OPTIMAL ENERGY HARVESTING BY VORTEX-INDUCED VIBRATIONS IN CABLES

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ABSTRACT
The issue of global climate change and the growing energy demand induce a need for innovative energy harvesting devices. The possibility to harvest energy using VIV of a long tensioned cable or of an elastically-mounted rigid cylinder is investigated throughout this paper. A simple wake-oscillator model is used to represent the major characteristics of the complex dynamics of such structures. The optimal efficiency of the two devices are similar and are reached when the solid and its fluctuating wake are in lock-in condition. The sensitivity of the optimal of such energy harvesters with flow velocity is also discussed.

INTRODUCTION
Geophysical flows represent a widely available source of clean energy, useful to tackle the global energy demand using for example wind turbines, marine turbines or wave energy converters.

An original way to extract energy from these flows is to take advantage of flow-induced vibrations, reviewed for example in Ref. [1]. For instance, several devices based on fluid-elastic instabilities like transverse galloping or flutter have already been introduced in Refs. [2–5]. Another kind of flow-induced oscillations that can be useful to harvest energy from a flow is the vortex-induced vibrations (VIV) of a bluff body [6]. The strong coupling between a solid and its fluctuating wake may lead to a lock-in phenomenon between the solid dynamics and the vortex shedding, resulting in high amplitude oscillations, that can be used for energy harvesting [7,8].

Yet, the energy density in geophysical flows is small, and large systems are required in order to harvest significant amount of energy. Besides, VIV of long cables like oil rigs anchors or risers have been extensively investigated since they are of capital importance for offshore industry [9]. The possibility to harvest energy from a flow using VIV of long tensioned cables is consequently studied in this paper [10].

Extensive experimental and numerical analysis have shown that the dynamics of slender structures in VIV are very rich and complex [11–13]. The most important features of these dynamics (frequencies, wavenumbers) are however well predicted by a simple wake oscillator approach [14–16]. This approach is used in the present paper to investigate the energy harvesting using VIV of long tensioned cables.

In a first section, the model is presented and the generic case of energy extraction using VIV of an elastically-mounted short rigid cylinder is analyzed. Energy harvesting using VIV of an infinite tensioned cable is investigated in a second section.

THE ELASTICALLY-MOUNTED RIGID CYLINDER
The energy harvesting from an elastically-mounted rigid cylinder VIV is first investigated, see Fig. 1. The fluid density and velocity are respectively noted \( \rho \) and \( U \), while \( D \) and \( m_s \) stand for the rigid cylinder diameter and mass per unit length. Let also \( r \) and \( h \) be the damping and stiffness coefficients of the elastic support.

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The fluid-solid model for energy harvesting

The cross-flow displacement $Y$ of the cylinder is described by

$$m_t \frac{\partial^2 Y}{\partial T^2} + (r + r_a) \frac{\partial Y}{\partial T} + hY = F_{wake},$$

where $m_t = m_s + m_u, m_s = \pi \rho D^2 C_{Mo}/4$ being the added mass per unit length and $C_{Mo}$ the added mass coefficient. The fluid added damping is defined by $r_a = \rho DU C_D/2$, where $C_D$ is the cylinder drag coefficient and $F_{wake}$ denotes the wake forcing on the body [14]. Two frequencies appear: (i) the natural frequency of the solid in still fluid $\omega_0 = \sqrt{h/m_t}$ and (ii) the vortex shedding frequency behind a fixed cylinder, $\omega_f = 2\pi St U/D, St$ being the Strouhal number [1]. Using $y = Y/D$ and $t = \omega_f T$, the dimensionless equation for the solid motion reads

$$\ddot{y} + \left(\xi + \frac{\gamma}{\mu}\right) \dot{y} + \delta^2 y = f_{wake},$$

where $\delta = \omega_0/\omega_f$ is the frequency ratio and $\xi = r/m_t \omega_f$ the damping coefficient. The dimensionless fluid added damping coefficient $\gamma/\mu$ is defined by the stall parameter $\gamma = C_D/4\pi St$ [17] and the mass ratio $\mu = m_t/\rho D^2$ [14]. As far as the cylinder is concerned, harvesting energy from its motion comes to a loss of energy, which is modelled by the damping term $r \partial Y/\partial T$, or $\frac{\gamma}{\mu}$ in dimensionless form. The efficiency of such an harvesting device is defined as the ratio between the time-averaged extracted power and the energy flux through the section of the cylinder $\rho DU^2/2 [3, 8, 10, 18]$. In dimensionless form, it reduces to

$$\eta = 16 \mu \pi^3 St^3 \langle \xi y^2 \rangle.$$  

Following [14], a wake-oscillator approach is used to model $f_{wake}$. The fluctuating load due to the wake is assumed to be proportional to a fluctuating lift coefficient $q = 2CL/C_{LD}$, which satisfies a Van der Pol oscillator equation, $C_{LD}$ being a fluctuating lift coefficient. This second equation is then coupled with Eqn. (2) by an inertial forcing to define the fluid-solid model

$$\ddot{y} + \left(\xi + \frac{\gamma}{\mu}\right) \dot{y} + \delta^2 y = Mq, \quad (4a)$$

$$\dot{q} + \epsilon (q^2 - 1) \dot{q} + q = A\dot{y}, \quad (4b)$$

where $\langle \rangle$ denotes derivation with respect to dimensionless time $t$, coefficient $M$ being defined as $M = C_{LD}/16\mu \pi^2 St^2$ and $A$ and $\epsilon$ are parameters based on experiments. In all the paper, the values $A = 12, \epsilon = 0.3, C_D = 2, C_{LD} = 0.8, St = 0.17, \mu = 2.79$ and $C_{Mo} = 1$ are fixed as in [10, 16], so that $M = 0.06$ and $\gamma/\mu = 0.34$. Equations (4) are integrated using finite differences and the limit cycle is analyzed in terms of efficiency.

Optimal energy harvesting and lock-in

The map of the efficiency as a function of the frequency ratio $\delta$ and the damping coefficient $\xi$ is displayed on Fig. 2.

There is an optimal harvesting configuration leading to maximum efficiency, corresponding to a frequency ratio of $\delta = 0.89$ and a damping coefficient of $\xi = 0.20$. The corresponding optimal efficiency $\eta_{opt}^2 = 0.23$ is close to the value found in [7] for a similar system.

The efficiency vanishes for both small and large damping, as expected. For small damping, the amplitude saturates according to the Skop-Griffin diagram [6] and $\eta$ thus varies linearly with $\xi$. On the other hand, large damping inhibits the solid motion leading to an amplitude varying as $1/\xi$: the efficiency $\eta \propto \xi y^2$ consequently also vanishes as $1/\xi$ for large values of $\xi$.

The optimal frequency ratio $\delta = \omega_0/\omega_f = 0.89$ is close to 1, which corresponds to the synchronization of the wake and solid natural frequencies or lock-in [6],...
known to lead to high amplitude oscillations. The optimal configuration for energy harvesting from VIV of an elastically-mounted short rigid cylinder corresponds to a lock-in condition for the frequency ratio \( \delta \) and a well-balanced value of the damping coefficient, or harvesting intensity, \( \xi \).

THE INFINITE TENSIONED CABLE

The possibility to harvest energy from a flow using VIV of an infinite tensioned cable is now investigated [10]. The same approach as in the previous section is followed, the vortex-induced vibrations of the structure are modelled by a wake-oscillator. This model is identical to the one used for the rigid cylinder case, except that the stiffness force \( bhY \) is now replaced by the stiffness induced by the uniform tension \( \Theta \) of the cable: \(-\Theta \partial^2 Y / \partial Z^2 \). The spanwise coordinate \( Z \) is scaled using a characteristic length based on the waves phase velocity \( c = Z \omega f \sqrt{m_f / \Theta} \) and the dimensionless form of the model reads

\[
\ddot{y} + \frac{2}{\mu} \dot{y} - \dddot{y} = Mq, \\
\dot{q} + \varepsilon (q^2 - 1) \dot{q} + q = A\ddot{y},
\]

where \( (\cdot)' \) denotes derivation with respect to dimensionless spanwise coordinate \( z \). The extraction of energy from a flow is still modelled by damping. In this configuration, discrete harvesters/dashpots are periodically distributed over the span of the cable, see Fig. 1, with a distance \( L \) between two harvesters of damping coefficient \( R \).

\[
\dot{y}'(0,t) - \dot{y}'(l,t) = l \xi \dddot{y}(l,t),
\]

where \( \xi = R/Lm_f \omega_f \) is the dimensionless damping per unit length and \( l = Lo_f \sqrt{m_f / \Theta} \) the dimensionless distance between two harvesters.

The efficiency is again defined as the ratio between the time-averaged power extracted by a dashpot and the energy flux through a length \( L \) of the cable, which is still written using the dimensionless variables as Eqn. (3). The system of Eqns. (5) and (6) is integrated in space and time using finite differences on a spatially periodic domain, with the cable initially at rest \( y = 0 \) and a small random perturbation of the fluctuating lift \( q \). The harvesting is studied as a function of the length between two dampers \( l \) and the damping per unit length \( \xi \).

Optimal efficiency

In order to compare the energy harvesting using either an elastically-mounted rigid cylinder or an infinite tensioned cable, two optimal efficiency curves are derived, giving the maximum achievable efficiency for any value of the damping density \( \xi \). For the two-dimensional case of the rigid cylinder, \( \eta_{2D}(\xi) \) corresponds to the efficiency obtained for an optimal value of the frequency ratio \( \delta \) at a given damping density \( \xi \). For the infinite tensioned cable case, \( \eta_{3D}(\xi) \) is the optimal efficiency given by the choice of an optimal value of \( l \) at a given \( \xi \). The two curves are shown on Fig. 3.
The optimal performances of these two systems are similar, $\eta_{2D}^{opt} = 0.23$ while $\eta_{3D}^{opt} = 0.19$, justifying a deeper investigation of the new concept of energy harvesting using a tensioned cable.

The differences in the shape of the two curves $\eta_{2D}$ and $\eta_{3D}$ are yet striking. Contrary to the case of the rigid cylinder, the efficiency of energy harvesting via an infinite tensioned cable in VIV is far from a classical bell-shape curve. The optimal damping per unit length is also very different for the two configurations as, for the elastically-mounted rigid cylinder $z_{2D}^{opt} = 0.20$, whereas in the case of an infinite tensioned cable, $z_{3D}^{opt} = 3.65$. A deeper analysis of the efficiency dependence of the latter configuration with the two harvesting parameters $l$ and $\xi$ is thus necessary. Figure 4 displays the map of the efficiency as a function of these two parameters.

This map is actually much more complex than that of Fig. 2. Three zones may be defined to explain its structure.

The first one, zone $A$, corresponds to small distances between two dashpots for any damping. Within this zone, the classical influence of damping on the efficiency is retrieved. Actually, $\eta$ vanishes for small and large damping, even if the peak is not at the same location for every values of $l$. Moreover, the overall optimal harvesting configuration lies in this zone: $\xi = 3.65$ and $l = 1.09\pi$ lead to $\eta = 0.19$. As shown on Fig. 5, this zone explains most of the efficiency curve $\eta_{3D}(\xi)$.

The second zone, zone $B$, corresponds to the left part of the map, typically $\xi < 0.4$, and larger values of $l$. The efficiency within this zone is rather small, it corresponds to the left low peak of the efficiency curve, Fig. 5. Within this zone, the motion of the cable is close to travelling waves of wavelength $\lambda = 2\pi, 4\pi, 6\pi$, etc... Yet, the efficiency is low in zone $B$ so it shall not be discussed any further.

Within zone $C$, the efficiency depends very strongly on the two parameters, especially on $l$. High efficiency tongues are surrounded with inefficient harvesting configurations, resulting in the discontinuities that can be seen on the efficiency curve, Fig. 5.

Mode shapes and efficiency

To gain some understanding of the efficient part of this complex energy map (zone $A$ and $C$), the dynamics of the cable in these two zones are compared and shown on Fig. 6. Contrary to zone $B$, the motion of the cable is close to stationary waves.

The optimal cable displacement for energy harvesting is shown on Fig. 6(a), and the corresponding values of the parameters are reported in Tab. 1. The motion resembles a mode 1 vibration of a tensioned cable, with slight displacement of the dashpots.

If a fixed value of the damping $\xi$ is considered, the cable motion evolves with an increasing length $l$, but it is always close to one of the classical harmonics of a tensioned cable. The cable dynamics actually jumps from even modes in zones where the harvesting efficiency is very low, Fig. 6(b), to odd modes in the high efficiency regions of zone $C$, Fig. 6(c).

The motion of the dashpot is forced by the jump in the cable slope between the two sides of the harvester, Eqn. (6). Odd modes are thus more likely than even ones to lead to high jumps in the cable slope, then to high efficiency, Fig. 7.

The mode number is defined as $n = 2l/\lambda$, where $\lambda$ denotes the wavelength of the cable motion. This mode
number may not be an integer, in that case, the closest integer is used. Following Ref. [16], the linearized version of the model is used to derive the characteristics of the most unstable linear mode which will dominate the non-linear response of the model. The linearized version of the model reads

\[
\ddot{y} + \frac{\gamma}{\mu} \dot{y} - y'' = Mq, \quad (7a)
\]

\[
\ddot{q} - \varepsilon \dot{q} + q = A\ddot{y}. \quad (7b)
\]

The dominance zones of every mode number \( n \) are reported on the efficiency map of the infinite tensioned cable, see Fig. 8. There is a very good agreement between the discontinuities of the efficiency map and the frontiers between the dominance zones of two different mode numbers. In particular, the high efficiency tongues indeed correspond with regions where the dominant mode number is odd. In zone B, only even modes exist, which are expected to be less efficient.

Finally, the damping coefficient plays a double role in energy harvesting from VIV of an infinite tensioned cable as it controls both the local dynamics of the harvesters and the global mode shape of the solid displacement.

**DISCUSSION**

Energy harvesting using VIV of (i) an elastically-mounted rigid cylinder and (ii) an infinite tensioned cable was investigated in this paper and exhibit similar optimal efficiencies, of the order of 0.2.

**Optimal harvesting and lock-in**

For the two devices, the optimal harvesting configuration corresponds to a lock-in condition. For an elastically-mounted rigid cylinder, this lock-in condition takes the classical form of synchronization between the vortex shedding frequency and the solid natural frequency, \( \delta = \omega_s / \omega_f = 1 \).

In the case of an infinite tensioned cable, Refs. [15, 16] showed that lock-in corresponds to the highest growth rates of the fluid/solid coupled mode instability. The modes dominance zones shown on Fig. 8 can then also be regarded as the lock-in regions of each mode. The dispersion relation between frequency \( \omega \) and wavenumber \( k \) derived from Eqn. 7 consequently results in a lock-in condition for an infinite cable, \( k = 1 \). This condi-
tion is yet modulated by the periodic boundary condition, Eqn. (6), leading to a lock-in condition for each mode. The solid adapts its own dynamics via its wavenumber in order to be always at lock-in, under the restrictions of the periodic harvesting boundary conditions. Considering an ideal lock-in condition \( k = 1 \) and the definition of the wavenumber, one may nevertheless derive optimal lengths corresponding with the lock-in condition for every mode. For the odd modes, which were shown to be the efficient ones, this results in the lock-in conditions \( l = (2n + 1)\pi \), which agree well on the location of the several peaks on Fig.5. For each mode number \( 2n + 1 \), it also defines a frequency ratio \( \delta_{2n+1} = (2n + 1)\pi / l \), playing a similar role as the frequency ratio \( \delta = \omega_s / \omega_f \) of the rigid cylinder case. Reminding that the exact optimal configurations differ slightly from the exact lock-in conditions, see Tab. 1, the optimal frequency ratio indeed reads for the elastically-mounted rigid cylinder

\[
\delta = \left( \frac{D}{2\pi S U} \right) \omega_s = 0.89, \tag{8}
\]

and for the infinite tensioned cable

\[
\delta_1 = \frac{\pi}{l} = \left( \frac{D}{2\pi S U} \right) \frac{\pi}{L} \sqrt{\frac{\Theta}{m_t}} = 0.92. \tag{9}
\]

These two conditions are of the same form and actually correspond to lock-in between the vortex shedding frequency and the natural frequency of the solid motion, the natural frequency of the rigid cylinder \( \omega_s \) being replaced by the natural frequency of the cable vibrations

\[
\omega_c = \frac{\pi}{L} \sqrt{\frac{\Theta}{m_t}}. \tag{10}
\]

The major difference between the two optimal configurations lies in the values of the optimal damping intensity \( \xi \), which differ by more than an order of magnitude between the two analyzed devices. This is due to the fact that for the tensioned cable with periodically distributed harvesting devices, \( \xi \) does not only drive the local dynamics of the dashpots, as it is the case for the short rigid cylinder, but it also controls the choice of the overall dynamics of the cable. This double role of the harvesters is of capital importance since the harvesting efficiency depends a lot on the cable overall motion, especially on the selected mode number.

### TABLE 1. OPTIMAL CONFIGURATIONS FOR THE TWO CONSIDERED ENERGY HARVESTING DEVICES USING VIV.

<table>
<thead>
<tr>
<th></th>
<th>Rigid cylinder</th>
<th>Tensioned cable</th>
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</thead>
<tbody>
<tr>
<td>( \eta )</td>
<td>0.23</td>
<td>0.19</td>
</tr>
<tr>
<td>( \xi )</td>
<td>0.20</td>
<td>3.65</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.89</td>
<td>0.92</td>
</tr>
</tbody>
</table>

**Optimal design of a real energy harvester**

The present study may then be used to design an energy harvester based on VIV, using Tab. 1. The definition of the efficiency \( \eta \) provides an estimation of the maximum power extracted by a cable of length \( L_{tot} = 100 \) m and diameter \( D = 4 \) cm in a flow of mean velocity \( U = 1.5 \) m.s\(^{-1} \), namely \( P = 1282.5 \) W. It is interesting to note that this is far from being negligible and that it can be achieved for many different triplets \( (\Theta, L, R) \).

For an elastically-mounted rigid cylinder of diameter \( D = 4 \) cm and mass ratio \( \mu = 2.79 \) placed in a flow of mean velocity \( U = 1.5 \) m.s\(^{-1} \), the dimensional optimal damping and stiffness are \( r = 35.8 \) kg.s\(^{-1}\).m\(^{-1} \) and \( h = 5673.2 \) kg.s\(^{-2}\).m\(^{-1} \). For the tensioned cable mentioned above \( (L_{tot} = 100 \) m), under a tension \( \Theta = 10 \) kN, the damping coefficient of each dashpot is \( R = 2645 \) kg.s\(^{-1} \) and they are separated by a distance \( L = 4.05 \) m.

**Influence of the velocity variations on the efficiency in real operating conditions**

The dimensional parameters are fixed once and for all, but the operating conditions may drift from the optimal ones because of a varying flow velocity \( U \). The influence of a varying flow speed on the performances of a harvesting device is now discussed. Three cases are considered : (i) an elastically-mounted rigid cylinder, (ii) an infinite cable with constant tension and (iii) an infinite cable with drag-induced tension. The values of the harvesting parameters are derived so that the optimal is reached for the mean flow speed \( U \). The evolution of the efficiency with the flow speed for each case is then plotted on Fig.9. In order to quantify the influence of flow speed fluctuations on the actual efficiency, the peak width \( w \) is defined as the ratio between the length of the velocity interval for which the efficiency is above 75% of the peak efficiency \( \Delta U \) and the mean velocity \( U \), see Tab. 2.

For the rigid cylinder, it comes from the dimensionless parameters definitions that \( \delta \propto 1/U \) and \( \xi \propto 1/U \).
FIGURE 9. EVOLUTION OF THE EFFICIENCY WITH FLOW VELOCITY. (RED - DASHED) RIGID CYLINDER, (BLUE - SOLID) CABLE WITH DRAG-INDUCED TENSION AND (BLACK - DASH-DOT) CABLE WITH FIXED TENSION.

The curve of the evolution of $\eta$ with $U$ hence corresponds to the values of $\eta$ along a curve $\delta \propto \xi$ passing through the optimal configuration. Even if the best efficiency is achieved as expected by this configuration, the width of the peak is small, see Tab. 2. The efficiency dramatically drops down as the current speed drifts away from its mean value.

For the long tensioned cable, if the tension is constant (induced for instance by a buoy on top of a cable anchored at its bottom), $\xi \propto 1/U$ and $l$ varies linearly with $U$. The efficiency depends a lot on the flow speed, Fig.9. The peak width is even smaller than for the rigid cylinder, $w = 0.54$, and $\eta$ even falls down to zero for some velocities. This solution should be avoided because of this high sensitivity to current velocity.

In the last case considered, the tension is due to a drag force acting on a well-chosen area $A$, which may differ from the cable area $DL_{tot}$. The area $A$ has here been chosen so that the optimal harvesting parameters are identical to the constant tension case in order to compare this configuration with previous ones ($\Theta = 10$ kN for $U = \bar{U}$). In that case, $\Theta \propto U^2$, $\xi \propto 1/U$ and $l$ is constant ($l = 3.43$), which sounds a valuable option as the efficiency depends a lot more on $l$ than on $\xi$, Fig. 4. It actually appears as the best solution since the efficiency stays high for a wide range of velocity, Fig. 9. The peak value is lower than for the rigid cylinder, but the peak width is much larger, $w = 2.47$, see Tab. 2. The efficiency decrease as $U$ deviates from $\bar{U}$ is slow, it even overtakes the rigid cylinder efficiency for $U \leq 1.1$ m.s$^{-1}$ and $U \geq 2.2$ m.s$^{-1}$.

TABLE 2. WIDTH OF THE PEAK AROUND THE OPTIMAL FLOW VELOCITY $w = \Delta U / \bar{U}$.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Peak width $w$</th>
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<tbody>
<tr>
<td>Rigid cylinder</td>
<td>0.73</td>
</tr>
<tr>
<td>Cable - constant tension</td>
<td>0.54</td>
</tr>
<tr>
<td>Cable - drag-induced tension</td>
<td>2.47</td>
</tr>
</tbody>
</table>

As a conclusion, the analysis performed throughout this paper has shown that energy harvesting using vortex-induced vibrations of a long tensioned cable seems, at least, as promising as using those of an elastically-mounted short rigid cylinder as in [7] or [8]. This new configuration may even have some advantages like the possibility to imagine very long cables able to harvest large amounts of energy, and to adapt their dynamics so that they are always near lock-in conditions, if their tension is induced by drag.

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