A theoretical study of the frequency selection mechanisms in afterbody unsteadiness

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The hydrodynamic stability theory is used to investigate the frequency selection in the flow behind an axisymmetric blunt based body. We present two theoretical approaches, resulting in different frequencies that are compared to the natural frequency observed in experimental configurations: we use the framework of local stability, in which vortex shedding arises from the transition to absolute instability of a large scale oscillating mode of azimuthal wave number m=1, and that of global stability accounting for the non-parallelism of the flow in the recirculating area. The differences between both approaches are discussed and an interpretation is proposed, based on the non-linear saturation effects.

I. Introduction

LAUNCHER afterbody flows are characterized by a massive separated area that generates strong low frequency wall-pressure fluctuations and induces aerodynamic excitation, resulting in high dynamic loads. These oscillations can trigger a response of the structural modes termed buffeting that can be critical during the transonic phase of flight. With the constant need of improving the aerodynamic performances and the reliability of launch vehicles, a good knowledge of the mechanisms responsible for the onset of the buffeting phenomenon is therefore needed to guide the design of afterbodies.

Within the last decades, flows around various launcher configurations have been investigated experimentally in the high subsonic regime, so as to obtain extensive unsteady wall pressure measurements in the afterbody region[6,7]. The results obtained on axisymmetric blunt based body configurations show that the spectra in the whole separated region exhibit a well-defined peak at a Strouhal number St_D~0.2 (based on the diameter of the afterbody and the free stream velocity), corresponding to the shedding of large-scale vortices in the wake. Furthermore, a two-point correlation analysis of the turbulent signals by Deprés *et al.*[6] shows that the flow is dominated by a highly coherent helical mode of azimuthal wavenumber m=1 at the vortex shedding frequency. This comes in support of other experimental studies of the wake behind axisymmetric obstacles, such as spheres and circular disks, showing that the dynamics of these flows is dominated by the shedding of large-scale vortical structures in the form of two superimposed helical modes m=±1[1,3].

In this paper, we study the unsteady properties of the flow past an axisymmetric blunt based body in the light of the stability theory: the aerodynamic flow field is decomposed into the following asymptotic expansion

$$q = q_0 + \varepsilon^{1/2} q_1 + \varepsilon q_2 + \varepsilon^{3/2} q_3 ... \quad (1)$$

The scaling of ε is given in section III-B. The flow field is hence considered as the superposition of a base flow q_0, searched as an axisymmetric solution of the governing equations, and of successive perturbations q_i of amplitude $\varepsilon^{i/2}$.

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Following the pioneering work of Pier in the context of two-dimensional wakes[15], we consider that the synchronized oscillations observed experimentally arise as a self-sustained resonance phenomenon resulting from a non-linear global instability. In such situations, it turns out that linear analyses often allow to determine the associated propagation velocity of this non-linear resonance[8,11,15]. We hence firstly carry out an analysis in the framework of linear stability, in which the asymptotic development (1) is truncated at order 1/2. This linear analysis itself will be divided in two steps, the first of which relying on a local formalism, *i.e.* the stability analysis is either performed on a theoretical parallel flow or on a more realistic flow supposed locally parallel. Although this may appear constraining, this formalism allows to introduce the concept of absolute instability, which plays a crucial role in the onset of such intrinsic periodic dynamics. In particular, it will be shown that the local analysis of model flows can be strikingly connected to the non-linear global frequency selection in more representative flows. The second step of this linear stability analysis will consist in an extension to fully spatially developing flows, using a global stability approach. Finally, we will use a weakly nonlinear global stability analysis to interpret the different frequencies selected by both linear approaches.

II. Local stability theory

The local stability theory investigates the stability of flows with one direction of inhomogeneity. A large body of work has been devoted to this formalism in the last decades, as 1D-problems demand very low computational costs. In particular in the context of afterbody flows, it allows to consider inhomogeneous compressible flows, as required by the applications. When applied to open flows, the local stability theory aims at identifying modes of zero group velocity, that are shown to trigger the asymptotic impulse response of the flow at large times: if the mode is damped, the flow is said to be convectively unstable, and the small amplitude wave packet generated by an initially localized impulse propagates downstream under the effect of advection by the base flow. If the mode is amplified, the flow is said to be absolutely unstable and the wave packet propagates both upstream and downstream and grows in time at any fixed location. Such flows are characterized by an intrinsic dynamics and can undergo self-sustained resonance phenomena at a well defined frequency. Extensive work has been carried out on a broad variety of shear flows, strong evidence has been collected to show that synchronized oscillations, such as those observed in the wake of a cylinder, or in hot jets, for instance, is related to the onset of a region of absolute instability in the flow.

We first investigate modes of zero group velocity on model wake profiles to address the points of azimuthal wavenumber and frequency selection. The flow is assumed to be governed by the compressible equations of continuity, momentum and energy. All flow variables are written in non-dimensional form, scaled with respect to the jet radius *R* and the upstream flow quantities. Provided that the streamwise variations of the base flow take place on a sufficiently large time scale compared to viscous diffusion time scale, the base flow can be taken as a solution of the inviscid equations. The energy equation for the base flow is then replaced by the Crocco-Busemann relation, derived from the boundary layer equations, so that it is possible to consider analytic base flow profiles. We use here the model family of axisymmetric wakes introduced by Monkewitz and Sohn[12] and Monkewitz[13], characterized by the ratio of the centerline to free stream velocities Λ, the dimensionless momentum thickness θ/D (represented by the parameter N in the following expression), and the ratio of the centerline to free stream densities S (see figure 1a)

$$w_0(r) = 1 - \Lambda + \frac{2\Lambda}{1 + (2^{r^2} - 1)^N} \tag{2a}$$

$$T_0(r) = \rho_0(r)^{-1} = 1 + \left(\frac{1}{S} - 1\right)\left(\frac{w_0(r) - 1 + \Lambda}{2\Lambda}\right) + \frac{\gamma - 1}{2} M_\infty^2 \left(\frac{w_0 - 1 + \Lambda}{1 - \Lambda}\right)\left(\frac{w_0 - 1 - \Lambda}{1 - \Lambda}\right). \tag{2b}$$

Since all quantities are 2π-periodic in the azimuthal direction, all perturbations are chosen in the form of normal modes

$$q_1(r,\theta,z,t) = q_1(r) e^{i(m\theta + kz - \omega t)} + cc \tag{3}$$

where $q_1=(u_1,v_1,w_1,\rho_1,T_1,p_1)$ is the radially varying part of the mode. m and k are the azimuthal and axial wavenumbers, and ω is the complex pulsation, ω_r and ω_i being respectively the frequency and the growth/damping rate of the mode (ω_i>0 for an unstable mode). The Strouhal number is therefore defined as $St_D=\omega_r D/2\pi U_\infty$. For a given set of control parameters, the system of equations determining ω and q_1 can then be written as a complex generalized eigenvalue problem, which we solve with a collocation method based on Chebyshev polynoms. This yields a complete set of eigenvalues and associated eigenvectors.

It is shown in the framework of fronts velocity dynamics, only the mode prevailing at the threshold of absolute instability is of interest when one considers a non-linear global instability triggered by a local absolute instability[4]. We find that for very light wakes of density ratios S<0.3, the convective-absolute transition is led by a short scale axisymmetric mode (m=0), whose frequency scales on the momentum thickness. Obviously, neither the structure, nor the frequency of the selected mode, correspond to the unsteady flow observed in wind tunnel tests. However, for all other parameters, the transition is led by a low-frequency large-scale helical mode (m=1), whose frequency varies within the range $0.1\leq St_D\leq 0.3$ and is extremely robust with respect to the variations of the control parameters. Figure 1b shows the variations of the helical mode frequency at the transition to absolute instability as a function of the Reynolds number, obtained for a generic wake ($\theta/D=1/50$, S=1, centerline velocity corresponding to backflow of 9% of the free stream velocity) for different Mach numbers. The frequency of the mode is weakly affected by the variations of the Reynolds number, whereas an increase of the Mach number results in a slight decrease of the mode frequency. One thus observes that local stability analysis performed on model flow profiles is in good agreement with that found in wind tunnel tests.

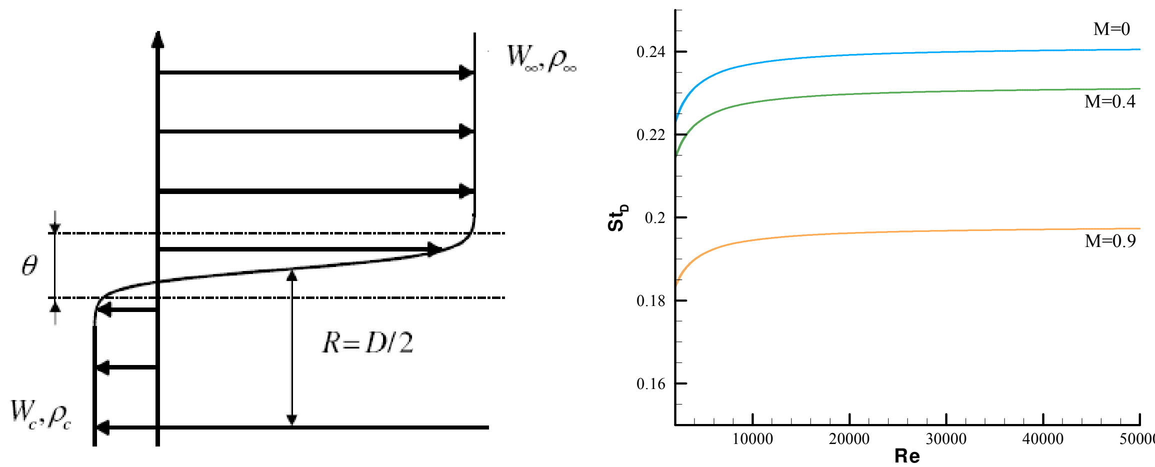

Figure 1. a) Model family of axisymmetric wakes considered in this study. All non dimensional parameters are based on the free stream quantities and the radius of the wake. b) Absolute frequency of the zero group velocity helical mode (m=1) as a function of the Reynolds number, for M=0, 0.4 and 0.9.

Although this first approach gives satisfying results, one must remember that these model profiles do not account for the streamwise variations of the flow past a real afterbody. In the last decades, a large body of work has therefore been conducted to connect these local stability properties to the global dynamics of spatially developing flows. It has been shown that many of the local instability properties obtained for parallel profiles can be used to describe the evolution of perturbations of the spatially developing flow, provided that its streamwise variations take place on a sufficiently large length scale compared to the instability wavelength. Under this condition, instability modes at a given streamwise station can be expected to evolve as if the baseflow were *locally parallel*. Practically, for each streamwise station, the spatially developping base flow is considered parallel and a local stability analysis is then performed to determine the absolute frequency and growth rate of the instability. If one wants to predict the frequency of the unstable global mode, additional criteria are needed, whose degree of success depends on the flow under considerations[10,15]. We use here a non-linear front criterion: from investigations of non-linear Ginzburg–Landau model equations, Pier *et al.*[14] have found that the global frequency of synchronized oscillations is imposed by a stationary wave front, pinned at the first streamwise position to undergo an absolute instability, that imparts its linearly selected frequency ω_{ca0} to the entire downstream nonlinear wavetrain. This criterion was shown to provide a 10% accurate prediction over the range of Reynolds numbers $100\leq Re\leq 180$. Similar results make plausible the interpretation of the axisymmetric oscillations in hot jets and of the double-helix structure arising in swirling jets as a non-linear global instabilities[8,11].

We apply this criterion to the case of a complex turbulent transsonic afterbody flow. We use a time and space averaged base flow issued from a large eddy simulation carried out at ONERA's Applied Aerodynamics Department[5] for a Reynolds number Re~5.10^6 and a Mach number M=0.7. Absolute frequencies and amplification rates of the first helical mode are computed as a function of the streamwise position (figure 2). Considering the most upstream point of marginal absolute instability, the front criterion predicts a frequency St_D=0.22 in excellent agreement with the experimental frequency St_D=0.21. A possible explanation for this striking result is that the analysis is conducted on the mean flow, rather than on the base flow. As discussed by Barkley[2], the mean flow is not a solution of the steady Navier-Stokes equations, but is a solution of the steady equations forced by the Reynolds stresses arising from the non-linear interactions. Therefore, the mean flow partially accounts for the non-linear saturation effect at this high Reynolds number and is more representative of the dynamics of the real flow.

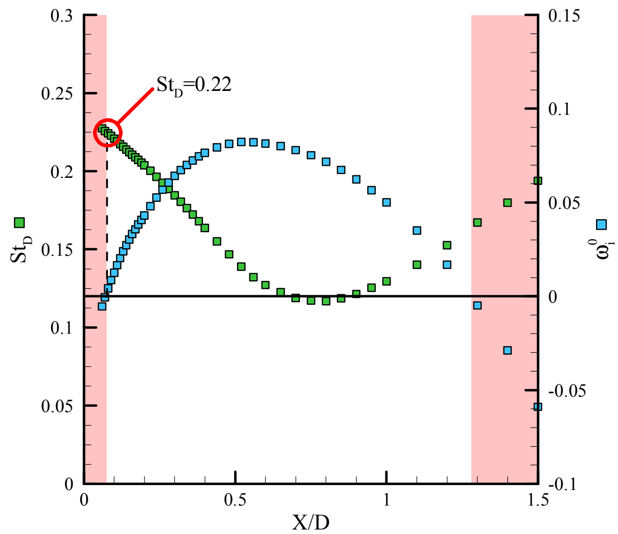

Figure 2. Local linear analysis of the mean flow issuing from a large eddy simulation (Re~5.10^6, M=0.7). Absolute frequencies (green symbols) and growth rates (blue symbols) of the first helical mode (m=1) as a function of the streamwise position. The instability is convective in the red areas and absolute otherwise.

III. Global stability theory

A. Linear analysis

The global stability formalism is a more realistic approach relaxing the slow streamwise variations hypothesis, as it considers base flows and disturbances with two directions of inhomogeneity, namely the radial and axial directions. Therefore, this formalism is particularly well adapted to the case of separated flows, where the quasi-parallel approximation does not hold in the recirculating area. As will be shown in the following, another motivation for this approach is to evaluate the accuracy of the frequency selection criteria based on the local theory. This can be done by considering the case of unperturbed base flows that can be computed only at low Reynolds numbers. We are by now limited to the incompressible case for a reason of computational costs.

Because all axisymmetric modes (m=0) are stable at the low Reynolds numbers considered here, the base flow can be obtained from time-dependent simulations based on a finite-element spatial discretization and a Lagrange-Galerkin temporal discretization. The perturbations are chosen under the form of *global* normal modes

$$q_1(t,r,\theta,z) = q_1(r,z)e^{\sigma t + im\theta} + cc. \tag{4}$$

where σ is the complex pulsation, σ_r and σ_i being respectively the growth/damping rate and the frequency of the mode (σ_r>0 for an unstable mode). The Strouhal number is therefore defined as $St_D=\sigma_i D/2\pi U_\infty$. The system of equations determining σ and q_1 can again be written as a generalized eigenvalue problem. A selected number of eigenvalues close to a given guess-value, and the associated set of eigenvectors are obtained by use of an Arnoldi method based on a shift-invert strategy. In this problem, the control parameters are the Reynolds number and the ratio L/D (and consequently the shear layer thickness at the base), where L and D are respectively the length and diameter of the axisymmetric body. In the following, we consider a wide range of aspect ratios, 1≤L/D≤15. Figure 3 presents the streamlines of the base flow obtained for L/D=4 at Re=666.

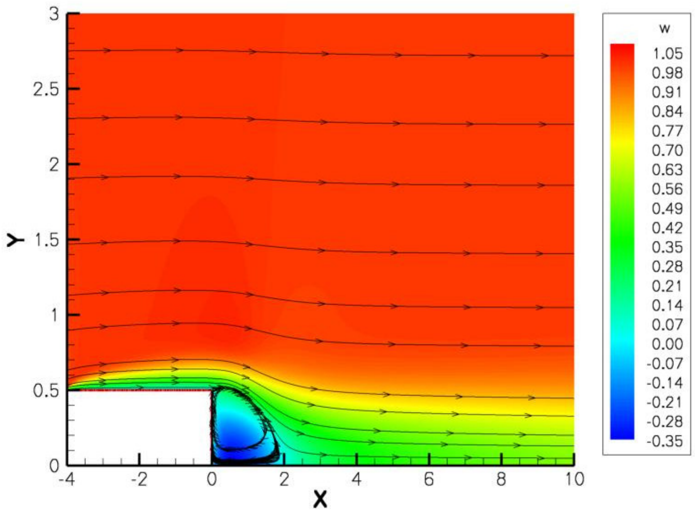

Figure 3. Base flow axial velocity and streamlines, for L/D=4 and Re=666.

For all values of L/D, we show that when the Reynolds number is increased, a first instability occurs for a non-oscillating m=1 mode. When the Reynolds number is further increased from this critical value, another oscillating helical mode becomes unstable, whose eigenmode exhibits the spatially periodic downstream structure characteristic of the oscillatory wake instability (see figure 4). An interesting result is that the frequency of the marginally unstable mode is significantly affected by the aspect ratio L/D, i.e. a change of the boundary layer thickness at the separation, whereas the frequency obtained in wind tunnel tests is robust: as a matter of fact, at such low Reynolds numbers, the global linear theory predicts frequencies in a range $0.06 \leq St_D \leq 0.13$ that only represent the linear approximation of the natural frequency of the flow for small variations on the Reynolds number around the critical value, and thus are unable to approach the frequency of the flow observed in wind tunnels at higher Reynolds numbers.

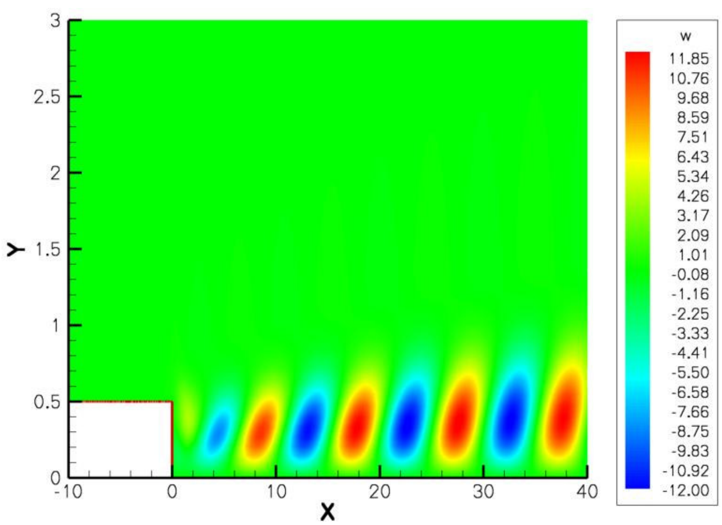

Figure 4. Axial velocity perturbations for the neutrally unstable oscillating mode (L/D=4, Re=666).

We now propose to use the spatially developing base flows issuing from the finite-elements computations to compare the frequency arising from the global stability analysis to that predicted by the front criterion. Figure 5 presents the absolute frequencies and amplification rates of the low-frequency large-scale helical mode (m=1) as a function of the streamwise position for the case L/D=4 at the critical Reynolds number Re_c=666. The local analysis confirms that the global base flow is locally absolutely unstable in the vicinity of the afterbody for $0.028 \leq X/D \leq 1.614$, and predicts a frequency St_D=0.17 whereas the global analysis predicts a frequency St_D=0.09. Results for the whole range of aspect ratios L/D under consideration are provided in table 1: it turns out that the front criterion now predicts a frequency almost independent on L/D, varying between 0.15 and 0.17. This may be due to the fact that the upstream station of marginal absolute instability is located very close to the base (x/D<0.05) and is therefore very weakly dependent on the incoming flow. Note that the position of the absolutely unstable domain varies slowly with L/D. Finally, although absolute instability is the physical mechanism responsible for the onset of the global unsteadiness, it turns out that the global and local theory do not meet at such low Reynolds numbers, due to the non-parallelism of the separated area.

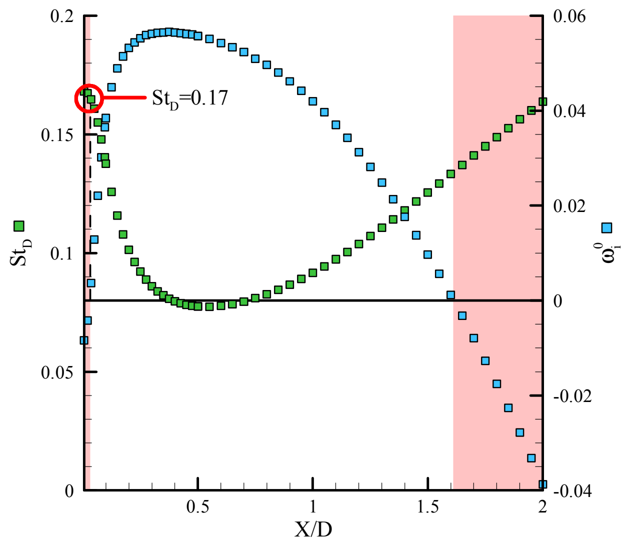

Figure 5. Linear local analysis of the base flow issuing from the FEM computations (L/D=4, Re_c=666). Absolute frequencies (green symbols) and growth rates (blue symbols) of the m=1 mode as a function of the streamwise position. The instability is convective in the red areas and absolute otherwise.

L/D	Absolutely unstable domain	Front criterion frequency prediction	Global frequency prediction
1	$0.044 \leq X/D \leq 1.307$	0.17	0.13
2.5	$0.033 \leq X/D \leq 1.497$	0.17	0.10
4	$0.028 \leq X/D \leq 1.614$	0.17	0.09
6	$0.022 \leq X/D \leq 1.716$	0.16	0.08
8	$0.018 \leq X/D \leq 1.788$	0.16	0.07
10	$0.016 \leq X/D \leq 1.838$	0.15	0.07
12	$0.014 \leq X/D \leq 1.874$	0.15	0.06
15	$0.012 \leq X/D \leq 1.915$	0.15	0.06

Table 1. Properties of the unstable flow predicted by the front criterion. The frequency predicted by the global analysis is also provided for comparison.

B. Weakly non-linear analysis

In this section, we focus on the oscillatory instability. Since the linear analysis fails to predict the frequency prevailing in the fully non-linear flow, it makes sense to perform a weakly non-linear analysis so as to estimate the leading order correction due to the saturation. This is done by substituting the full asymptotic expansion

$$q = q_0 + \varepsilon^{1/2} q_1 + \varepsilon q_2 + \varepsilon^{3/2} q_3 \ldots \tag{5}$$

in the governing equations, where ε is the small parameter $\varepsilon = Re_c^{-1} - Re^{-1}$. This yields a series of equations of successive order $\varepsilon^{i/2}$. At order 0, we find the non-linear equation specifying that q_0 is a steady solution of the Navier-Stokes equations at the critical Reynolds number Re_c. At order 1, we obtain the homogeneous linear equation specifying that q_1 may be taken as a superposition of global modes of the steady flow field q_0 at Re_c. We can therefore choose q_1 as the superposition of the marginal eigenmodes existing at the critical Reynolds number, each mode being multiplied by some complex scalar amplitude. Note that for symmetry considerations, two global modes are to be considered, *i.e.* the system undergoes a codimension-two bifurcation, as we have to superimpose two modes of azimuthal wavenumbers m=±1. q_1 can therefore be written in the form

$$q_1 = \left(A^+ q_1^+ e^{i\theta} + A^- q_1^- e^{-i\theta}\right) e^{-i\sigma_0 t} + cc \tag{6}$$

where A^+ (resp. A^-) is the complex amplitude of the m=1 mode (resp. m=-1). At order 2, we derive a system of coupled Stuart-Landau amplitude equations which reads

$$\frac{dA^+}{dt} = \varepsilon\lambda A^+ - \varepsilon A^+\left(\mu\left|A^+\right|^2 + \nu\left|A^-\right|^2\right) \qquad \frac{dA^-}{dt} = \varepsilon\lambda A^- - \varepsilon A^-\left(\mu\left|A^-\right|^2 + \nu\left|A^+\right|^2\right) \tag{7}$$

For more details, the reader may refer to the study of Sipp and Lebedev[16]. For symmetric initial conditions in A^+ and A^-, it can be shown that q_1 can be written as

$$q_1 = |A| \exp\{i(\sigma_0 + \varepsilon\sigma_l + \varepsilon\sigma_{nl})t\}\left(q_1^+ e^{i\theta} + q_1^- e^{-i\theta}\right) + cc \tag{8}$$

with

$$\sigma_l = \varepsilon\lambda_i \qquad \sigma_{nl} = -\varepsilon\lambda_r \frac{\mu_i + \nu_i}{\mu_r + \nu_r}. \tag{9}$$

The non-linear frequency of the saturated flow now reads $\sigma = \sigma_0 + \sigma_l + \sigma_{nl}$, where σ_0 is the frequency of the marginally unstable global mode issuing from the linear theory. σ_l and σ_{nl} are respectively the shifts in frequency due to linear and non-linear mechanisms as the Reynolds numbers gets supercritical. Figure 6 shows the evolution of frequencies issuing from the linear and the non-linear amplitude equations. The dashed lines used at high values of Re are a reminder that the domain of validity of the weakly non-linear analysis is limited in terms of Reynolds numbers, namely the stronger the non-linear saturation, the more questionable the relevance of the analysis. In figure 6, the weakly non-linear analysis allows to calculate the exact shape of the tangent line to the St_D(Re) curve at Re= Re_c (black solid lines) but not its curvature, as this would require to consider the following terms of order 4 and 5. Note however that the leading-order non-linear effect results in a significant increase in the frequency. Again, this illustrates that the synchronized oscillations observed experimentally may be triggered by a non-linear global instability, since the fully non-linear interactions have to be considered to understand the increase from the linear marginal frequency to the natural frequency in high Reynolds number flows. Figure 7 presents the generalization of this non-linear effect to other aspect ratios in the range 1≤L/D≤15: interestingly, the effect of L/D is still significant. To this day, several hypotheses are still to be considered in order to present a clear picture of this phenomenon. For instance, it is possible that the characteristic length controlling the size of the separated area in the high Reynolds number flow is that of small-scale turbulence, instead of that of the incoming boundary layer, as it was found in this theoretical study.

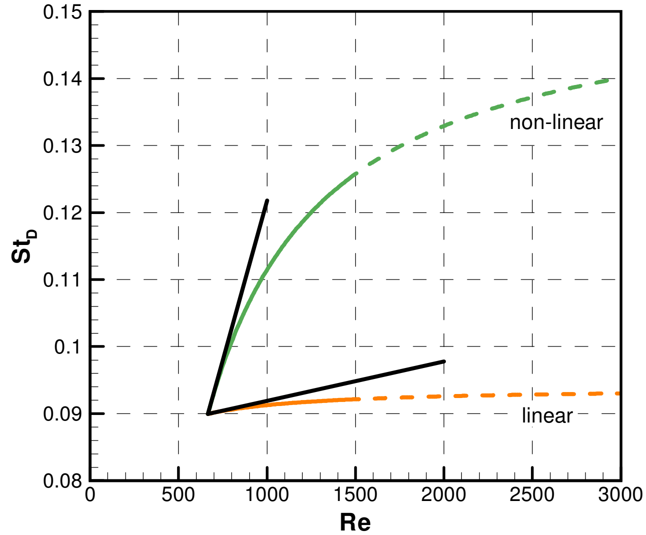

Figure 6. Linear global frequency (orange line) and non-linearly corrected frequency (green line) as a function of the Reynolds number, for L/D=4). The black solid line is the tangent line to the $St_D(Re)$ curve at the critical Reynolds, and the grey line is the tangent to the linear frequency curve.

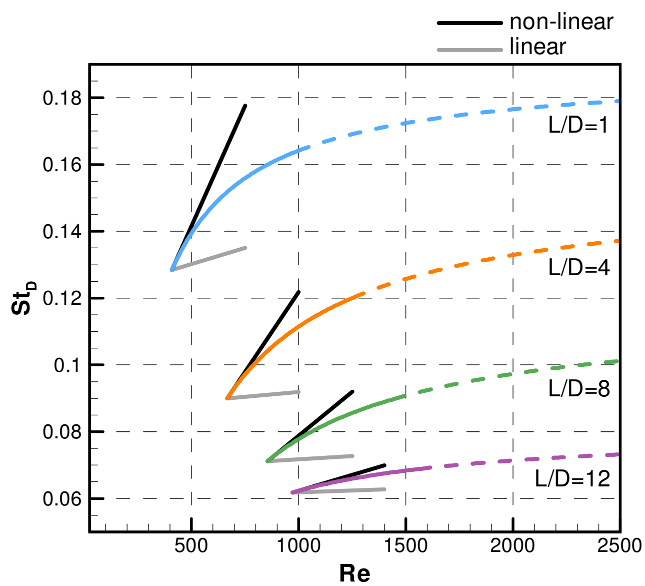

Figure 7. Non-linearly corrected frequency as a function of the Reynolds number, for various aspect ratios. The black solid line is the tangent line to the $St_D(Re)$ curve at the critical Reynolds, and the grey line is the tangent to the linear frequency curve (not shown).

IV. Conclusion

In this study, we have used the frameworks of both the local linear and global linear stability to investigate the frequency selection in the unsteady flow past an axisymmetric blunt base body. Consistently with previous numerical and experimental studies, both approaches predict that the dynamics of the flow is dominated by an instability of the first helical mode (m=1). At low Reynolds numbers, the frequency obtained from the global linear stability analysis of axisymmetric base flows leads to low-frequencies depending on the thickness of the boundary layer at the separation. A front criterion based on the local convective-absolute stability properties of the base flow leads to higher and more robust frequencies closer to the natural frequency observed in wind tunnel tests. At high Reynolds numbers, the front criterion used on the time and space mean flow issuing from a large eddy simulation provides a strikingly accurate frequency, making plausible the interpretation of the vortex shedding unsteadiness behind an axisymmetric blunt based body as the result of a non-linear global instability. This is confirmed by a weakly non-linear global analysis that shows that the leading-order correction on the global frequency due to the non-linear saturation leads to a significant increase.

These results are also very useful in the context of flow control: we now plan to use them in order to control the onset of unsteadiness by determining an optimal control law based on stationary base and wall bleed/suction methods.

Acknowledgments

The authors thank the Centre National d'Etudes Spatiales (CNES) for financial support within the framework of the research and technology program ATAC (Aérodynamique des Tuyères et Arrière-Corps). The Ph.D. dissertation of P. Meliga is funded by CNES and ONERA.

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