Habilitation à diriger des recherches

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Instability dynamics in jets and plumes

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par

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1 Introduction

1.1 My research activities

The study of instability dynamics in jets, already the topic of my Ph.D. thesis [L65], has remained the focus of my research over the last nine years. My postdoctoral work (2007–2008) was concerned with sedimentation from river outflows and from near-coastal turbidity currents, and occasional collaborations with Eckart Meiburg's group at UC Santa Barbara on these topics continue [L20], but these studies are not included in the present manuscript for the sake of a coherent presentation of my main line of research.

My interests lie in the conceptual description of instability dynamics in jets and in jet-like open shear flows, based on a variety of methodological approaches befitting the different aspects of flow behaviour that these flows can exhibit. These aspects include oscillator and amplifier behaviour, laminar deterministic and turbulent stochastic dynamics, primary as well as secondary instability phenomena. The methodology employed for their characterisation relies on local and global formulations, modal and non-modal perturbation growth, statistical state dynamics, and Floquet theory. In all instances, the objective in my research is to uncover physical mechanisms and to identify the most appropriate framework for a conceptual modelling of the flow dynamics.

This first section of the manuscript provides a short overview of the flow phenomena that have been treated in my past research projects since 2009, and how these have been approached from a methodological point of view. A brief chronological account is attempted in §1.2, offering a deliberately subjective perspective on the context in which our current understanding of jet instability has evolved. This context is presented in broad strokes; references to the literature outside of jet studies are given sparingly and only in so far as they influenced my own direction of research.

1.2 Trends in jet instability over the last ten years

Global eigenmode analysis Scientific development, even within one discipline as close-knit as open flow instability, does not usually advance in lockstep. When I took my position at LadHyX in 2009, global eigenmode analysis (or "BiGlobal" in the diction of [57]) was the fashion of the time. This framework, which is based on the computation of temporal eigenmodes of the linear operator formulated for a non-parallel base flow, had been introduced already in the late 1980s (see the review by Theofilis [57]), yet its application to jet flows remained virgin territory. The Ph.D. work of Nichols [44], Coenen [12] and myself [L65] on jet instability was still entirely built on the *local* analysis of laminar steady base flows, even though all these works were concerned with self-sustained global oscillations in low-density jets. Our studies demonstrated that such oscillator behaviour is well characterised by the onset of absolute instability above a critical ambient-to-jet density ratio, although the general validity of such a critical value was challenged [13], and by means of adjoint-based optimisation indeed was shown not to exist [L6].

Meanwhile, the success of linear global eigenmode analysis of flows exhibiting self-sustained oscillations generated great enthusiasm in the instability community. Noack *et al.* [46], Barkley [3] and Sipp & Lebedev [55] had demonstrated that linear global instability in a laminar cylinder wake sets in at $Re = 47$, in perfect agreement with experimental observations. Marquet *et al.* [35] showed how adjoint-based sensitivity analysis of global eigenmodes can be leveraged for passive control design, and Giannetti & Luchini [20] proposed a global "wavemaker" definition that served as a basis for the physical discussion of instability mechanisms. Global eigenmode analysis, base flow sensitivity and the "wavemaker" formalism provided the blueprint for instability studies over many years.

The study of jets has benefited from this global toolbox with significant delay: linear global eigenspectra of supersonic jets were finally presented by Nichols & Lele [45], and for subsonic settings by Garnaud *et al.* [L10]. Both of these studies were carried out in parameter regimes where jets behave as *amplifiers* of external noise, as opposed to their *oscillator* behaviour in the presence of strong density gradients. Linear global spectra of jets in their oscillator regime, at low Reynolds number and high density ratio, have been published only very recently [L16],[L18].

Input-output analysis For several years, global eigenmode analysis was the standard tool of choice for the study of oscillator- as well as amplifier-type flow configurations. On the one hand, the qualitative characterisation of amplifier flows as being *stable* from this point of view is certainly a matter of poor semantics; on the other hand, decaying eigenmodes are not particularly useful objects for a quantitative analysis of amplifying flow behaviour. The least stable eigenmodes describe the asymptotic perturbation dynamics at long times after an initial perturbation, and in an amplifier flow that asymptotic limit is ultimately zero[1]. Short-time growth is *non-modal* and can be characterised by the gain of the optimal initial perturbation, as done for jets in references [45] and [L10].

In the context of local theory, it had been established long ago that amplifier behaviour is consistently described in the *frequency domain* (the "spatial problem", see Michalke [37]). A non-modal formalism in the frequency domain, suitable for non-parallel flow instability problems, had been introduced by Trefethen *et al.* [59], but it was only hesitantly accepted in the global instability community, following its application to boundary layers [34, 42, 24]. Based on singular value decomposition (SVD)

[1] "But this long run is a misleading guide to current affairs. In the long run we are all dead. [Scientists] set themselves too easy, too useless a task if in tempestuous seasons they can only tell us that when the storm is past the ocean is flat again." Keynes [27].

of the resolvent operator, this analysis method is known under many names in the literature: optimal forcing, frequency response, resolvent or input-output analysis — the latter will be used throughout this manuscript. Garnaud *et al.* [L11] performed such input-output analysis of jet flows, both in incompressible and in compressible settings (see [L46] for compressible jets with acoustic radiation). These studies used laminar steady solutions as well as turbulent mean flows as base flows, even though the theoretical justification for mean flow analysis seemed unclear at the time.

Linear wavepackets in turbulent mean flow Frequency-domain analysis of linear perturbation wavepackets in turbulent jet mean flows has been pursued at Pprime and at Caltech over the last ten years, motivated by the prospect of obtaining dynamical models for the prediction and the control of jet noise.

The classical framework of flow instability analysis is based on the linearisation of governing equations about a steady flow state, which provides a consistent description of small-amplitude perturbation dynamics. In practice, however, such a steady solution is often replaced with a time-averaged mean flow; the motivation may be simply that a mean flow is more readily available, or that a laminar steady state poorly represents the spatial features of a turbulent flow. In the case of a turbulent jet at high Reynolds number, the mean flow spreads much more rapidly than the corresponding laminar steady state, and it seems inappropriate to model perturbations as if they evolved in a nearly parallel base flow.

A large body of literature suggests that the dominant large-scale fluctuations in turbulent jets behave like linear instability waves developing in the jet mean flow [26]. Such instability waves, or indeed “wavepackets”, have in the past been modelled by local spatial theory [56] and by PSE [21]. Application of resolvent-based input-output analysis to an experimental mean flow at high Reynolds number, and validation of the results against unsteady flow measurements, was the proposition of the “Cool Jazz” project (2013–2016). Cool Jazz was funded by the Agence Nationale de la Recherche, and associated researchers at LadHyX, Pprime and Limsi. Towards the end of this project, it was realised that the analogy between spectral POD modes of the turbulent jet and linear input-output modes has a clear mathematical foundation. The theory of statistical state dynamics, pioneered by Farrell & Ioannou [16], provides the adequate formalism for the analysis of mean flow and “jittering” wavepackets [9] as *statistical* objects. The implications of this description for turbulent shear flows are currently being explored by various research teams [5, 8], particularly for the case of jets [L57][58, 53][L22].

Instability and control of periodic flows The above developments indicate that trends are driven to a large extent by available methodology. The current Ph.D. work of Léopold Shaabani Ardali aims to extend the established techniques for non-modal instability analysis and optimal control to the class of periodic base flows. Similar efforts are currently being conducted by other research groups [50, 43]. A pulsed jet is chosen as the object of our study; subject to axisymmetric harmonic forcing, such a jet forms ring vortices at the forcing frequency. Secondary instabilities arise in the form of vortex pairing, which may be analysed by way of Floquet theory. Additional non-axisymmetric forcing is known to give rise to the phenomenon of "bifurcation" [49]. Optimisation of such active control requires strategies different from singular value decomposition, as it is used in the case of steady base flows.

Tools developed in this context may prove useful in a wide array of applications, such as periodic flow in turbo engines, behind flapping wings, and in blood vessels.

1.3 Organisation of this manuscript

Results of my research on jets and plumes from the past nine years are summarised in the following sections, grouped into the three categories

1. Extrinsically driven oscillations (§2), described by input-output relations developed around a steady base flow,

2. Intrinsic oscillations (§3), described by modal instabilities of a steady base flow,

3. Secondary instabilities (§4), described by the modal and non-modal evolution of perturbations in a periodic base flow.

Perspectives on the application of these concepts, to reacting flows and to jet noise predictions in complex settings, are given in §5. Publications and conference presentations that I have co-authored are listed in §6, followed by general literature references.

2 Extrinsically driven oscillations in jets

The amplifier behaviour of jets, i.e. their linear response to external perturbation input, in the absence of strong density variations, was the topic of Xavier Garnaud's Ph.D. work [19]. Laminar steady states as well as turbulent mean states were considered, both under incompressible and under compressible conditions. These studies were continued by Onofrio Semeraro, during his postdoc project 2013–2016, who used experimental mean flows and compared quantitatively the linear flow response to experimentally measured spectral POD modes.

Note that all flow configurations in this section represent jets that are of the same chemical composition and the same temperature as the ambient fluid. If furthermore the incompressible limit $Ma = 0$ is considered, the density is strictly constant in these flows; however, at non-zero Mach number, small density variations arise from compressibility effects. In the following, these configurations (incompressible as well as compressible) will be denoted as *homogeneous* jets, in order to distinguish them from the *inhomogeneous* settings investigated in §3, which involve strong density variations between the jet interior and the ambient atmosphere.

2.1 Linear eigenmodes of homogeneous jets

[L9] X. Garnaud, L. Lesshafft, P. Schmid & J.-M. Chomaz (2012): A relaxation method for large eigenvalue problems, with an application to flow stability analysis. *J. Comp. Phys.* vol. 231, p. 3912–3927

[L10] X. Garnaud, L. Lesshafft, P. Schmid & P. Huerre (2013): Modal and transient dynamics of jet flows. *Phys. Fluids* vol. 25, art. 044103

Motivation Xavier Garnaud's investigation of the forcing response in jets set out from the hypothesis that a *slightly damped eigenmode* exists, which would be easily excited by low-amplitude forcing input. This conception, suggested by Huerre & Monkewitz [23], would be consistent with the typical observation of a "preferred mode" in amplifier jets, characterised by a distinct maximum of perturbation amplitude in a turbulent jet around a Strouhal number of 0.3 [14].

Methodology The computation of converged global jet eigenmodes in *compressible* settings turned out to be fraught with technical difficulties. Due to high-order finite-difference schemes and large flow domains, which were used in order to capture the acoustic field, the need for matrix inversion as part of the classical shift-invert technique led to computational resource requirements that appeared to be too restrictive.

A new computational method, named the "shift and relax" technique [L9] was developed for these compressible eigenmode calculations. The method is matrix-free and only requires the memory needed for time-stepping of an augmented system. Akin to the “selective frequency damping” [1], a set of auxiliary filter equations is added to the regular system of linear flow equations, and a coupling is prescribed in such a way that eigenmodes are damped with increasing distance from a chosen shift value. The resulting eigenmode calculations are rather time-intensive, but very light on memory, such that they can be run on workstations.

Incompressible problems were discretised on an unstructured grid with finite elements (FreeFEM++), and eigenmode spectra were efficiently found with standard ARPACK and SLEPc routines. These calculations are easily performed on a single processor.

Results Stable spectra were obtained in all cases, mostly characterised by a more or less flat branch of eigenvalues. The spectrum for a laminar incompressible jet at $Re = 1000$ is shown in figure 1: modes plotted as black crosses form an ‘arc branch’ [L19], brought about by spurious feedback from the outflow boundary (see §3.3.2). The modes shown as red plus signs were linked to the limited accuracy of the numerical scheme, which allows the resolution of global amplitude variations over not more than 15 decades [L10].

Eigenvalues of a laminar compressible jet, computed with the “shift and relax” technique, are shown in figure 2. The base flow, at $Re = 100$ and $Ma = 0.75$, was obtained using selective frequency damping. The eigenvalues again form a branch that is likely to arise from spurious boundary feedback. Eigenmode perturbations of vorticity are represented in figure 3, corresponding to the labels in figure 2. These structures have the typical appearance of Kelvin-Helmholtz wavepackets that originate at the nozzle ($x = 0$), coupled with Tollmien–Schlichting waves in the boundary layers of the inflow pipe.

None of the computed spectra exhibited a slightly stable discrete eigenmode that could convincingly be interpreted as the origin of the “preferred mode” flow response to low-amplitude forcing. It had become clear that temporal eigenmodes are not an adequate basis for the description of extrinsically driven flow oscillations.

2.2 Deterministic input-output analysis of incompressible jets

[L11] X. Garnaud, L. Lesshafft, P. Schmid & P. Huerre (2013): The preferred mode of incompressible jets: linear frequency response analysis. *J. Fluid Mech.*, vol. 716, p. 189–202

Motivation Extrinsically driven flow oscillations in a jet are to be expressed in a modal basis that genuinely reflects their input-output character, and ideally, such a basis would be orthogonal. These are the properties of *singular value decomposition*

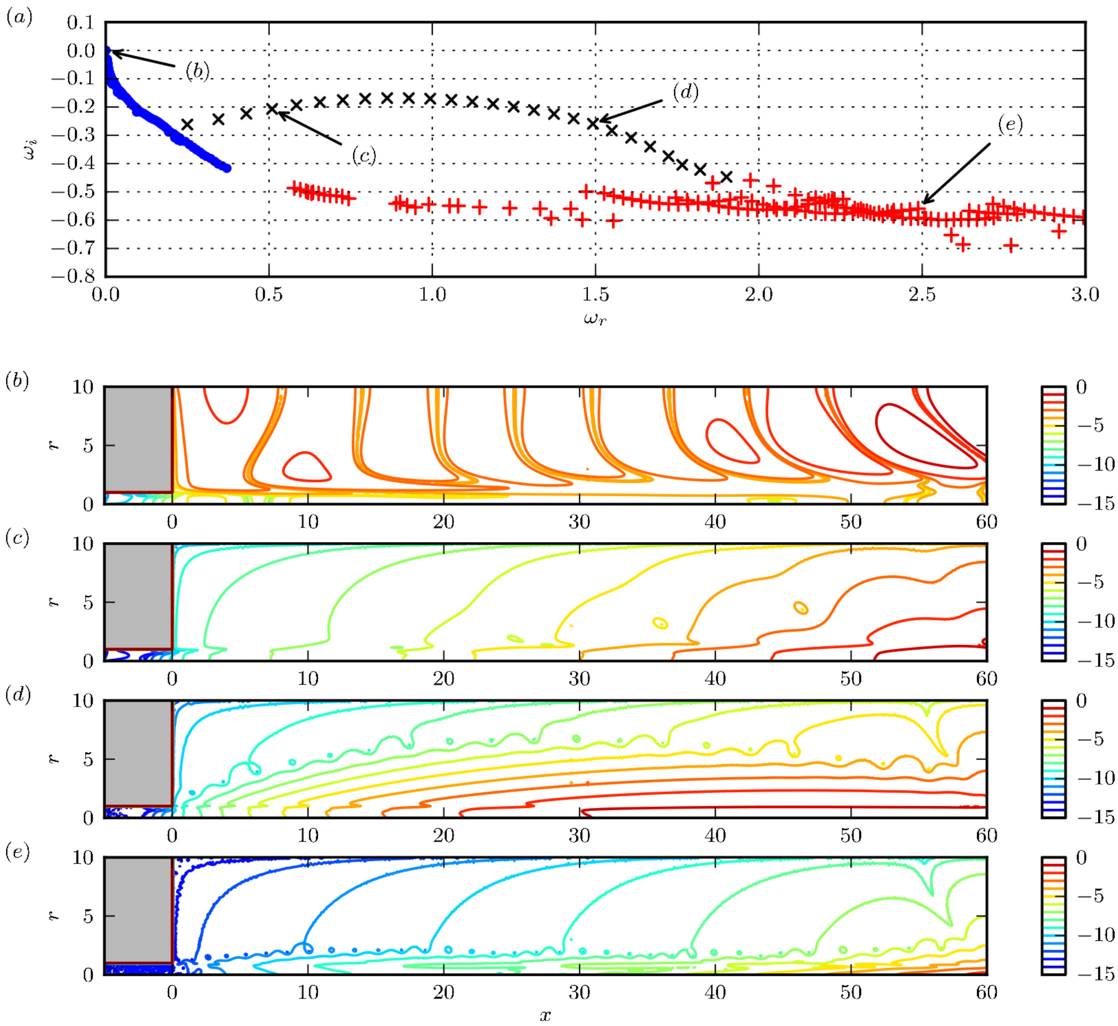


Figure 1: Eigenmodes of axisymmetric perturbations in a laminar incompressible jet at $Re = 1000$. a) Eigenvalues; b)-e) modulus of axial velocity perturbations, $\log_{10}|u|$, of individual modes, as labeled in (a). From reference [L10].

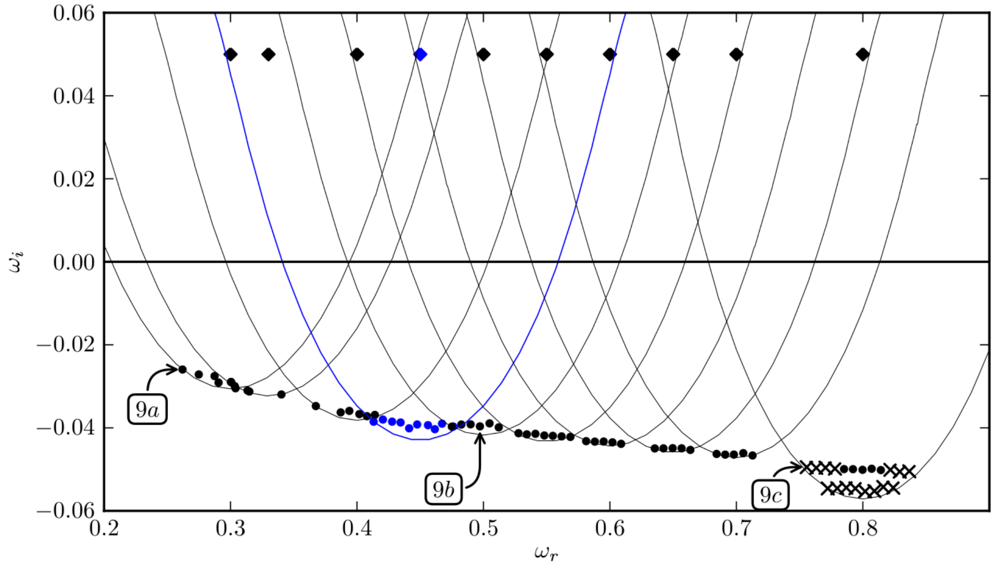


Figure 2: Eigenvalues of axisymmetric perturbations in an isothermal jet at $Re = 100$ and $Ma = 0.75$, computed using the "shift and relax" method. Diamond markers indicate shift values. From reference [L9].

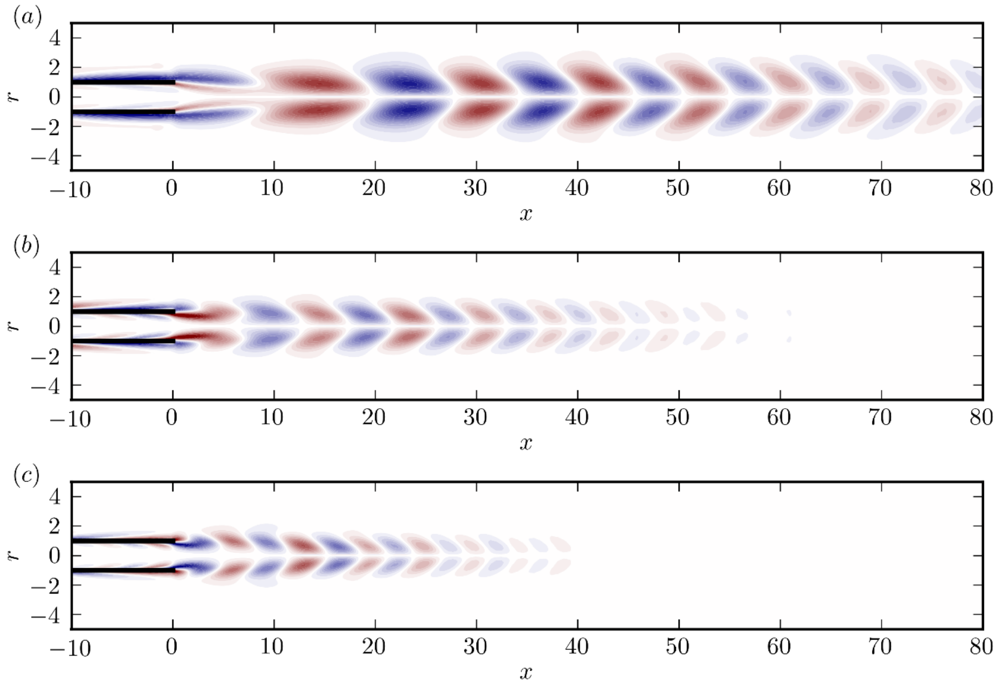


Figure 3: Real part of the vorticity perturbations of typical eigenmodes, labeled in figure 2. From reference [L9].

(SVD). Following the eigenspectrum analysis described in the previous section, Xavier Garnaud proceeded to apply an SVD-based formalism to an incompressible jet, in order to test if the "preferred mode" at $St \approx 0.3$ could be captured and characterised in this way.

Methodology SVD-based input-output analysis has been introduced for flow instability problems by Trefethen *et al.* [59], and has been applied to a number of *parallel* flow situations in the following years. The application of such a formalism to *non-parallel* problems has been demonstrated, to my knowledge, for the first time by Alizard *et al.* [2] for a separated boundary layer, using a reduced-order eigenmode representation of the linear flow system. Monokrousos *et al.* [42] computed leading singular modes in a Blasius boundary layer without the need for eigenmode expansion. In the context of Xavier Garnaud's PhD thesis, our goal was to use the same formalism on jet flows.

The governing equations are linearised around a steady base flow. These equations include a source term f that represents a volume force, as a model for external perturbation input. After a temporal Fourier transform, the linear system is written as

$$(-i\omega B + L)\hat{q} = \hat{f}. \tag{1}$$

The operator $(-i\omega B + L)^{-1}$ that maps any given forcing $\hat{f}e^{-i\omega t}$ onto its time-asymptotic linear flow response $\hat{q}e^{-i\omega t}$ is called the *resolvent* operator [52]. Its matrix SVD representation, $(-i\omega B + L)^{-1} = U\Sigma V^H$, associates each column vector v_i of matrix V with a real gain value σ_i and a column vector u_i of matrix U. The sets of v_i and u_i are both orthonormal among themselves. It follows that the pair (v_i, u_i) with highest associated gain σ_i represents the *optimal forcing* and *response* structures of frequency ω. The resolvent operator can be tweaked prior to the SVD in order to account for specific gain definitions and forcing restrictions.

For any given frequency, a discrete resolvent matrix was constructed with FreeFEM++, using similar tools as in the incompressible eigenmode computations described in §2.1. Singular value decomposition was performed on regular workstations, with routines from the MUMPS and SLEPc libraries, called from Python.

Results An analytical model of a turbulent jet mean flow [41] was chosen as a base flow, with a straight pipe section upstream of the nozzle, and the Reynolds number was fixed at $Re = 1000$ for the linear perturbation computations. Forcing input was restricted to the interior of the pipe, the rationale being that stochastic random fluctuations enter the flow from the nozzle, whereas no volume forces are present in the free jet. The jet was treated as a purely linear flow system, for lack of clear ideas on how to model the effect of nonlinearity, especially in the face of the incoherent nature of turbulent fluctuations.

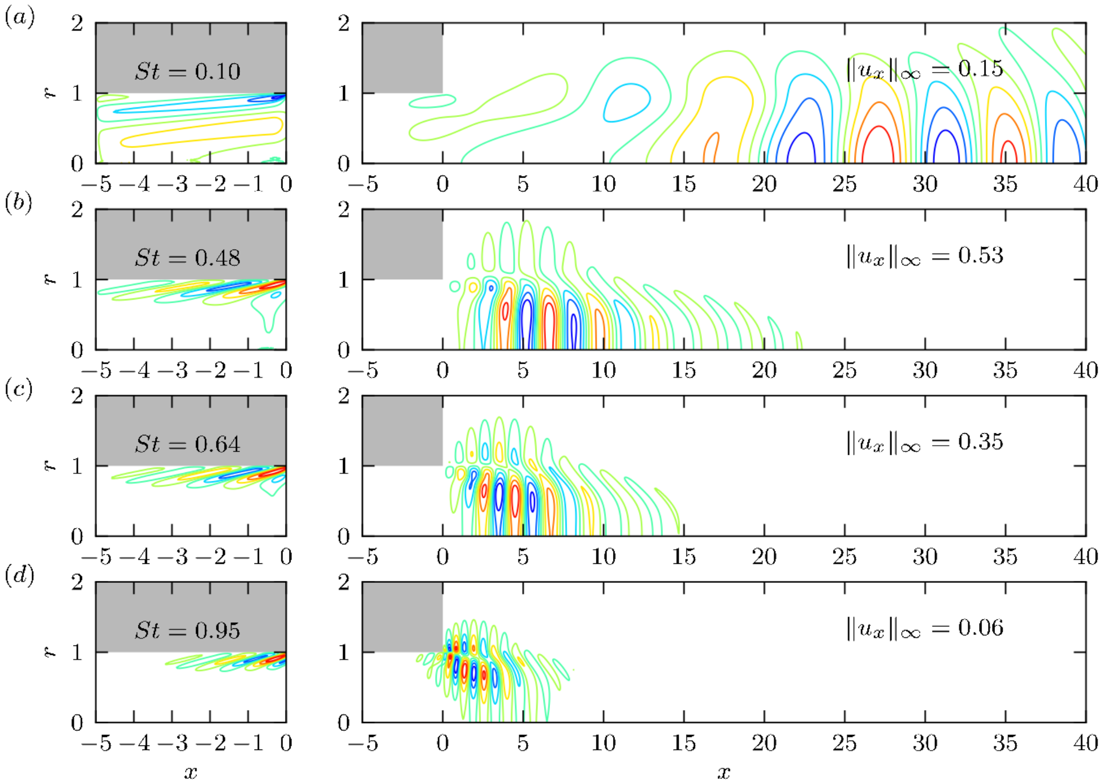


Figure 4: Input-output analysis of an incompressible turbulent jet mean flow: spatial structures associated with optimal body forcing at different Strouhal numbers, indicated in the figures. Left column: axial component of optimal forcing; right column: axial velocity of associated flow response. From reference [L11].

Optimal forcing in this base flow, at all frequencies, takes the shape of tilted vortical structures near the pipe wall, with maximum amplitude at the nozzle exit. These structures, shown on the left side in figure 4, are suggestive of the Orr-mechanism in boundary layers, which give rise to strong energy growth over short convection distances. The ensuing flow response, shown on the right side in figure 4, clearly represents a free-jet wavepacket that grows and eventually decays due to shear instability. It was demonstrated that the initial spatial growth of perturbation amplitude in the response wavepacket corresponds well to the local growth rate of a spatial shear instability mode. The maximum input-output energy gain was achieved at a Strouhal number around $St_D = 0.45$, in decent agreement with typical measurements of the "preferred mode" in turbulent jets.

2.3 Deterministic input-output analysis of compressible jets

[L46] X. Garnaud, R. Sandberg & L. Lesshafft (2013): Global response to forcing in a subsonic jet: instability wavepackets and acoustic radiation. *AIAA Paper* 2013-2232

[L15] O. Semeraro, L. Lesshafft, V. Jaunet & P. Jordan (2016): Modeling of coherent structures in a turbulent jet as global linear instability wavepackets: theory and experiment. *Int. J. Heat Fluid Flow* vol. 62, p. 24–32

Methodology As the resolvent matrix of highly-resolved compressible flow systems is unpractically large, a direct-adjoint time-stepping strategy [31] was chosen for the computations presented in this section. Jet noise analysis was one objective of these studies from the outset; we therefore opted for high-order finite-difference discretisation on orthogonal grids. Accurate adjoint time-stepping is achieved by a modular construction technique [18]. These compressible computations require significantly more resources than the incompressible studies presented in the preceding sections, and they were all run on the HPC platforms of TGCC and CINES.

Results In two separate studies, we investigated optimal forcing and associated linear response structures in the mean flow of turbulent compressible jets. The mean flow used by Garnaud *et al.* [L46] was provided by Richard Sandberg, obtained by direct numerical simulation of a jet issuing from a long straight pipe, at $Re = 3691$ and $Ma = 0.84$, and with significant co-flow. The study by Semeraro *et al.* [L15] used an experimentally measured mean flow at $Re = 10^6$ and $Ma = 0.9$, without co-flow, made available by Peter Jordan and his group.

Following the same conceptions as in the incompressible analysis of section 2.2, forcing input was again restricted to the interior of the pipe. Both compressible studies led to similar results. A clear peak in the energy gain is found, at $St_D = 0.8$ in the co-flowing configuration [L46] and at $St_D = 0.4$ in the case of [L15]. Optimal forcing structures as well as flow response wavepackets resemble those identified in the incompressible case, except for the added presence of acoustic waves in both.

Garnaud *et al.* [L46] compared the linear response wavepackets to Fourier modes extracted from the DNS by Richard Sandberg and found good agreement in the near-nozzle region of the free jet. More strikingly, the *acoustic far field* from the DNS is well reproduced, displaying beam-like radiation patterns, as shown in figure 5. Only at the highest frequency $\omega = 4$ (or $St_D = 1.27$), the linear flow response in figure 5c exhibits a strong upstream lobe, which is absent in the DNS Fourier mode.

An additional point of interest in the analysis of the experimental base flow [L15] lies in the use of a spatially distributed *turbulent viscosity*, deduced from Reynolds stress measurements. Including this in the linear model has the predictable effect of

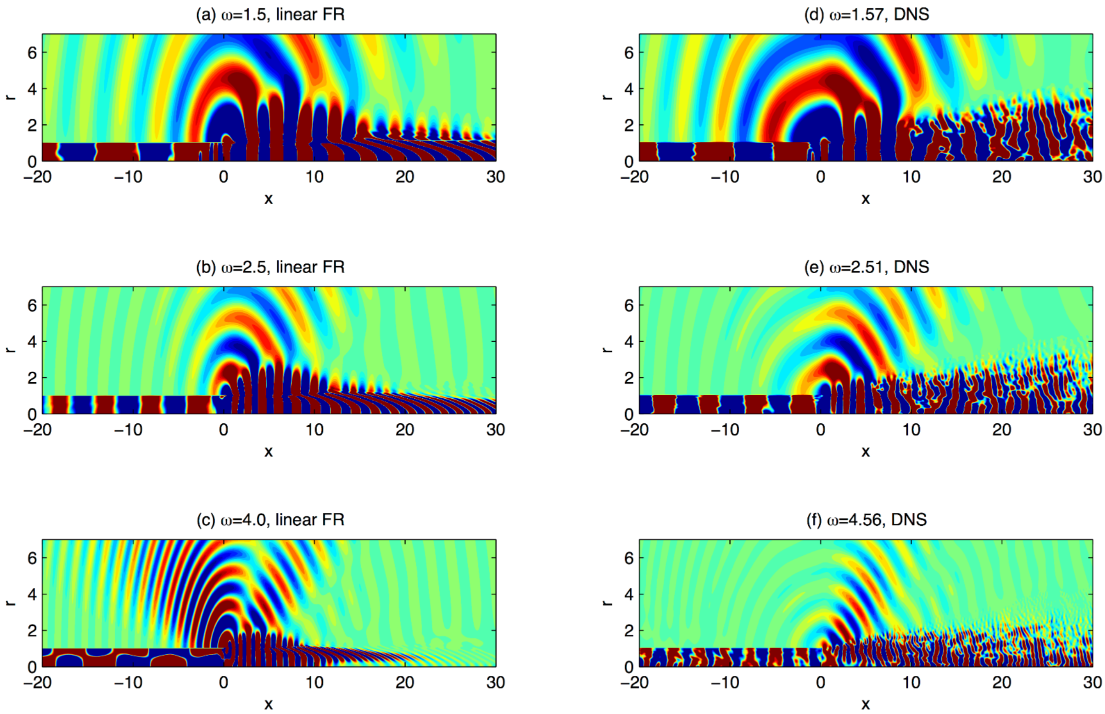


Figure 5: Comparison between the linear flow response to optimal forcing (left column) and Fourier modes extracted from DNS data (right column): density fluctuations at various frequencies $\omega = \pi St_D$. Contour values are chosen such as to visualise the acoustic field. From reference [L46].

lowering the global energy gain and shortening the streamwise extent of the response wavepackets.

2.4 Stochastic input-output analysis of compressible jets

[L57] O. Semeraro, V. Jaunet, P. Jordan, A.V.G. Cavalieri & L. Lesshafft (2016): Stochastic and harmonic optimal forcing in subsonic jets. *AIAA Paper* 2016-2935

[L22] L. Lesshafft, O. Semeraro, V. Jaunet, A.V.G. Cavalieri & P. Jordan (2018): Resolvent-based modelling of coherent wavepackets in a turbulent jet. ArXiv preprint 1810.09340, submitted to *Phys. Rev. Fluids*

An important step forward in jet instability research has been made over the last two years, following the realisation that optimal linear input-output structures are closely related to *spectral POD* modes of the fluctuations in turbulent jets. This term is used in the sense of reference [47], denoting eigenmodes of the cross-spectral density tensor, and thereby characterising two-point statistics in frequency-space. The theoretical basis for this relation is partially described by Dergham *et al.* [15], Hwang & Cossu [24] and Beneddine *et al.* [5]. More complete developments are given in references [L57], [58] and [L22].

Motivation While it had been acknowledged for a while, and the evidence reviewed in detail by Jordan & Colonius [26], that coherent structures in fully turbulent jets strongly resemble linear instability wavepackets developing in the *mean flow*, the theoretical justification for this analogy remained elusive until very recently. PSE calculations of perturbations in mean flows had been shown to reproduce spectral POD modes, or at least filtered power-spectral density distributions [21, 54], with high accuracy over several jet diameters downstream of the nozzle. Our own linear response wavepackets, obtained from fully global input-output analysis (see the previous section), promised to allow even more accurate predictions. Beneddine *et al.* [L14] had just demonstrated very good agreement between global response wavepackets and spectral POD modes in turbulent flow over a backward-facing step. Based on the argument that the anonymous forcing term $\hat{f}$ in the linear input-output relation (1) can be interpreted as a representation of turbulent Reynolds stress fluctuations, they had worked out that such agreement may be expected in flow regimes where the largest singular value is much greater than the second-largest ("gain separation").

Furthermore, together with André Cavalieri and Peter Jordan, we elaborated an approach to jet turbulence based on the dynamics of covariances [16], in parallel with colleagues at Caltech and Stanford. It is easily demonstrated that spectral POD modes and the singular modes of the mean-flow resolvent are indeed identical under the strongly idealising hypothesis that the Reynolds stress fluctuations consist of white noise [L57],[58]. This assumption, however, is not required in cases of large gain separation. Jets with thin initial shear layers represent such a case.

The overarching question of our stochastic jet analysis is therefore: can the dominant coherent structures in jet turbulence be accurately modelled by linear input-output analysis, which requires the turbulent mean flow as the only *a priori* information?

Methodology The stochastic input-output analysis requires the same computational tools as the deterministic studies discussed in section 2.3. A hierarchical set of orthogonal forcing input and response output structures was computed, for the resolvent operator that stems from linearisation of the flow equations around the mean flow. As developed in [L57], and more fully in [L22], the deterministic resolvent operator provides the relation between the cross-spectral densities of the forcing (the Reynolds stresses) and of the response (the turbulent fluctuations).

The acquisition of experimental reference data for a spatially resolved representa-

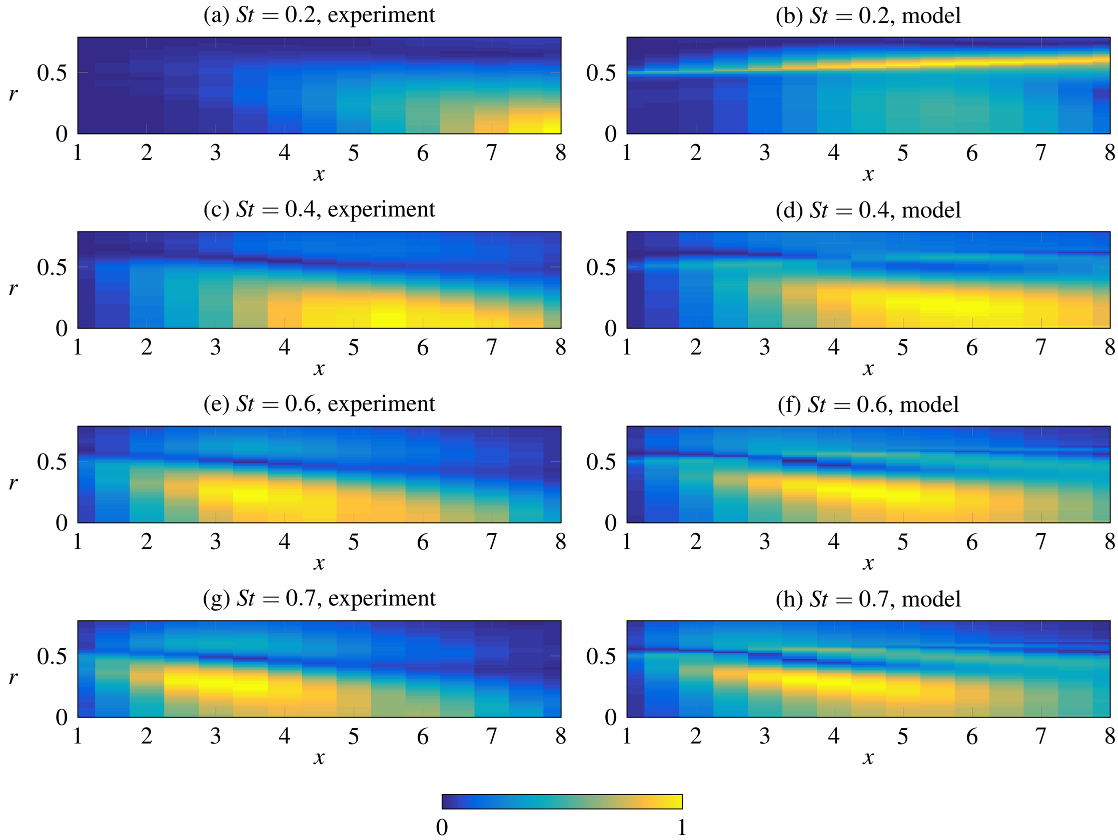


Figure 6: Comparison between the leading spectral POD modes extracted from experiments (left column) and from the linear model, based on the first five output modes (right column). Absolute values of axial velocity fluctuations are shown in linear scale. Taken from [L22].

tion of two-point correlations is an ambitious undertaking. Vincent Jaunet and Peter Jordan at the Pprime Institute performed synchronised dual-plane TR-PIV measurements in a jet at $Ma = 0.4$ and $Re = 460\,000$, gathering correlated data in cross-stream planes at 15 different streamwise locations. About 20 terabytes of image data were recorded [25].

A model cross-spectral density matrix was then constructed, at various Strouhal numbers, from the five leading linear output modes, under the strong assumption that the forcing is given by spatially uncorrelated noise. Eigenvectors of this linear model matrix provide our prediction for spectral POD modes of the turbulent jet.

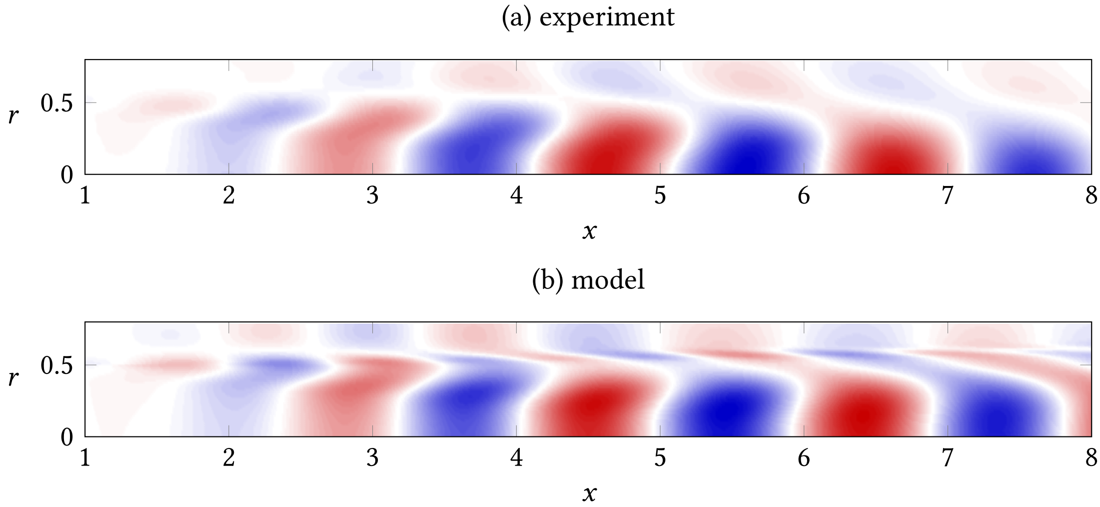


Figure 7: Real parts of spectral POD (experiment and model) at the dominant Strouhal number, corresponding to figures 6c and 6d. The data is interpolated between measurement locations in x. Taken from [L22].

Results The mean flow of the jet experiment was used as a basis for linearisation, and its careful inter- and extrapolation onto the numerical mesh was an important step towards clean analysis results. Frequency-resolved wavepackets corresponding to the dominant coherent structures (leading spectral POD modes) in the experimental data are compared to their numerical counterparts, computed as the leading eigenvectors of the model cross-spectral density matrix, in figure 6. Except at the lowest Strouhal number (figures 6a and 6b), the agreement between the two is remarkable. Figure 7 gives a more detailed comparison at the most amplified Strouhal number $St_D = 0.4$ (the "preferred mode" [14]), showing the real part of axial velocity fluctuations. The data had to be interpolated between the 15 measurement positions in x for this plot.

It can be noted that the computed wavepackets extend slightly further in the streamwise direction than the experimental ones; including the effect of turbulent dissipation would help to improve the agreement further. However, at present it is not obvious how such turbulent dissipation is best modelled. The principal limitation of the analysis at this point lies in the assumption of perfectly uncorrelated Reynolds stress fluctuations ("white noise forcing"). Current efforts, together with our collaborators, aim at a more realistic characterisation of the Reynolds stress statistics.

3 Intrinsically driven oscillations in jets and plumes

If the density of a jet is significantly lower than that of the ambient fluid at rest, a spontaneous onset of synchronised flow oscillations may be observed. Such oscillator behaviour, characterised as a global instability, has been observed experimentally in situations where the density variations were either due to heating [40] or due to the mixing of air with helium [28, 22, 61]. In all these studies, intrinsic oscillations appear in the form of a regular roll-up of axisymmetric ring vortices.

The link between these oscillations and absolute instability, a *local* concept, was the subject of my Ph.D. thesis [L65]. My more recent research, which will be presented in the present section, focused on a *global* characterisation of the linear instability behind intrinsic oscillations in jets.

The Ph.D. thesis of Chakravarthy covered both local and global instability in circular *plumes*. These flows differ from jets in the one important aspect, that their momentum is generated by buoyancy. The question to what extent the instability dynamics of plumes are determined by buoyancy was largely unexplored before Chakravarthy's analysis.

Motivation The earlier local instability studies of jets necessarily relied on the assumption of slow streamwise flow variations. In jets, this assumption is questionable, because the region near the nozzle exit, where local instability is most pronounced, presents strong shear layer growth. Also, the nozzle itself represents a geometrical singularity in the flow, which may be very important for global pressure feedback effects [L24]. The objective of linear global instability analysis was to fully account for the non-parallelism of the base flow, and to characterise the mechanisms by which light jets may become globally unstable.

A second important question concerned the role of buoyancy as an instability mechanism in jets and plumes. In previous local jet instability studies, buoyancy, if it was at all included in the governing equations, had been found to be unimportant for the instability dynamics. Would it become an important ingredient in plumes?

Methodology Light jets and plumes were treated as two branches of one family of flows, arising from the injection of low-density fluid into a higher-density ambient through a circular orifice. Light jets are then characterised by a low Richardson number, which measures the effect of buoyant acceleration with respect to the injected momentum, whereas plumes are characterised by a high Richardson number. The same set of equations was used for both flow regimes: the so-called low-Mach-number approximation of the compressible Navier–Stokes equations [36] fully accounts for the dynamic effects of density variations, but it does not allow density to increase due to compressibility by pressure. This corresponds in fact to the limit of *zero* Mach number.

For all global calculations, the linearised low-Mach-number equations were discretised by finite elements in the FreeFEM++ environment, by extending the code originally developed by Xavier Garnaud for incompressible homogeneous jets (see §2.1). The parameters in the equations were the Richardson number, the density ratio $S = \rho_{jet}/\rho_\infty$ and the Reynolds number; the inflow condition introduced the boundary layer thickness at the orifice as an additional parameter.

The identification of *physical mechanisms* is an essential element of the global instability studies presented in this section. In some cases, such mechanisms may be inferred in an *ad hoc* fashion from an inspection of the eigenmode structures, if plausible narratives about the cause-and-effect relationship between different perturbation quantities may be constructed. (In the context of instability, which always relies on positive feedback effects, the relation between cause and effect is analogous to the relation between hen and egg.) However, a more universal formal approach for the identification of instability mechanisms in eigenmodes has been proposed in an unpublished article [L67]. This formalism is briefly outlined in section §3.3, together with a study on spurious eigenmodes that are regularly encountered in global spectra of open shear flows, owing to unphysical pressure feedback from imperfectly transparent outflow conditions.

3.1 Linear global instability of weakly buoyant jets

[L16] W. Coenen, L. Lesshafft, X. Garnaud & A. Sevilla (2017): Global instability of low-density jets. *J. Fluid Mech.* vol. 820, p. 187–207

The choice of parameter values for this study was guided by the helium jet experiments by Hallberg & Strykowski [22]: laminar jets with Reynolds number values $Re_D \leqslant 1000$ and with density ratios $0.143 \leqslant S \leqslant 0.5$ were considered. The jet exits from a straight pipe, included in the numerical domain, with a shear layer momentum thickness θ_0 between 2.8% and 6.7% of the nozzle diameter.

In settings corresponding to pure helium injected into air, $S = 0.143$, an isolated eigenvalue was found to dominate the spectrum. Convergence tests demonstrated that this was indeed the only eigenvalue in our calculations that was independent of the computational domain size (see figure 8). Through systematic variations of the Reynolds number and of the nozzle-exit shear layer thickness, the neutral curve of this dominant eigenmode was traced, as shown in figure 9, and could thus be compared to the experimental results [22].

While the general trend of the experimental neutral curve is well captured by our linear calculations, and decent agreement is found between measured and computed Strouhal numbers (indicated by text labels in figure 9), a quite significant offset in the critical Reynolds number remains. In order to find an explanation for this discrepancy, the numerical model was extended to include the effects of buoyancy as well as

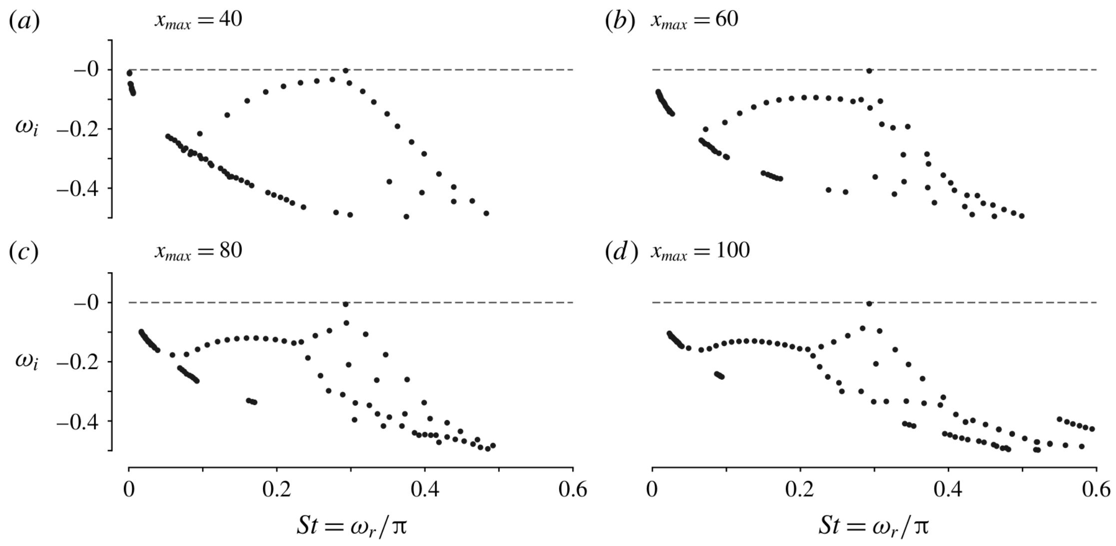


Figure 8: Spectrum of a jet at $Re = 360$, $S = 0.143$ and $D/\theta_0 = 24.3$, computed on numerical domains of different streamwise length x_{max}. From reference [L16].

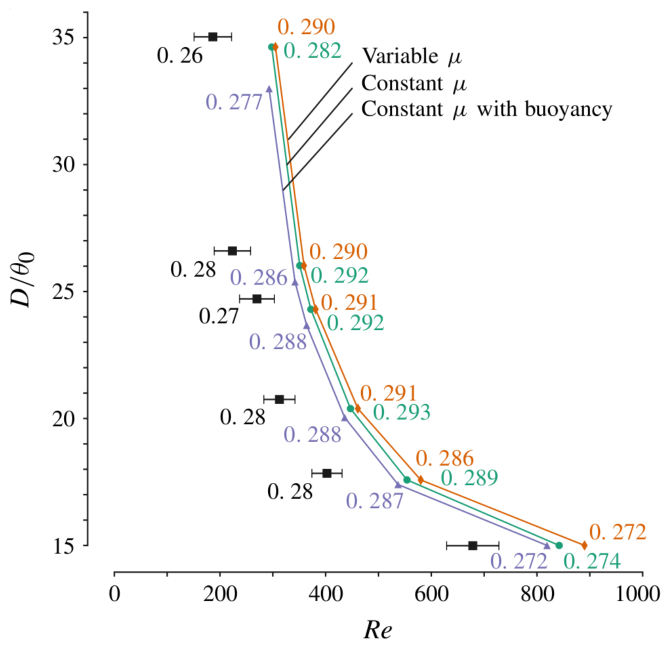


Figure 9: Comparison of neutral curves obtained from linear analysis (colour) with those obtained experimentally (black squares and error bars) by Hallberg & Strykowski [22] for pure helium jets. Numbers indicate the Strouhal values of dominant jet oscillations. From reference [L16].

viscosity variations, but both effects are seen to be rather negligible.

The onset of linear global instability was successfully linked to the presence of absolute instability near the nozzle. Both the structural sensitivity [20] and the sensitivity to base flow variations [35] was computed, but neither one of these provided clear indications about the physical mechanisms that are involved in the global destabilisation of the jet. This discussion was revisited in the investigation of plumes (see §3.2).

At the highest Reynolds number, $Re = 1000$, no isolated eigenvalue could be detected, and the entire spectrum was found to be strongly dependent on the length of the computational domain. This observation, consistent with our earlier homogeneous jet calculations [L10], motivated a more detailed investigation of the effect of domain truncation (see §3.3.2).

3.2 Linear local and global instability of strongly buoyant plumes

[L13] R.V.K. Chakravarthy, L. Lesshafft & P. Huerre (2015): Local linear stability of laminar axisymmetric plumes. *J. Fluid Mech.* vol. 780, p. 344–369

[L12] L. Lesshafft (2015): Linear global stability of a confined plume. *Theor. Appl. Mech. Lett.* vol. 5, p. 126–128

[L18] R.V.K. Chakravarthy, L. Lesshafft & P. Huerre (2018): Global stability of buoyant jets and plumes. *J. Fluid Mech.* vol. 835, p. 654–673

In 2013, at the beginning of Chakravarthy's Ph.D. project, the literature on linear instability of plumes was limited to a few local studies, mostly from the 1980s. Temporal and spatial analyses had been performed for self-similar base flows in parameter regimes that were numerically accessible at the time. Prandtl numbers for these self-similar profiles were limited to values 1 and 2. Yet several experiments and numerical simulations had established the presence of intrinsic oscillations, in the form of axisymmetric vortex formation.

The first step towards a linear description of self-sustained oscillations in plumes had to be made in terms of *local* theory. Our first article [L13] describes a numerical procedure for the construction of self-similar plume profiles, under the Boussinesq approximation, for arbitrary Prandtl and Grashof number values, which were then used for temporal and spatio-temporal instability analysis. New instability mechanisms were described, both for axisymmetric and for helical modes, based on the interplay between vorticity and temperature perturbations in the self-similar plume far from its source. Absolute instability in such profiles was shown to occur for helical, but never for axisymmetric perturbations. The absolute helical mode is characterised by a very long wavelength, small frequency and small positive growth rate.

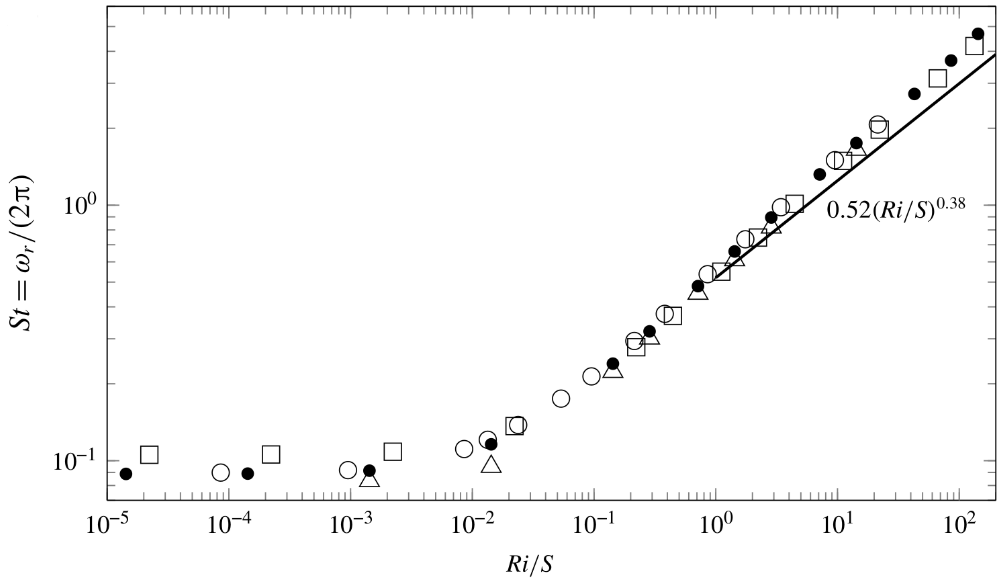


Figure 10: Strouhal number of the dominant global eigenvalue, as a function of Ri/S. Legend: (•) Re = 200, S = 7; (◦) Re = 500, S = 7; (□) Re = 200, S = 4.5; (△) Re = 200, S = 7, with parabolic inlet velocity profile. Solid line: power law from the Cetegen & Kasper [10] experiments, rescaled to match the present definition of Ri. From reference [L18].

These local results, valid in the self-similar regime, do not explain the observed intrinsic plume oscillations, which are consistently reported to be axisymmetric. A *global* eigenmode analysis was therefore undertaken, for laminar base flows that include the buoyancy source region, obtained by Newton–Raphson iteration of the low-Mach-number flow equations. This set of equations does not invoke the Boussinesq approximation, and is valid for arbitrarily high density variations. The principal influence parameters in this study were the density ratio, in this study defined as $S = \rho_\infty/\rho_{plume}$ (reciprocally to our previous definition), and the Richardson number Ri. A low value of Ri denotes a base flow which is dominated by the injected momentum, and therefore is classified as a *jet*, whereas a high value of Ri denotes a flow that is dominated by the effect of the buoyancy force, characteristic of a *plume*.

The global spectra of these non-self-similar base flows revealed the presence of several strongly growing eigenmodes in the high-Ri plume regime, distinct from the instability already documented for low-Ri jets (§ 3.1). Across the entire interval of investigated Richardson number values, $10^{-4} \leqslant Ri \leqslant 10^3$, only *axisymmetric* perturbations were found to exhibit global instability, fully consistent with empirical observations. All globally unstable base flows were shown to be absolutely unstable at the inflow, but convectively unstable in the downstream self-similar flow region. The absolute instability of helical perturbations turned out to be too weak in order to trigger global instability.

The first significant result of Chakravarthy's global stability analysis was the precise recovery of the experimental scaling law, given by Cetegen & Kasper [10], that relates the Strouhal number of flow oscillations to the ratio Ri/S. This scaling is shown in figure 10: symbols represent the Strouhal number of the dominant instability mode in various flow configurations, and a solid line indicates the power law reported from experiments [10]. This result has been corroborated in a parallel study by Bharadwaj & Das [6].

Another significant result was the formal demonstration that global instability in the plume regime is underpinned by the effect of *buoyancy*, whereas in the jet regime it is caused by the *baroclinic torque*. This analysis of physical mechanisms was based on the formalism developed by Marquet & Lesshafft [L67] (see §3.3.1), slipped into our publication [L18] in the guise of a sensitivity analysis.

A short study of internal plumes, confined inside a cylindrical box with solid, isothermal walls, indicated that global instability in such a configuration is driven by non-local feedback between a cooled top and a heated bottom boundary [L12]. Despite the absence of absolute instability, global instability was observed in such a setting. The critical Rayleigh number for the onset of nonlinear oscillations, determined to be $Ra_c = 3.85 \times 10^7$ in direct numerical simulations [32], was recovered as $Ra_c = 3.80 \times 10^7$ for the threshold of linear global instability; the nonlinear and linear frequencies at this threshold were found to match within 0.5%.

3.3 Interpretation of instability mechanisms

3.3.1 A refined definition of the global wavemaker

[L67] O. Marquet & L. Lesshafft (2015): Identifying the active flow regions that drive linear and nonlinear instabilities. arXiv:1508.07620

The "wavemaker" (Monkewitz [38]) associated with a global instability mode, as a notional concept, denotes the flow region where oscillations are *generated*, as opposed to the flow region where they may reach their amplitude maximum after further *amplification*. In the context of weakly non-parallel flows, the wavemaker has been identified with the location of a saddle point in the analytic continuation of the absolute frequency as a function of the streamwise coordinate [11]. In the context of global eigenmode analysis, the wavemaker definitions by Luchini *et al.* [20, 33], based on the structural sensitivity of a given eigenmode, has been widely used for the discussion of instability dynamics.

Olivier Marquet and myself proposed a similar but different definition of the wavemaker [L67]. The definition starts from the simple observation that the linear operator

L of a given eigenvalue problem

$$\omega B q = L q \tag{2}$$

has a unique diagonal representation $L = BQ\Omega Q^{-1}$, and that the matrix $Q^{\dagger}$ formed by the adjoint eigenvectors satisfies $Q^{-1} = Q^{\dagger,H}B$. The diagonal eigenvalue matrix Ω is then uniquely related to L as

$$\Omega = Q^{\dagger,H} L Q. \tag{3}$$

Suppose one can formulate a *physically meaningful* decomposition of the linear operator, $L = L_1 + L_2 + \cdots + L_n$, a given eigenvalue ω_1 with associated direct and adjoint eigenvectors q_1 and $q_1^{\dagger}$ is precisely determined by

$$\omega_1 = q_1^{\dagger,H} L q_1 = q_1^{\dagger,H}(L_1 + L_2 + \cdots + L_n) q_1, \tag{4}$$

such that the contribution of each component L_i to the eigenvalue ω_1 is quantified as $q_1^{\dagger,H} L_i q_1$.

The decomposition of the operator L can be performed to denote spatial locations, individual terms in the flow equations, or both. Contributions of different flow regions, as well as different physical mechanisms, to the frequency and the growth rate of an eigenmode may therefore be quantified. The original paper [L67] demonstrated this concept for the Ginzburg–Landau equation and for the 2D cylinder wake, both in linear and nonlinear contexts. It was shown for these examples that the ensuing wavemaker definition is consistent with those given by Chomaz *et al.*[11] and by Luchini *et al.* [20, 33]. In contrast to those established definitions, however, our formalism provides a straightforward framework for a discussion of physical mechanisms, in so far as they can be related to individual terms in the flow equations. Its potential has since been tested in several flow configurations, including the plume study discussed in the previous section, the dynamics of a spring-mounted cylinder [L59](c), and the instability of a premixed flame [L58](a). A new submission of the manuscript [L67] is in preparation.

3.3.2 Spurious feedback from boundary conditions

[L19] L. Lesshafft (2018): Artificial eigenmodes in truncated flow domains. *Theor. Comp. Fluid Dyn.*, vol. 32, p. 245-262.

The prominent branch of evenly spaced eigenmodes, which has been found to dominate most jet and plume spectra, is in fact regularly encountered in the global spectral analysis of open shear flows. We have named it the "arc branch" [L19]. The arc branch has in several instances been discussed as the spectral manifestation of amplifier flow behaviour, but such a conception is problematic — first, because amplifier behaviour is appropriately described by the *pseudospectrum*, which does not require the presence

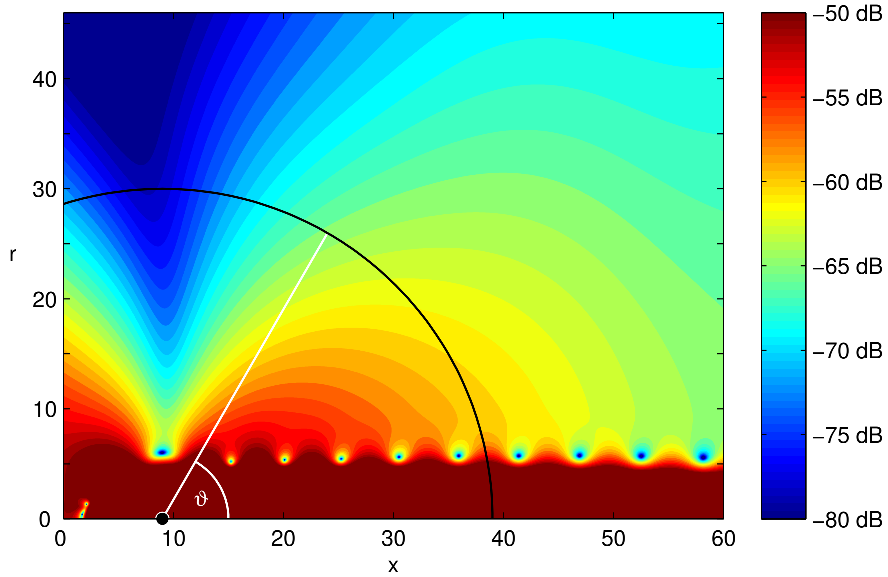


Figure 11: Isocontours of the pressure amplitude in the acoustic far field of a jet with $S = 0.3$. The apparent sound source on the jet axis is located at $x = 9$. The directivity pattern in figure 12 is extracted along the arc of radius 30. From reference [L5].

of such eigenmodes, second, because the arc branch is notoriously dependent on the computational domain size. The nature of these modes craved an explanation.

It was demonstrated that arc branch modes in jets arise from the coupling of downstream-propagating shear instability waves and upstream-reaching pressure feedback [L19]. The latter originates as a spurious effect at the numerical outflow, and it provokes perturbations at the numerical inflow, where shear instabilities can be triggered. The study shows that explicit inflow-outflow coupling in a Ginzburg–Landau model produces an arc branch very similar to the one found in open shear flows. It is further confirmed that arc branch modes of a parallel jet depend on the presence of spurious forcing of a local k^+ instability wave at the inflow, caused by pressure signals that appear to be generated at the outflow. Absorbing layers, or sponge zones, are suggested and tested as a technical means to reduce the effect of spurious pressure feedback from artificial domain boundaries.

3.4 Acoustic radiation from oscillating hot jets

[L5] L. Lesshafft, P. Huerre & P. Sagaut (2010): Aerodynamic sound generation by global modes in hot jets. *J. Fluid Mech.* vol. 647, p. 473–489

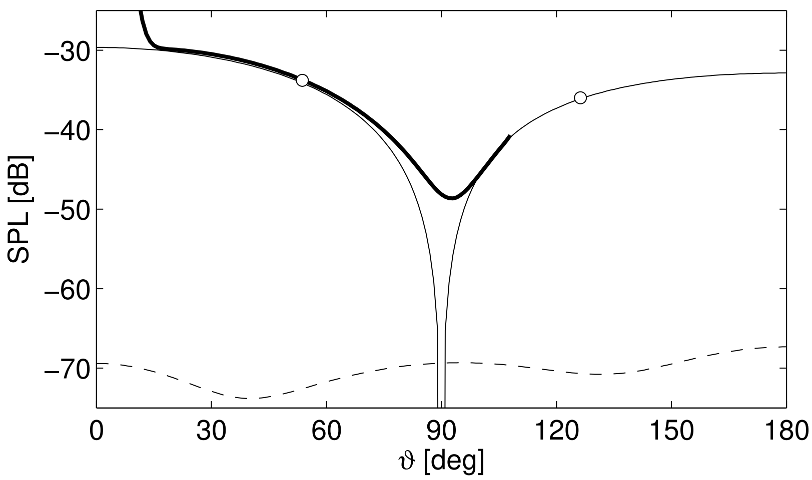


Figure 12: Directivity of the acoustic far field, comparison between direct numerical simulation and Lighthill solution. Thick solid line: directly computed sound; thin solid line: Lighthill solution from enthalpy-flux term alone; dashed line: combined radiation from all other terms. The absolute SPL level is not adjusted, but follows directly from the data analysis. From reference [L5].

Jet instability studies are in large measure motivated by the problem of jet noise. The noise that is emitted by the regular formation of vortex rings in a hot jet is available from the direct numerical simulations performed during my Ph.D., and it is accessible for an investigation into the underlying acoustic source mechanisms.

A configuration with Reynolds number $Re = 1000$, Mach number $Ma = 0.1$ and density ratio $S = 0.3$ is chosen as the baseline case (density ratios 0.1 and 0.2 are also considered. The acoustic far field, extracted from the DNS and shown in figure 11, is found to be of dipole character: the pressure amplitude varies with the observation angle ϑ (measured from the jet axis) as $\hat{p} \propto \cos\vartheta$.

In order to identify the acoustic source mechanisms, far-field solutions of the Lighthill equation are constructed, under the Fraunhofer approximation and under the assumption of radially compact, axisymmetric near-field source distributions. These solutions permit the isolation of contributions from individual source terms to the total acoustic far field. The source distributions are evaluated from the numerical simulation data.

A first attempt [L65], based on the original source terms of the Lighthill equation, gave unsatisfactory results, because (i) the Lighthill equation only contains monopole and quadrupole sources, (ii) as a result, the acoustic extinction angle was inaccurately reproduced and (iii) the dominant source term was found to be the apparent ‘entropy’ fluctuation, which does not lead to a clear physical interpretation in the presence of strong density variations. Instead, Lilley's [30] reformulation of the Lighthill source terms was employed with success. Dipole components appear explicitly in this formulation, and it was shown that the dominant contribution by far arises from the dipole source related to the axial flux of enthalpy in the oscillating jet. This isolated component is compared to the total sound field in figure 12 (solid lines). In this highly synchronised flow case, the acoustic source region is quite compact in the axial direction, and antenna effects are therefore not pronounced.

4 Secondary global instabilities of incompressible jets

Moving onwards from linear analysis of primary instabilities in steady jet base flows, the study project of Léopold Shaabani Ardali's Ph.D. work targets the *secondary* instability of axisymmetric vortex streets in a jet. Two particularly striking instances of secondary instability phenomena are considered:

a) **vortex pairing** as a self-sustained process,

b) **jet bifurcation** [49] as an extrinsically forced process.

Both scenarios, as observed in experiments and in numerical simulations, appear to be of a fundamentally nonlinear nature, yet we approach them in a linear framework. Vortex pairing, arising from inherent mechanisms, is formalised as a modal Floquet problem and complemented by an analysis of transient growth. Jet bifurcation, relying on subharmonic actuation at the nozzle, is investigated as a non-modal optimal forcing problem. The analysis in both cases is based on a *time-periodic* base flow, represented by the axisymmetric T-periodic vortex street resulting from T-periodic forcing of the primary instability at the inlet.

4.1 Vortex pairing as a Floquet instability

[L17] L. Shaabani-Ardali, D. Sipp & L. Lesshafft (2017): Time-delayed feedback technique for suppressing instabilities in time-periodic flow. *Phys. Rev. Fluids* vol. 2, no. 113904

[L21] L. Shaabani Ardali, D. Sipp & L. Lesshafft (2018): Vortex pairing in jets as a global Floquet instability: modal and transient dynamics. *J. Fluid Mech.*, in press

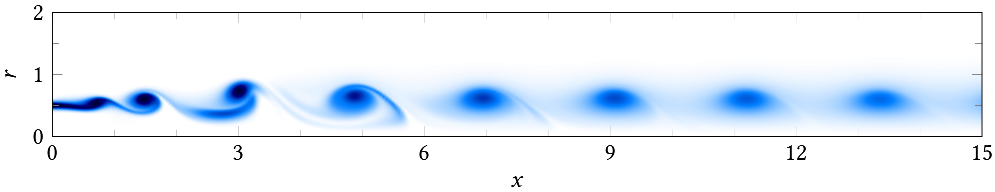


Figure 13: Vortex pairing in a harmonically forced jet, for $St_D = 0.6$, $Re = 2000$ and an inflow forcing amplitude $A = 0.05$. From [L21].

Motivation Vortex pairing, visualised in figure 13 has long been described as a secondary instability of a regular vortex street, both in plane shear layers and in jets. The underlying vortex street arises from the primary shear instability, typically in response to harmonic forcing at the nozzle. If this primary forcing is T-periodic, characterised by the *fundamental* Strouhal number $St_D = D/TU_j$ based on jet diameter and exit velocity, the pairing process is $2T$-periodic, and therefore a *subharmonic* instability is expected. Much work in the 1980s and 1990s was directed at the conditions under which subharmonic perturbations can grow in vortex streets, principally based on the resonance criterion formulated by Monkewitz [39]; it was even suspected that the global feedback mechanism behind vortex pairing underpinned the development of jet turbulence [29]. Yet no quantitative *global* stability analysis of the vortex pairing phenomenon had ever been undertaken.

Our study analyses the instability properties of a spatially developing T-periodic vortex street, as it arises due to harmonic forcing at the inflow, in the framework of Floquet theory [17].

Methodology Prior to performing instability analysis, the T-periodic base flow is obtained from nonlinear DNS. However, as this base flow may be unstable with respect to pairing, all non-T-periodic perturbations must be artificially stabilised. Harmonic modulations of the inlet jet velocity are imposed, with Strouhal number St_D and forcing amplitude A, such that the time-dependent inflow condition is prescribed as $U(r, z = 0, t) = [1 + A\sin(2\pi St_D t)]\tilde{U}(r)$. Similar to the technique of *selected frequency damping*, commonly applied in order to compute unstable *steady* base flows, Léopold Shaabani Ardali devised a method based on *time-delay control* [L17], which damps differences between the flow states at times t and $t - T$, and which is maximally efficient for eliminating subharmonic fluctuations. By the time of submission of this first article, we realised that this technique constitutes a special case of the delayed feedback control method described by Pyragas [48], used in the context of low-dimensional chaotic systems.

Floquet instability is characterised by the presence of Floquet multipliers μ_i with an absolute value larger than unity, denoting modal perturbation growth over one

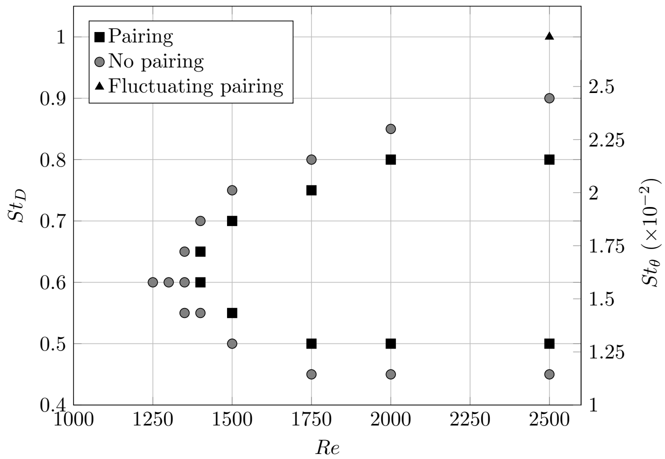


Figure 14: Occurrence of vortex pairing in direct numerical simulations, for an inflow forcing amplitude $A = 0.05$. From reference [L21].

flow period T. These multipliers are found as the eigenvalues of the linear time-shift operator Φ that propagates a small perturbation from time 0 to T. The eigenvalues are computed by projecting Φ onto an orthonormal basis of a Krylov subspace, using only linear time-stepping of the linearised flow equations. The linear time-stepping is implemented in FreeFEM++, and a block-Arnoldi algorithm [51] is employed in order to construct the orthonormal Krylov basis with maximum efficiency.

The possibility of transient perturbation growth in the time-periodic base flow is again explored by means of singular value decomposition, as described by Barkley *et al.* [4]. A special twist of the numerical procedure permits us to construct the leading singular modes solely based on the same Krylov basis that is already available from the modal analysis, without the need for further time-stepping. In particular, contrary to the procedure given by Barkley *et al.* [4], no adjoint time-stepping is required.

Results The study starts out from a parametric survey of the spontaneous occurrence of vortex pairing in direct numerical simulations, in the absence of artificial stabilisation. Simulations of the nonlinear flow development are performed with the Nek5000 code, restricted to an axisymmetric geometry. Both the Strouhal and the Reynolds number are varied systematically, for three different values $A = 0.01$, 0.05 and 0.1. As a result of the inflow modulations and the primary jet instability, the shear layer rolls up into a regular street of ring vortices, with a passage Strouhal number equal to St_D. Self-sustained vortex pairing is observed in these simulations in a specific region of the St_D/Re plane, delineated by a “neutral curve”, which depends on A. Figure 14 shows these empirical results for the standard forcing amplitude values $A = 0.05$.

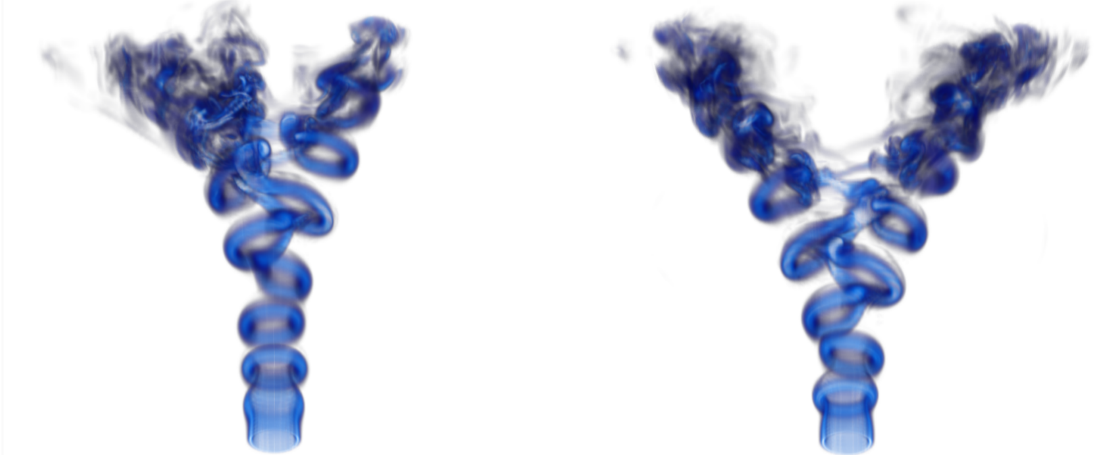
The first question is whether the occurrence of self-sustained vortex pairing is linked to the presence of a linear instability of the T-periodic (i.e. unpaired) base flow. Using the time-delay stabilisation technique [L17], which only involves adding a control force that depends linearly on the difference of the flow state at times t and $t - T$, strictly T-periodic flows are computed. Modal Floquet analysis is performed for $A = 0.05$, along two paths in the St_D/Re plane, once varying Re at constant $St_D = 0.6$ and once varying St_D at constant $Re = 2000$. Along both paths, unstable eigenvalues are found to arise precisely over the parameter regime where vortex pairing occurs in the DNS. Furthermore, these unstable eigenvalues are real and negative; in other words, their complex phase is π. This characterises the associated perturbation mode as being *subharmonic* with respect to the T-periodic forcing, as one would expect for the vortex pairing instability. It is concluded that vortex pairing, as a $2T$-periodic limit cycle, is indeed the result of a subharmonic Floquet instability.

However, the transition from an unstable unpaired towards a paired state, in typical simulations, exhibits stronger growth and different spatial distributions than what the modal analysis predicts. In order to better describe the transient dynamics by which this bifurcation takes place, the *optimal perturbation* for transient growth is computed. Non-modal analysis predicts strong transient growth of perturbations close to the jet inlet, in good agreement with DNS observations. At $St_D = 0.6$ and $Re = 2000$, a modally unstable setting, the optimal perturbation provides an amplitude gain of five orders of magnitude over the purely modal growth.

4.2 Optimal forcing of jet bifurcation

[L23] L. Shaabani-Ardali, L. Lesshafft & D. Sipp: Optimal triggering of jet bifurcation. In preparation for *J. Fluid Mech.*

Motivation The phenomenon of jet bifurcation is chosen as a particularly interesting effect of active flow control exploiting a secondary instability of a periodic flow. Under suitable actuation at the inflow, a jet splits into two separate streams of vortex rings in a zipper-like fashion (see figure 15). The actuation is composed of an axisymmetric component of Strouhal number St_D and an added helical subharmonic component of $St_D/2$. While the axisymmetric component sets up the basic T-periodic vortex street, as in the previous section, the helical component imparts a left/right displacement to each vortex ring, which is amplified as the vortices propagate downstream. The rather drastic split-up of the fundamental vortex street occurs once the subharmonic perturbation reaches nonlinear amplitude levels, but we suspect that linear instability mechanisms acting within the periodic base flow provide the necessary amplification that leads up to the parting of the streams.



(a) nozzle flapping (b) optimal forcing

Figure 15: Direct numerical simulation of jet bifurcation at $St_D = 0.5$ and $Re = 2000$, snapshots of vorticity contours. (a) "traditional" subharmonic forcing in the form of nozzle flapping; (b) optimal subharmonic forcing, as identified by linear analysis. From [L23].

The unstable vortex dynamics can be worked out qualitatively by three-fingered hand-wringing, as explained by Reynolds *et al.* [49]. Yet the quantitative analysis requires global input-output computations, similar to those discussed in §2.2, adapted to time-periodic base flows. Hitherto unexplored, optimal forcing strategies for jet bifurcation can then be identified.

Methodology The base flow computations are performed in the same way as described in the previous section, using the Nek5000 code for nonlinear axisymmetric DNS, with added stabilisation of non-T-periodic components [L17]. These computations fully account for the axisymmetric forcing that leads to the formation of the basic vortex street. The evolution of *linear* helical subharmonic perturbations within this axisymmetric and T-periodic base flow is calculated via linear time-stepping in FreeFEM++.

Continuous subharmonic forcing is applied only in the inlet plane $z = 0$, by prescribing helical perturbations in all three velocity components as a boundary condition. Radial distributions are chosen in the form of Bessel functions (J_0, J_1 and J_2), combined such as to respect the compatibility conditions on the axis and as to ensure that the velocity field is divergence-free. Linear time-stepping is performed for a large number of such boundary conditions, which form an orthogonal basis for inflow velocity perturbations, until the long-time asymptotic flow response is obtained for each of them. Coefficients for the optimal superposition of forcing basis functions are readily obtained for any given objective in the linear flow response.

Results The objective of optimal triggering of jet bifurcation is formalised in two ways: first, we aim to maximise the standard L_2 norm of subharmonic velocity perturbations in the flow response to unit-L_2-norm forcing. This represents an integral measure of subharmonic kinetic energy gain in the flow domain. Second, we consider a specifically tailored norm of the flow response that measures the *radial displacement* of base flow vortices, which corresponds more directly to the intended effect of triggering bifurcation of the vortex street. It is found however that both formulations lead to nearly identical shapes of the optimal forcing.

In previous numerical simulations of jet bifurcation, for instance by [60], the shape of helical inflow forcing was prescribed such as to represent a low-amplitude left/right flapping of the jet nozzle. Using our optimised forcing distribution in three-dimensional DNS, it is found that the splitting of the vortex street is more vigorous, and achievable over a larger range of St_D, than with simple nozzle flapping. Figure 15 compares snapshots from simulations, at $St_D = 0.5$ and $Re = 2000$, with flapping and with optimal forcing. The injected kinetic energy of the subharmonic velocity perturbations is identical in both cases.

5 Perspectives

5.1 Flame instability

Flames constitute a family of flows that are similar to jets and plumes in many respects, with the added ingredient of chemical reaction and heat release. Unsteadiness in combustion processes, due to instability phenomena, is a cause for loss of performance, increased pollution, and structural damage of combustion engines. These phenomena involve multi-physics and multi-scale mechanisms, through the coupling of heat release, gas flow and acoustics.

Over the past three years, I have attempted linear analysis of several flame configurations. Together with postdoc Onofrio Semeraro, building on the Ph.D. work of Mathieu Blanchard at LadHyX, we first investigated the instability of a premixed "M-flame" in an annular burner [7], by means of modal as well as input-output analysis [L58]. The annular burner consists of a pipe, from where the premixed fuel-air stream exits into a large combustion chamber, and a thin cylindrical rod, concentrically fixed inside the pipe. A flame of the M-type attaches to the exterior rim of the inflow pipe and to the interior rim of the rod. Mesh and base flow (methane volume fraction) in the flame region are shown in figure 16. An Arrhenius law is used to model the reaction rate [7].

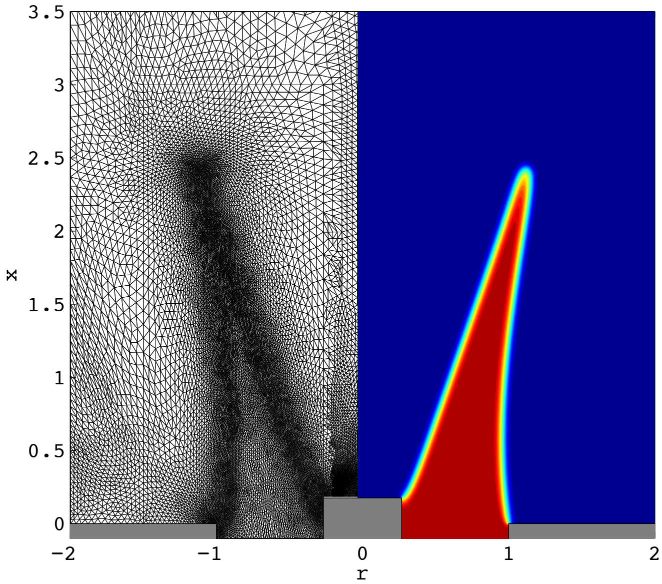


Figure 16: Geometry, mesh and base flow (methane volume fraction) of an M-flame. The base flow is taken from [7] and interpolated onto the FEM mesh.

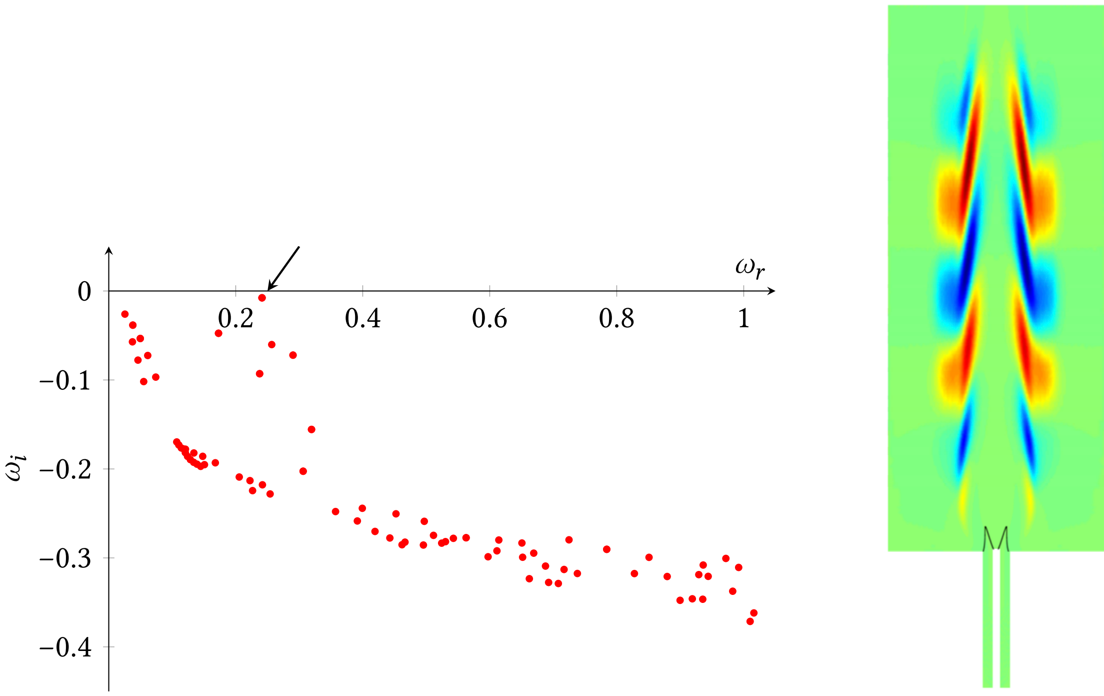


Figure 17: Left: eigenvalue spectrum of the M-flame (axisymmetric perturbations only). Right: Snapshot of temperature fluctuations associated with the least stable mode. The flame front is drawn as a black line.

Experiments indicate that this flame configuration does not exhibit self-excited behaviour, but that it is highly receptive to incoming perturbations. Our linear analysis reproduces this receptivity in a narrow frequency band, where the energy gain between flow response and applied forcing peaks sharply. This behaviour is the result of a resonance, caused by a slightly stable eigenmode of the flame spectrum displayed in figure 17. This eigenmode is accessible to a detailed analysis of its intrinsic mechanisms, by way of the wavemaker in the sense of section 3.3.1. It is thus found that the instability is dominated by simple shear mechanisms, which act mainly outside the flame region and give rise to strong oscillations ("puffing") in the plume. Fluctuations of reaction rate and heat release only play a passive part in driving this instability. The role of combustion in this context, as it turns out, is only to set up the basic shear flow state through buoyancy.

We carried out similar calculations for laminar "V-flames" and turbulent swirl flames, with base flows provided by Kilian Oberleithner at TU Berlin. All these analyses suffer from severe uncertainty about the appropriate chemistry modelling, and from the unavailability of accurate density and temperature fields. Our current efforts, led by Léopold Shaabani Ardali and myself, are concentrated on a simple premixed conical flame of a Bunsen burner, for which the base flow is contributed by our partners Bénédicte Cuenot and Laurent Gicqel at CERFACS, computed in direct numerical simulations with the AVBP code.

5.2 Semi-empirical modelling of noise from installed jets in flight

Following up on the mostly fundamental research within the ANR Cool Jazz project, our new project "DARETOMODEL" in the H2020 CleanSky2 program is a step higher up on the TR-scale. This project is led by Peter Jordan (Institut Pprime), with Anurag Agarwal (University of Cambridge), Jérôme Huber (Airbus) and myself as partners.

The objective is to construct low-rank models for the prediction of noise radiated from engine jets, in the presence of a wing, and with co-flow as in flight conditions. The noise source is to be modelled as a stochastic wavepacket, with an amplitude envelope function obtained from parabolised Navier–Stokes computations. The associated sound field may then be constructed from the Green's function, which can be modified in order to account for the effect of a wing surface and co-flow. An important unanswered question in this context is how turbulence may realistically be modelled in the form of non-white forcing of a linear system, and if the concept of turbulent viscosity may be adapted in order to capture a portion of turbulence effects on coherent perturbation statistics.

6 Publications and Conferences

6.1 Journal publications

[L1] L. Lesshafft, P. Huerre, P. Sagaut & M. Terracol, 2006:
Nonlinear global modes in hot jets
J. Fluid Mech., vol. 554 (50^{th} anniversary volume), 393–409

[L2] L. Lesshafft & P. Huerre, 2007:
Linear impulse response in hot round jets
Phys. Fluids, vol. 19, no. 024102

[L3] L. Lesshafft, P. Huerre & P. Sagaut, 2007:
Frequency selection in globally unstable round jets
Phys. Fluids, vol. 19, no. 054108

[L4] B. Selvam, L. Talon, L. Lesshafft & E. Meiburg, 2009:
Convective/absolute instability in miscible core–annular flow. Part 2. Numerical simulations and nonlinear global modes
J. Fluid Mech., vol. 618, 323–348

[L5] L. Lesshafft, P. Huerre & P. Sagaut, 2010:
Aerodynamic sound generation by global modes in hot jets
J. Fluid Mech., vol. 647, 473–48

[L6] L. Lesshafft & O. Marquet, 2010:
Optimal velocity and density profiles for the onset of absolute instability in jets
J. Fluid Mech., vol. 662, 398–408

[L7] L. Lesshafft, E. Meiburg, B. Kneller & A. Marsden, 2011:
Towards inverse modeling of turbidity currents: the inverse lock-exchange problem
Comput. Geosci., vol. 37, 521–529

[L8] L. Lesshafft, B. Hall, E. Meiburg & B. Kneller, 2011:
Deep-water sediment wave formation: linear stability analysis of coupled flow/bed interaction
J. Fluid Mech., vol. 680, 435–458

[L9] X. Garnaud, L. Lesshafft, P. Schmid & J.-M. Chomaz, 2012:
A relaxation method for large eigenvalue problems, with an application to flow stability analysis
J. Comput. Phys., vol. 231(10), 3912–3927

[L10] X. Garnaud, L. Lesshafft, P. Schmid & P. Huerre, 2013:
Modal and transient dynamics of jet flows
Phys. Fluids, vol. 25, no. 044103

[L11] X. Garnaud, L. Lesshafft, P. Schmid & P. Huerre, 2013:
The preferred mode of incompressible jets: linear frequency response analysis
J. Fluid Mech., vol. 716, 189–202

[L12] L. Lesshafft, 2015:
Linear global stability of a confined plume
Theor. Appl. Mech. Lett., vol. 5, p. 126-128

[L13] R. Chakravarthy, L. Lesshafft & P. Huerre, 2015:
Local linear stability of laminar axisymmetric plumes
J. Fluid Mech., vol. 780, p. 344-369

[L14] S. Beneddine, D. Sipp, A. Arnault, J. Dandois & L. Lesshafft, 2016:
Conditions for validity of mean flow stability analysis
J. Fluid Mech., vol. 798, p. 485-504

[L15] O. Semeraro, L. Lesshafft, V. Jaunet & P. Jordan, 2016:
Modeling of coherent structures in a turbulent jet as global linear instability wavepackets: theory and experiment
Int. J. Heat Fluid Flow, vol. 62A, p. 24-32

[L16] W. Coenen, L. Lesshafft, X. Garnaud & A. Sevilla, 2017:
Global instability of low-density jets
J. Fluid Mech., vol. 820, p. 187-207

[L17] L. Shaabani Ardali, D. Sipp & L. Lesshafft, 2017:
Time-delayed feedback technique for suppressing instabilities in time-periodic flow
Phys. Rev. Fluids, vol. 2, 113904

[L18] R. Chakravarthy, L. Lesshafft & P. Huerre, 2018:
Global stability of buoyant jets and plumes
J. Fluid Mech., vol. 835, p. 654-673

[L19] L. Lesshafft, 2018:
Artificial eigenmodes in truncated flow domains
Theor. Comp. Fluid Dyn., vol. 32, p. 245-262

[L20] N. Konopliv, L. Lesshafft & E. Meiburg, 2018:
The influence of shear on double-diffusive and settling-driven instabilities
J. Fluid Mech., vol. 849, p. 902-926

[L21] L. Shaabani Ardali, D. Sipp & L. Lesshafft 2018:
Vortex pairing in jets as a global Floquet instability: modal and transient dynamics
J. Fluid Mech., in press

[L22] L. Lesshafft, O. Semeraro, V. Jaunet, A.V.G. Cavalieri & P. Jordan:
Resolvent-based modelling of coherent wavepackets in a turbulent jet
arXiv 1810.09340, submitted to *Phys. Rev. Fluids*

[L23] L. Shaabani Ardali, L. Lesshafft & D. Sipp:
Optimal triggering of jet bifurcation
in preparation for *J. Fluid Mech.*

6.2 Conference presentations

[L24] L. Lesshafft, P. Huerre, P. Sagaut & M. Terracol:
Global modes in hot jets, absolute/convective instabilities and acoustic feedback
AIAA Paper 2005-3040, presented at the 11^{th} AIAA Aeroacoustics Conference
May 2005, Monterey, California

[L25] L. Lesshafft, P. Huerre, P. Sagaut & M. Terracol:
Global modes in hot jets and their acoustic far field
58^{th} annual meeting of the APS Division of Fluid Dynamics
November 2005, Chicago, Illinois

[L26] L. Lesshafft, P. Huerre & P. Sagaut:
Sound radiation from instability waves in subsonic jets: entropy sound and superdirectivity
59^{th} annual meeting of the APS Division of Fluid Dynamics
November 2006, Tampa, Florida

[L27] L. Lesshafft, E. Meiburg & B. Kneller:
Sediment deposition from buoyant river plumes
60^{th} annual meeting of the APS Division of Fluid Dynamics
November 2007, Salt Lake City, Utah

[L28] L. Lesshafft, E. Meiburg & B. Kneller:
Numerical modelling of sediment deposition from buoyant river plumes
Euromech Coll. 501 "Mixing of coastal, estuarine and riverine shallow flows"
June 2008, Ancona, Italie

[L29] a) L. Lesshafft, B. Hall, E. Meiburg & B. Kneller:
Sediment waves: coupled flow/sediment-bed instability in turbidity currents
b) L. Lesshafft, P. Huerre & P. Sagaut:
Sound radiation from self-sustained oscillations in a hot jet
7^{th} Euromech Fluid Mechanics Conference (EFMC7)
September 2008, Manchester, UK

[L30] a) L. Lesshafft, E. Meiburg & B. Kneller:
Inverse modeling: reconstructing the initial conditions of a turbidity current
b) B. Hall, L. Lesshafft, E. Meiburg & B. Kneller:
Sediment wave formation by turbidity currents: a Navier-Stokes based linear stability analysis
61^{st} annual meeting of the APS Division of Fluid Dynamics
November 2008, San Antonio, Texas

[L31] a) L. Lesshafft, E. Meiburg & B. Kneller:
Inverse modeling of sediment deposition from turbidity currents
b) L. Lesshafft, B. Hall, E. Meiburg & B. Kneller:
Linear instabilities in a turbidity current boundary layer
CSDMS Workshop "Modeling of turbidity currents and related gravity currents"
June 2009, Santa Barbara, California

[L32] a) L. Lesshafft, B. Hall, E. Meiburg & B. Kneller:
Sediment wave formation by unstable internal waves in a turbidity current boundary layer
b) P. Burns, L. Lesshafft & E. Meiburg:
Instability Phenomena in Stratified, Particle-laden Flow
62^{nd} annual meeting of the APS Division of Fluid Dynamics
November 2009, Minneapolis, Minnesota

[L33] L. Lesshafft, E. Meiburg & B. Kneller:
Sediment deposition from turbidity currents: inverse modeling and instability phenomena
Hydrodynamique des Lacs et Approximation de St Venant
January 2010, Paris, France

[L34] L. Lesshafft, B. Hall, E. Meiburg & B. Kneller:
Hydrodynamic instabilities in a turbidity current boundary layer as a mechanism for sediment wave formation
Deep-water circulation: processes & products
Juin 2010, Baiona, Spain

[L35] L. Lesshafft & O. Marquet:
Sensitivity-based optimization of jet profiles for the onset of absolute instability
8^{th} Euromech Fluid Mechanics Conference (EFMC8)
September 2010, Bad Reichenhall, Germany

[L36] X. Garnaud, L. Lesshafft, P. Schmid & P. Huerre:
Global modes of compressible subsonic jets
63^{th} annual meeting of the APS Division of Fluid Dynamics
November 2010, Long Beach, California

[L37] X. Garnaud, L. Lesshafft & P. Huerre:
Global linear stability of a model subsonic jet
AIAA Paper 2011-3608, presented at the 41^{st} AIAA Fluid Dynamics Conference
June 2011, Honolulu, Hawaii

[L38] L. Lesshafft, B. Hall, E. Meiburg & B. Kneller:
Hydrodynamic instabilities in turbidity currents as mechanisms for bed-form creation
7th Int. Symp. on Stratified Flows
August 2011, Rome, Italy

[L39] L. Lesshafft, X. Garnaud, P. Huerre & P. Schmid:
Global stability of subsonic jets
ERCOFTAC Workshop
September 2011, Toledo, Spain

[L40] X. Garnaud, L. Lesshafft, P. Schmid &P. Huerre :
Stabilité globale d'un jet laminaire subsonique
20^{e} Congrès Français de Mécanique
August 2011, Besançon, France

[L41] L. Lesshafft, X. Garnaud, P. Huerre & P. Schmid:
Linear forcing response of subsonic jets
64^{th} annual meeting of the APS Division of Fluid Dynamics
November 2011, Baltimore, Maryland

[L42] X. Garnaud, L. Lesshafft, P. Schmid & P. Huerre:
Linear mechanisms of the sensitivity of jets to external forcing
23^{rd} ICTAM, August 2012, Beijing, China

[L43] a) X. Garnaud, L. Lesshafft, P. Schmid & P. Huerre:
Global frequency response in subsonic jets
b) X. Garnaud, L. Lesshafft, P. Schmid & P. Huerre:
Global instability of jets: limitations of a modal approach
9th Euromech Fluid Mechanics Conference
September 2012, Rome, Italy

[L44] X. Garnaud, L. Lesshafft, P. Schmid & P. Huerre:
Preferred modes in jets: comparison between different measures of receptivity
65^{th} annual meeting of the APS Division of Fluid Dynamics
November 2012, San Diego, California

[L45] S. Derebail, X. Garnaud, L. Lesshafft, P. Schmid & P. Huerre:
Instabilités dans les jets et les panaches
16^{e} Rencontre du Non-linéaire, March 2013, Paris, France

[L46] X. Garnaud, R. Sandberg & L. Lesshafft:
Global response to forcing in a subsonic jet: instability wavepackets and acoustic radiation
AIAA Paper 2013-2232
19th AIAA/CEAS Aeroacoustics Conference
May 2013, Berlin, Germany

[L47] X. Garnaud, L. Lesshafft & P. Huerre:
Global flow response of a RANS jet to inflow forcing
Euromech Colloquium "Trends in Open Shear Flow Instability",
June 2013, École polytechnique, France

[L48] W. Coenen, A. Sevilla & L. Lesshafft:
The influence of the density ratio on the linear frequency response of low-density jets
66^{th} annual meeting of the APS Division of Fluid Dynamics
November 2013, Pittsburgh, Pennsylvania

[L49] a) L. Lesshafft & O. Marquet:
"Wavemakers" — how do we identify what drives an eigenmode?
b) O. Marquet & L. Lesshafft:
Global stability of three-dimensional wake flows developing behind

rectangular flat plates
Interdisciplinary Fluids Meeting, Madingley Hall, Cambridge, July 2014.

[L50] L. Lesshafft, W. Coenen, X. Garnaud & A. Sevilla:
Modal instability analysis of light jets
IUTAM-ABCM Symposium on Laminar-Turbulent Transition, Rio de Janeiro, September 2014.

[L51] a) L. Lesshafft, R.V.K. Chakravarthy & P. Huerre:
Local and global instability of free and confined plumes
b) O. Semeraro & L. Lesshafft:
Optimal forcing of subsonic jets
c) W. Coenen, L. Lesshafft, X. Garnaud & A. Sevilla:
Modal instability analysis of low-density jets
d) O. Marquet & L. Lesshafft:
Steady and unsteady three-dimensional bifurcations in separated flow
10th European Fluid Mechanics Conference, Copenhagen, September 2014.

[L52] a) O. Semeraro, L. Lesshafft & R. Sandberg:
Wavepackets in subsonic jets using optimal forcing
b) W. Coenen, L. Lesshafft, X. Garnaud & A. Sevilla:
Global mode and frequency response analysis of low-density jets
c) R.V.K. Chakravarthy, L. Lesshafft & P. Huerre:
Local stability of axisymmetric plumes
59th meeting of the American Physical Society, San Francisco, November 2014.

[L53] L. Lesshafft & O. Marquet:
Unsteady wavemakers: how to identify instability mechanisms in propagating nonlinear wavefronts
11th ERCOFTAC SIG 33 Workshop on Progress in Transition Modelling and Control, Jersey, 2015

[L54] a) O. Marquet & L. Lesshafft:
Is the frequency of finite-amplitude time- periodic instabilities selected by the mean flow or the eddies interaction?
b) L. Lesshafft & O. Marquet:
How to characterize the instability source in linear and nonlinear global modes
c) O. Semeraro, L. Lesshafft, V. Jaunet, P. Jordan & R. Sandberg:
Coherent structures in turbulent jets: a numerical-experimental analysis
Bifurcations and Instabilities in Fluid Dynamics, Paris, 2015

[L55] L. Lesshafft & O. Marquet:
Endogeneity analysis of linear and nonlinear global modes
ERCOFTAC Symposium on Global Flow Instability and Control, Crete, 2015

[L56] a) R.V.K. Chakravarthy, L. Lesshafft & P. Huerre:
Effect of Buoyancy on the Instability of Light Jets and Plumes
b) O. Semeraro, L. Lesshafft & R. Sandberg:
Can jet noise be predicted using linear instability wavepackets?
c) L. Lesshafft, O. Semeraro, V. Jaunet & P. Jordan:
Modeling of coherent structures in a turbulent jet as global linear instability wavepackets: theory and experiment
Proceedings of the 5th International Conference on Jets, Wakes and Separated Flows, Stockholm, 2015

[L57] O. Semeraro, V. Jaunet, P. Jordan, A. Cavalieri & L. Lesshafft:
Stochastic and harmonic optimal forcing in subsonic jets
AIAA Paper 2016-2935
22th AIAA/CEAS Aeroacoustics Conference, Lyon, 2016

[L58] a) L. Lesshafft, M. Blanchard & O. Semeraro:
Instability response of a premixed M-flame to harmonic and stochastic forcing
b) S. Beneddine, D. Sipp, A. Arnault, J. Dandois & L. Lesshafft:
Conditions of validity for mean flow stability analysis and application to a turbulent backward-facing step case
c) L. Shaabani Ardali, L. Lesshafft & D. Sipp:
Vortex pairing in a jet as an unstable Floquet mode
d) O. Marquet & L. Lesshafft:
Identifying the frequency selection of fluid/solid modes when the interaction is large
e) O. Semeraro, V. Jaunet, P. Jordan & L. Lesshafft:
Stochastic and deterministic optimal forcing in subsonic jets: an experimental and numerical analysis
11th European Fluid Mechanics Conference, Sevilla, 2016

[L59] a) P. Huerre, R.V.K. Chakravarthy & L. Lesshafft:
Local and global instability of buoyant jets and plumes
b) S. Beneddine, A. Arnault, R. Yegavian, D. Sipp, J. Dandois, B. Leclaire & L. Lesshafft:
Stability analysis of the mean field to determine coherent structures in a turbulent backward-facing step flow
c) O. Marquet & L. Lesshafft:

A new formalism for identifying wavemaker regions of linear instabilities: application to a spring-mounted cylinder
XXIV International Congress of Theoretical and Applied Mechanics, Montréal, 2016

[L60] a) L. Shaabani Ardali, L. Lesshafft & D. Sipp:
Vortex pairing and Floquet instability
b) L. Lesshafft, O. Semeraro, V. Jaunet, A. Cavalieri & P. Jordan:
Modelling of large-scale dynamics in a stochastically driven jet
Euromech Colloquium and IUTAM Symposium "Jet Noise Modelling and Control", Palaiseau, 2016

[L61] a) L. Lesshafft, R.V.K. Chakravarthy & P. Huerre:
The multifold effects of density on the instability of jets, plumes and premixed flames
b) L. Shaabani Ardali, D. Sipp & L. Lesshafft:
Subharmonic instability mechanisms of the bifurcation phenomenon in harmonically forced jets
12th ERCOFTAC SIG 33 Workshop on Progress in Flow Instability, Transition and Control, Siena, 2017

[L62] a) L. Lesshafft, O. Semeraro, V. Jaunet, P. Jordan & A.V.G. Cavalieri:
Modelling of coherent structures in turbulent jets at high Reynolds number by mean flow instability analysis
b) L. Shaabani Ardali, D. Sipp & L. Lesshafft:
Optimal forcing for jet bifurcation
16th European Turbulence Conference, Stockholm, 2017

[L63] a) L. Lesshafft, O. Semeraro, V. Jaunet, P. Jordan & A.V.G. Cavalieri:
Success and open questions in the modelling of jet turbulence through Statistical State Dynamics
b) L. Shaabani Ardali, D. Sipp & L. Lesshafft:
Mixing enhancement through optimally controlled jet bifurcation
13th ERCOFTAC SIG 33 Workshop on Progress in Flow Instability, Transition and Control, Paraty, 2018

[L64] a) L. Lesshafft, L. Shaabani Ardali, O. Semeraro, V. Jaunet, P. Jordan, A.V.G. Cavalieri, B. Cuenot & L. Gicquel:
Recovering the transfer function of a turbulent jet and a laminar flame from linear mean flow analysis
b) L. Shaabani Ardali, L. Lesshafft & D. Sipp:
Triggering symmetry-breaking secondary instabilities in pulsed jets and

flames
12th European Fluid Mechanics Conference, Vienna, 2018

6.3 Other

[L65] L. Lesshafft, 2006:
Nonlinear global modes and sound generation in hot jets
Ph.D. thesis, École Polytechnique, defended 11/12/2006

[L66] L. Lesshafft, 2012:
Le génome du bananier épluché
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[L67] O. Marquet & L. Lesshafft, 2015:
Identifying the active flow regions that drive linear and nonlinear instabilities
arXiv:1508.07620

[L68] L. Lesshafft, 2015:
Preface to a Festschrift for Patrick Huerre
Eur. J. Mech. B/Fluids, vol. 49, p. 299–300

[L69] L. Lesshafft, P. Jordan & A. Agarwal 2018:
Foreword
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