsymbol{k},t) + 4\nu_\ell k^2\partial_t\breve{\zeta}(\boldsymbol{k},t) + g'k\breve{\zeta}(\boldsymbol{k},t) = -k\breve{p}_0(\boldsymbol{k},t) - i\boldsymbol{k}\cdot\breve{\sigma}_0(\boldsymbol{k},t)$$



$$\frac{\zeta_{\mathrm{rms}}}{\delta} \simeq C\frac{\rho_a}{\rho_\ell}\frac{u^{*3/2}}{(g\nu_\ell)^{1/2}}$$

Wrinkle = viscous-saturated Phillips regime!

Breakdown of linear (uncoupled) dynamics

Wrinkle amplitude viscous $\zeta \simeq A \times$ sublayer thickness $\delta_\nu = \nu_a/u^*$

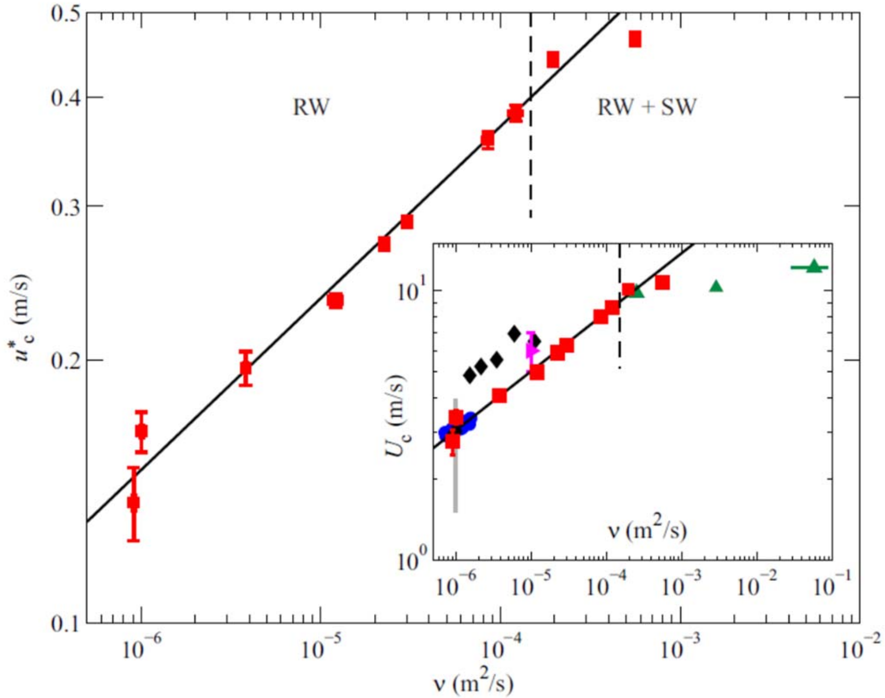
for a critical friction velocity

$$u_c^* = \left(\frac{A}{C}\right)^{2/5} \left(\frac{\rho_\ell}{\rho_a}\right)^{2/5} \left(\frac{g\nu_\ell\nu_a^2}{\delta^2}\right)^{1/5}$$

Excellent agreement with the

empiral law for regular wave onset!

$$u_c^* \simeq (2.3 \pm 0.2)\nu_\ell^{0.20}$$



Wrinkles = base state for regular wave growth

S. Perrard, A. Lozano-Duran, M. Rabaud, M. Benzaquen, subm. to JFM (2019)