Global stability and adjoint-based control of a confined impinging jet

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We investigate a laminar plane jet impinging on a flat plate in a channel. A global stability analysis is carried out and shows that, for a strong confinement, the two-dimensional steady flow is unstable to three-dimensional steady perturbations. Adjoint methods and sensitivity analyses are then used to assess the efficiency of a 3D harmonic or 2D steady control to stabilize the leading 3D mode, by means of either bulk or wall forcing.

Keywords: Impinging jet, Global instability, Adjoint methods, Control

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Extended Summary

Jet impingement is widely used in many industrial and engineering applications when an intense and rapid heat transfer is desired, for instance the tempering of metal sheets, the cooling of electronic components and turbine blades, or the drying of papers and textiles. Only a limited amount of studies is available about highly confined, low-Reynolds number jets. The steady and unsteady dynamics of a laminar jet impinging on a flat plate has been investigated for various Reynolds numbers and levels of confinement (Chiriac & Ortega, 2002). However, such studies restrict to 2D flows only, whereas three-dimensionality is crucial when spanwise homogeneous transfers are desired. We investigate here the stability of a two-dimensional jet of width e impacting a wall set normal to the jet and located at a distance H from the jet exit. We consider a highly confined jet of height ratio $H/e = 2$. The fluid motion is governed by the incompressible Navier-Stokes equations made non-dimensional by the height H and the jet centerline velocity. $\boldsymbol{u} = (u, v, w)^T$ is the fluid velocity where u, v and w are the streamwise, cross-stream and spanwise components, and p is the pressure. We consider a flow $\boldsymbol{q} = (\boldsymbol{u}, p)^T$ made of the superposition of a 2D steady base flow $\boldsymbol{Q} = (U, V, 0, P)^T$ and a three-dimensional perturbation $\boldsymbol{q'}$ of amplitude ϵ.

The base flow is obtained by carrying out time-dependent simulations with a parabolic velocity profile imposed at the inlet, the discretization being based on second order schemes (Taylor-Hood finite-elements and Lagrange-Galerkin discretizations). It is symmetric with respect to the jet axis, and consists in a first separation bubble extending at the upper wall from the edge of the nozzle, and, for Reynolds numbers $\mathrm{Re} \simeq 330$, in a second separation bubble at the lower wall. The instability problem for $\boldsymbol{q'}$ is formulated linearizing the governing equations about $\boldsymbol{Q}$. We use a global normal mode expansion

$$\boldsymbol{q'} = \hat{\boldsymbol{q}}(r, z) e^{(\sigma + i\omega)t + \mathrm{i}kz} + \text{ c.c.} \tag{0.1}$$

where $\hat{\boldsymbol{q}}$ is the so-called global mode of spanwise wavenumber k, growth rate σ and frequency ω. $\hat{\boldsymbol{q}}$ and the complex pulsation $(\sigma + i\omega)$ are solutions of an eigenvalue problem which is then solved numerically using an Arnoldi method. We find that the first instability occurs at $\mathrm{Re} = 858$ for a stationary ($\omega = 0$), antisymmetric mode of spanwise wavenumber $k = 3.49$ (corresponding to a wavelength $2\pi/k = 1.80$),

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indicating that the flow bifurcates towards a 3D steady state. Fig. 1 shows the spatial structure of the streamwise velocity component of the leading global mode, which is dominated by axially extended disturbances located downstream the stagnation point.

We then use adjoint methods to identify different regions of the flow that are of particular interest in the perspective of controlling the leading global mode. The results obtained suggest different locations of the actuator depending on the control method. In the case of a 3D harmonic bulk forcing, the actuator should be placed in the flow regions where the adjoint global mode has a large amplitude, the latter adjoint mode being computed as the solution of an eigenvalue problem. In the case of a 'force-velocity' coupling where a local bulk force applies a small feedback on the global mode through a sensor located as the actuator position (Giannetti & Luchini, 2007), the sensor/actuator should be placed in the overlapping region between the direct and adjoint global modes, which is presently located within the first recirculating bubble. The case of a 2D steady control is accounted using sensitivity analyses, where the variation of the growth rate induced by the control is expressed formally as the scalar product between the control itself, and a gradient or sensitivity function, depending on the adjoint global mode and on an adjoint base flow computed as the solution of a linear nonhomogeneous problem (Marquet *et al.*, 2008). We find that the forcing should act so as to modify the base flow in the first recirculating bubble to obtain a large impact on the dynamics. For both 3D harmonic and 2D steady forcing, we will also address the question of wall forcing. Finally, we will discuss the application of these concepts to two open-loop control strategies in which we introduce into the flow a small control device chosen as a cylinder or a flat-plate airfoil, modeled by the drag or lift force it exerts on the flow. Physical interpretation for the effect of such control methods will also be proposed, based on the competition between production and advection of disturbances.

References

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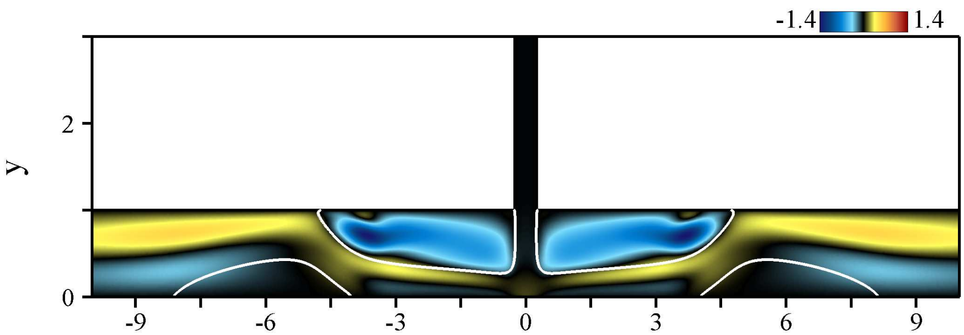

Figure 1: Spatial distribution of streamwise velocity for the leading 3D global mode at threshold of instability. The white solid lines stand for the separation lines of the base flow.