Experiments on vortex-induced traveling waves along a cable

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In the recent development of oil fields in deeper ocean sea, structures of length-to-diameter aspect ratio exceeding 1000 and complex geometry are used, and Vortex-Induced Vibrations (VIV) remain a challenging problem. Because of the high aspect ratio, boundary conditions at the sea bed and at the sea surface are less effective in reflecting bending waves, so that vortex-induced traveling waves are expected prior to standing ones [1, 2]. Moreover, new design concepts and configurations such as free hanging export lines, Simple Catenary Risers (SCR), Lazy Wave SCR (LW), Compliant Vertical Access Risers (CVAR), Steel Hybrid Risers for extended water depth (SHR'ewd), etc. have been recently introduced [3]. For structures whose geometry varies in time, e.g. due to the changing location of the touch-down point at the sea bottom, a traveling wave approach may therefore become a useful alternative to the standard modal analysis based on standing waves. Here we focus on traveling waves induced by vortex shedding in a free-hanging towed cable.

From an analytic point of view, an ideal infinite length structure allows any couple of angular frequency $\omega$ and wave velocity $c$ satisfying its dispersion relation. Conversely, the vortex shedding phenomenon is characterized by the Strouhal frequency $\omega_s = \frac{2\pi U}{D}$, where $D$ is the diameter of the structure, $U$ is the flow velocity and $S$ the Strouhal number. In a coupled VIV fluid-structure dynamical system of infinite length the fluid sets the Strouhal frequency and the structure fixes the corresponding wave velocity through its dispersion relation. This general framework is also for a low-order model of VIV of slender structures based on van der Pol wake oscillators in [4, 5].

Experiments are carried out in a water towing-tank. A long flexible cable, made of interwoven synthetic fibers and whose physical properties are summarized in table 1, is towed and thus submitted to vortex shedding excitation at low Reynolds number, of the order of 100, figure 1. A fixed digital video camera is aligned in the vertical towing plane: the provided numerical movie is processed in order to correct for parallax and perspective aberration, and the transverse displacement of the cable is obtained versus time, figure 2.

The upper end of the cable is connected to a towing carriage under the free surface, to avoid any interface effect. The lower end is unrestrained, in order to model a partially non-reflecting boundary condition, through the effect of vanishing tension. By observing the spatio-temporal evolution of the system, figure 2, there is definite evidence of traveling waves propagating through the towed cable towards the lower free end. In order to quantify the characteristics of the traveling waves, both standard spatio-temporal correlation analysis and Bi-Orthogonal Decomposition (BOD) are applied.
Table 1: Cable physical properties.

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<tbody>
<tr>
<td>length</td>
<td>L</td>
<td>500 mm</td>
</tr>
<tr>
<td>diameter</td>
<td>D</td>
<td>2 mm</td>
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<tr>
<td>mass in water</td>
<td>m</td>
<td>0.22 g/m</td>
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Hydro-elastic stability of the towed cable is insured by a length-to-diameter aspect ratio exceeding the critical value of $\pi/2C_f = 78$, where $C_f = 0.02$ is the frictional coefficient of the cable [6]. Therefore, the cable motion is definitely due to VIV.

The angular frequency $\omega$ is computed by Fast Fourier Transform (FFT) at several spanwise location of the cable. Measurements show a uniform distribution of $\omega$ along the cable extent, meaning that the overall cable motion has a unique harmonic component. The cable being bent in the vertical towing plane at a constant angle $\theta$, figure 1, the significant Strouhal number is derived from the experiment by applying the cosine relationship [7], $St = \omega/2\pi$ $D/U$ $\cos$ $\theta$, and then compared to a universal $St(R_e)$ curve [8], figure 3a. Measurements show a reasonable agreement to the general VIV phenomenology. This supports the main idea of the fluid role in selecting the angular frequency through the Strouhal’s law.

The bending angle of the towed cable $\theta$, figure 1, is constant along the cable extent. According to the linearized dynamical model of towed flexible cylinders [9, 10], the mean cable position in the vertical towing plane is completely uncoupled from the transverse VIV oscillations and the inclination angle $\theta$ is set by the balance of the fluid-dynamical and the gravity forces, leading to a spanwise linear variable tension in the cable. We consider here the simple model where the fluid-dynamical forces are reduced to a sectional drag loading $F = 0.5\rho U^2 CD$. For stationary cylinders at $R_e$ of order 100 the sectional drag coefficient is assumed to be $C_{D_s} = 1.5$ [11] and a drag magnification depending on the cable transverse motion may be taken into account in the form $(1 + 2Y)C_{D_s}$ [11]. The cable dimensionless transverse oscillation amplitude is here of the order of $Y = 1$, so that $C_D = 4.5$. The static equilibrium in the direction normal to the cable yields the bending angle

$$\tan \theta = \frac{F_r^2 \rho U^2 L r CD}{2m}, \tag{1}$$

where $F_r = U/\sqrt{gL_r}$ is the Froude number and $L_r = L/2$ is a reference length. The static equilibrium in the tangential direction yields the constant tension gradient along the cable, namely

$$\frac{dT}{ds} = gL_r \left( F_r^2 \rho U^2 \frac{D}{2} CD \sin \theta + \frac{m}{L_r} \cos \theta \right). \tag{2}$$

A dimensionless wave velocity $c$ may be predicted as

$$c = \frac{c_r}{c_o}, \quad c_o = \sqrt{\frac{T_r}{m + m_g}}, \quad T_r = \left( \frac{dT}{ds} \right) L_r, \quad c_a = \sqrt{\frac{mgL_r}{m + m_g}}, \tag{3}$$
Figure 3: (a) Strouhal number $S_t$ as a function of Reynolds number $R_e$: measurements ‘—’ $S_t = 0.2665 - 1.018/\sqrt{R_e}$ [8]. Error bars are due to the accuracy in evaluating $\theta$. (b) Dimensionless wave velocity $c$ as a function of Froude number $F_r$: measurements ‘—’ are compared to the model (3).

where $m_a$ is the classical fluid added mass [11]. Measurements of wave velocity show a reasonable agreement to the proposed model, figure 3b. This supports the main idea of the structure role in setting the wave velocity, the angular frequency being fixed by the Strouhal’s law.

References


