LOW SPEED FLUTTER AND LIMIT CYCLE OSCILLATIONS OF A FLAT PLATE IN A WIND TUNNEL

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ABSTRACT
This paper focuses on the dynamical responses of a two degrees of freedom flat plate undergoing classical coupled-mode flutter in a wind tunnel at low Reynolds number Re~2.5 10⁴. The flat plate model, at zero angle of attack, was flexibly mounted in heave and pitch in an experimental setup that allow high amplitude oscillations. At the critical velocity associated to the merging of frequencies, the system undergoes an unstable transient behavior before reaching a low amplitude limit-cycle oscillation regime with slow time varying amplitude. For higher velocity the system branches-off to a higher and more stable limit cycle oscillation regime. At that context a by-pass transition to high limit cycle oscillations can be observed for hard perturbation below the critical velocity.

NOMENCLATURE
LCO Limit Cycle Oscillations
O Center of rotation
α(t) Pitch angle of attack about O (rad)
z(t) Position in heave (m)
M_o Aerodynamic momentum about O (N.m)
F_z Lift force (N)
L, c, t Span, chord and thickness of the plate model (m)
J_o Inertia of the system about O (kg.m²)
m Mass of the system (kg)
d Distance between the center of gravity and the center of rotation
k_o Stiffness in rotation (N.m/rad)
k_z Stiffness in bending (N/m)
f_o Natural frequency in rotation (Hz)
f_z Natural frequency in bending (Hz)
η_o Reduced structural damping in rotation (%)
η_z Reduced structural damping in bending (%)
U Mean wind velocity (m/s)

Re Reynolds number: Re = U × c/ν
ν Kinematic viscosity (m²/s)
U_c Critical velocity (m/s)
U_r Relative velocity: U_r = U/ν_c
U_{c*} Reduced critical velocity : U_{c*} = U_c/ f_c × c
f_c Merging frequency value at U=U_c
α_{LCO} Amplitude in pitch at LCO (deg)
z_{LCO} Amplitude in heave at LCO (m)
ϕ_{LCO} Phase lag between the pitch and the heave at LCO (deg)

INTRODUCTION
Among the fluid-structure instabilities that can be experienced by a slender body in cross flow, classical flutter and stall flutter are probably the most thoroughly investigated. Classical flutter is a dynamic instability for which the energy transfer from the flow to the body relies on elastic and/or aerodynamic coupling between two structural modes [1]. Often referred to coupled-mode flutter, this instability can lead to high oscillations and then structural failures of slender structures such as wings or bridge decks if not properly designed. Fortunately, the critical parameters for the onset of classical flutter can be easily predicted using linear stability analysis [2-3].

On the other hand, stall flutter is known be an instability not dependant on coupling for which a single mode body motion suffices to induce an aerodynamic loading in a way that energy is transferred from the flow to the body [4]. For slender structures such as wings or blades in stall flutter the most frequent mode of vibration involved is in torsion. The mechanism for energy transfer then relies on a dynamic stall process for which partial or complete detachment of the flow from the body occurs during each cycle of oscillation [5]. In that context, the essential feature of stall flutter is the non linear aerodynamic reaction of the body to the motion. This phenomenon is of particular importance for wing
operating at high angle of attack [6-7], helicopter rotor blades [8] and more recently for wind turbine blades [9].

Due the non linear behavior of the aerodynamic load, stall flutter is also known to be limited in amplitude [10]. But limit cycle oscillations can also occur for aeroelastic systems in post critical flutter conditions due to structural nonlinearities [11]. In a recent work a link between classical flutter and stall flutter has also been pointed out by Razak et al [12] during tests performed on an aeroelastic wing model in wind tunnel, highlighting limit cycle oscillations around a mean angle of attack close to the static stall angle.

In that context, if the prediction for the onset of classical flutter is well understood the investigation of the non-linear flutter dynamics suffers from a lack of study. The aim of this paper is then to provide experimental evidences of the LCO for a simple aeroelastic flat-plate model undergoing classical coupled mode flutter. The present paper is then organized as follows: the experimental setup is first presented. Dynamical response of the system is then characterized below, at, and beyond the critical velocity associated to the merging of frequencies. The LCO amplitude evolution with the flow velocity is then highlighted along with the influence of the initial perturbation.

EXPERIMENTAL SETUP

The experiments were performed on a flat rectangular steel plate of span \( L = 0.225 \text{m} \), chord length \( c = 0.035 \text{m} \) and thickness \( \delta = 0.015 \text{m} \), which give a thickness-to-chord ratio of 4.3%. Dimensions of the model are shown in Fig. 1. This rectangular configuration has been chosen in order to limit the effect of the Reynolds number.

The flat plate model is flexibly mounted in heave and pitch in a in a small Eiffel wind tunnel with a closed rectangular test-section of 0.26 m width and 0.24 m height. The chord dimension of the model is then less than 15% of the height of the wind tunnel cross section in order to avoid blockage effects for high amplitude oscillations. Two end plates are also mounted at both the extremities of the flat-plate model in order limit end effects. The setup is shown in Fig. 1. The vertical stiffness of the system is set by two long steel laminated springs supporting the axis of rotation of the model. The rotational stiffness is set by two series of linear springs.

A particular attention has been paid to the design of a setup that can allow high amplitude linear response in pitch and heave. Moreover the rotation centre of the model \( O \) has been chosen between aerodynamic centre (i.e. the first quarter chord at zero angle of attack), and the centre of gravity \( G \) (i.e. the mid-chord) so the flat plate section can be subject to classical flutter in the velocity range of the wind tunnel.

Tests were performed for a mean velocity in the test-section varying from 5 to 13 m/s, with a turbulence level less than 0.4% over this velocity range. In the present study the mean angle of attack of the model is set to zero.

The two degrees of freedom \( z(t) \) and \( \alpha(t) \) are measured using laser displacement sensors connected to an acquisition system.

![FIGURE 1: EXPERIMENTAL SETUP AND FLAT PLATE MODEL](image)

Structural parameters

The linearized equations of motion for this structurally coupled two degrees of freedom system can be expressed as following [13]:

\[
\begin{align*}
    m \ddot{z} + 2m \eta_z \omega_z \dot{z} + k_z z + md \ddot{\alpha} &= F_z \\
    J_\alpha \ddot{\alpha} + 2J_\alpha \eta_\alpha \omega_\alpha \dot{\alpha} + k_\alpha \alpha + md \ddot{z} &= M_\alpha
\end{align*}
\]

With \( d = GO \) the distance between the centre of gravity and the center of rotation \( d < 0 \).

Structural parameters of the system are identified under zero-wind velocity. A static weight calibration technique is used to assess the stiffness \( k_z \) and \( k_\alpha \). Results reported in Fig. 2 show that the bending stiffness of the system behaves linearly in the range of \(-0.3 \leq z/c \leq 0.3\).

![FIGURE 2: STIFFNESS STATIC WEIGHT CALIBRATION IN HEAVE AND PITCH](image)
On the other hand the linearity of the stiffness in rotation is only guaranteed in the range of 30 deg ≤ α ≤ 30 deg. For higher angles of rotation the rotational stiffness smoothly reduces until the critical limit angle of ±50 degrees. Above this critical angle the system is unable to properly restore a moment.

Free decay tests under zero wind conditions have been performed for each degree of freedom taken independently (the other one being locked). Natural frequencies \( f_n \) and \( f_s \) are then obtained by spectral analysis. Pure structural damping values \( \eta_s \) and \( \eta_n \) being determined using a standard decrement technique.

Assuming that the structural damping is small, the inertia \( J_0 \) and mass \( m \) are then deduced, using

\[
J_0 = k_s / (2\pi f_s)^2
\]

\[
m = k_z / (2\pi f_z)^2
\]

Free decay tests have also been performed for the two-degrees of freedom system under zero wind conditions. This procedure allows the identification of the distance \( d \) between the centre of gravity and the center of rotation using the following expression (solution of the eigenvalue problem for the coupled system) [14]:

\[
\frac{f_1^2 + f_2^2}{f_s^2 + f_2^2} = \frac{1}{1 - md^2 / J_0}
\]

Structural parameters of the system are summarized in Table 1.

**TABLE 1: STRUCTURAL PARAMETERS OF THE SYSTEM**

<table>
<thead>
<tr>
<th>( f_n )</th>
<th>( f_s )</th>
<th>( k_n )</th>
<th>( k_z )</th>
<th>( J_0 )</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.0</td>
<td>7.05</td>
<td>0.149</td>
<td>595.6</td>
<td>4.66 ( 10^5 )</td>
<td>0.304</td>
</tr>
</tbody>
</table>

### Aerodynamics of flat-plate

In situ measurements of the lift and moment coefficient has been performed using a static weight calibration technique under a wind velocity \( U = 10 m/s \) (i.e. a Reynolds number close to 2.3 \( 10^6 \)) at various angles of attack. Results are compared in Fig. 3 with the experiments of Fage and Johansen [15] (sharp-edged flat-plate model of thickness-to-chord ratio of 3% at Reynolds number \( \text{Re}=10^5 \)), along with those of Pelletier and Mueller [16] (flat-plate model of thickness-to-chord ratio of 1.93% at Reynolds number \( \text{Re}=8 \times 10^5 \)).

Those results show that the flat-plate model used in the present study follows the same trend with a lift-curve slope and moment-curve slope in the low-angle linear region respectively close to 6 and 1.5. The critical angle of attack at which the plate stalls can also be estimated between 7-8 degrees. As reported by Fage and Johansen [15] beyond this stall angle of attack the flow is completely separated from the upper surface.

### LOW SPEED FLUTTER RESULTS

Experiments were performed with the flat plate model at zero mean angle of attack for wind tunnel velocity ranging from 5 up to 13m/s. Increasing the velocity the system remains stable until a critical velocity \( U_c = 10.5 m/s \) is reached at which the merging of frequencies occurs. Beyond this critical velocity the system undergoes a flutter instability characterized by limit cycle oscillations that were studied up to \( U_c / U_r = 1.2 \). For higher velocities the dynamics of the system is corrupted by a static divergence in the pitching degree of freedom due to the structural limitation of the experimental setup.

### Frequencies evolution with the flow-velocity

Free decay tests have been performed for various velocities in stable and post-stable condition. Spectral analysis of the dynamical responses then allows to identify the heaving and pitching frequencies evolutions versus the wind velocity. Results are reported in Fig. 4. For \( U/U_r > 0.4 \) both frequencies smoothly approach each other (the heaving frequency increasing while the pitching one decreases), until the critical condition \( U/U_r = 1 \) where the pitching frequency sharply reduces and merges the heaving frequency. Beyond that critical condition, the system dynamics adopts one dominant frequency close to \( f_s = 8 Hz \) which slightly reduces with the velocity.

For this flat-plate aeroelastic system characterized by a natural frequencies ratio \( f_n / f_s = 1.28 \) the critical reduced velocity associated to flutter can then be approximated by \( U^* = U_c / (f_s \times c) \approx 37.5 \).
Analysis of the dynamical response

Below the critical velocity, i.e. for \( U/U_c < 1 \), heaving and pitching responses to any small initial perturbations are both damped to reach a small turbulence-induced vibration regime \( (z/c < 0.0025 \& \alpha < 0.1 \text{ deg.}) \).

At the critical velocity for which the frequency merging occurs, the system is unstable and any small initial perturbation is amplified. Heaving and pitching responses to a small initial deflection \( (z_0/c < 0.07) \) are reported on Fig. 5. It clearly shows a transient growth in pitch and heave that is followed by a small decay and another growth. A low amplitude limit-cycle oscillations regime with slow time varying amplitude then occurs beyond reduced time \( tU/c > 2 \). Mean amplitudes in heave and pitch are then close to \( z_{LCO}/c \approx 0.11 \) and \( \alpha_{LCO} \approx 6.5^\circ \).

In the LCO regime the mean phase angle \( \varphi_{LCO} \) by which the quasi-harmonic response in pitch lags the response in heave has been measured close to \( \varphi_{LCO} \approx -160^\circ \).

For higher velocity the dynamics of the response in heave and pitch change strongly. This can be seen in Fig 6. For \( U/U_c \approx 1.08 \) the initial transient growth of mechanical energy leads to a first regime of low amplitude oscillations in pitch \( (\alpha < 11^\circ) \) in the time domain \( 1.1 < tU/c < 1.4 \). In the same time the heaving response strongly decreases before growing again along with the pitching amplitude. The system then branches-off to a higher and more stable limit cycle oscillations regime characterized by high harmonic oscillations: \( z_{LCO}/c \approx 0.2 \) and \( \alpha_{LCO} \approx 34^\circ \).

A phase diagram associated to each of the dynamical responses reported in Figs. 5 and 6 has been plotted in Fig. 7. For \( U/U_c \approx 1 \) the phase diagram shows the alternative occurrence of a phase angle regime of \( \varphi \approx -160^\circ \) which can be associated to the growth of mechanical energy, and an out of phase regime \( \varphi \approx -180^\circ \) where the oscillations decrease. For \( U/U_c \approx 1.08 \) the phase diagram shows that the first initial transient growth (ITG) regime is characterized by a mean phase lag between the pitch and the heave close to \( \varphi_{LCO} \approx -160^\circ \). Then the system branches-off to a more stable limit cycle oscillations regime characterized a mean phase angle \( \varphi_{LCO} \approx -36^\circ \).
velocity” results clearly show a hysteretic behavior of the system which stay in a high amplitude LCO regime down to a relative velocity $U/U_c \approx 0.85$. For lower velocity the system is then damped. From $U/U_c \approx 0.85$ the heaving and pitching amplitudes of oscillations in the LCO branch also linearly increase with the relative velocity, reaching $z_{LCO}/c \approx 0.25$ and $\phi_{LCO} \approx 44^\circ$ at $U/U_c \approx 1.2$.

As shown in Fig. 10 the negative phase lag between the pitch and the heave also changes with the velocity ratio. Following the “decreasing velocity” results one can see that $\phi_{LCO} \approx -90^\circ$ for $U/U_c \approx 1.15$ and gradually increase to $\phi_{LCO} \approx -11^\circ$ for the lower relative velocity $U/U_c \approx 0.85$.

**Effect of the initial conditions**

It is known that initial perturbations can significantly affect the dynamic response of the system governed by nonlinear behavior. Tests have then been performed below and beyond the critical velocity with different sets of initial conditions.

Below the critical velocity, the system remains stable (i.e. its dynamical response is damped) for different sets of low or moderate initial perturbations. On the other hand, for relative velocities $0.85 < U/U_c < 1$, a subcritical transition has been observed. Indeed, the onset of strong vibrations leading to the high limit cycle oscillations values reported on Figures 8 and 9 can be triggered by initial pitch angle and/or heave deflection such as: $\alpha_0 > \alpha_{LCO}$ and/or $z_0 > z_{LCO}$.

Above the critical velocity (i.e. for $U/U_c > 1$) different sets of low, moderate or strong initial conditions have been tested. They showed that even though the initial perturbations can significantly affect the transient regime, the same stable limit cycle oscillations regime is reached, with amplitudes and phase angle values in accordance with those reported in Figs 8-10.

**CONCLUSIONS**

Dynamical response of a two degrees of freedom flat plate undergoing classical coupled-mode flutter in a wind tunnel has been studied. Tests have been performed at low Reynolds number $Re \sim 2.5 \times 10^6$ using an experimental setup that allow high amplitude linear response in pitch and heave for relative velocity up to $U/U_c \approx 1.2$. In this study the frequency merging critical velocity (i.e. the onset of flutter), is such as the associated reduced velocity is close to $U'_c = U_c / f_c \times c \approx 37.5$.

The results showed that beyond the critical velocity the system dynamical response is limited in amplitude. Indeed after a transient regime the system systematically branches off to a stable LCO regime characterized by high amplitudes in heave and pitch increasing linearly with the relative velocity, up to $z_{LCO}/c \approx 0.25$ and $\phi_{LCO} \approx 44^\circ$ for $U/U_c \approx 1.2$. A hysteretic behavior has also been pointed out decreasing the velocity from a stable post-critical LCO regime. Indeed, the system dynamics remains in a high amplitude LCO regime down to a relative velocity $U/U_c \approx 0.85$, for which $z_{LCO}/c \approx 0.12$ and $\phi_{LCO} \approx 18^\circ$.

In that context, a subcritical transition to high limit cycle oscillations can also be observed for hard perturbation of the system, starting from rest, for relative velocity $0.85 < U/U_c < 1$.

**REFERENCES**


