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Transient temporal response of a flexible bridge deck submitted to a single gust

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Abstract

Temporal simulations are increasingly performed for wind effects analysis of flexible structures. By comparison with classical techniques such as spectral methods, temporal simulations provide advantage of easily combining different kinds of load, can take nonlinearities into account and are also the only way to reproduce transient behaviours. In that context this study deals with the transient response of a two degrees-of-freedom streamlined bridge deck section submitted to a single gust. Experimental evidence of the potentially high level of transient energy amplification due to that kind of extraneously excitation have been recently demonstrated for an airfoil section [3, 4] and for a streamlined bridge deck section [5], below the critical flutter wind speed. The present study then focuses on the validation of a time-dependant model, based on a simple formulation of both the motion-dependant and buffeting forces, for catching that kind of transient behaviour. A parametric study is done in order to highlight the impact of the pitch-plunge frequency ratio on the energy amplification below the critical flutter wind speed.

1 Introduction

Temporal simulations are increasingly performed for wind effects analysis of flexible structures. By comparison with classical techniques such as spectral methods, temporal simulations provide advantage of easily combining different kinds of load, can take nonlinearities into account and are also the only way to reproduce transient behaviors. But in the case of wind-induced vibrations of flexible structures, such as bridge decks, the combination of wind turbulence excitation and aeroelastic effects can lead to new phenomena which are not always fully understood.

Especially for high turbulent wind the wind gusts act more as sudden transient excitations than as stationary excitation. In this context, a temporal simulation can be seen as a series of transient periods for which the response of the structure could be different than the response to statistically similar but stationary excitation. Temporal simulations have been studied for instance by Caracoglia & Jones [1] and recently by Costa *et al.* [2], but none of these approaches have considered the transient response of the structure. Therefore the careful study of transient phenomena is important. In that context the present study deals with the transient response of a two degrees-of-freedom bridge deck section submitted to a single gust, first in wind tunnel and then numerically.

A rigid bridge deck section is flexibly mounted in heave and pitch in a steady air flow. The velocity is maintained under the coupled mode flutter critical speed and a superimposed single gust produces an initial excitation of the deck. Similar experiments have already been conducted with a NACA airfoil [3, 4]. They have shown the existence of the mechanism named as transient growth of energy which was theoretically studied by Schmid & de Langre [5]. This mechanism can be described as an initial amplification of energy followed by a monotonic decay due to the asymptotic stability of the system. Schmid & Hennigson [6] showed that it is a consequence of non-orthogonal modes involved in the system. It is strongly dependent on the initial conditions. For an airfoil, transient growth of energy can lead to an amplification by a factor up to 10 of the initial energy and can even trigger the flutter instability in case of nonlinear structures [4]. However, the dynamics and the aeroelasticity of a bridge deck are quite different from those of an airfoil. Most bridge deck sections, except very streamlined, behave like bluff bodies. Despite theses differences, transient growth of energy has also been recently observed on a streamlined bridge deck section [7, 8].

The present study focuses on experimental results which are tentatively reproduced numerically using a time-dependant model. In a first step we recall the main points of the experimental study. Then the numerical model is described and the results are finally compared with those of the experiments.

2 Transient wind tunnel tests

2.1 Experimental setup and identification of parameters

Details concerning the experiments can be found in previous work [7, 8] and we recall here the main points. The bridge deck section is mounted in a closed wind tunnel with the setup shown in Figure 1. The deck can move in heave $z(t)$ and in pitch $\alpha(t)$. These two degrees of freedom are measured using laser displacement sensors connected to an acquisition system.

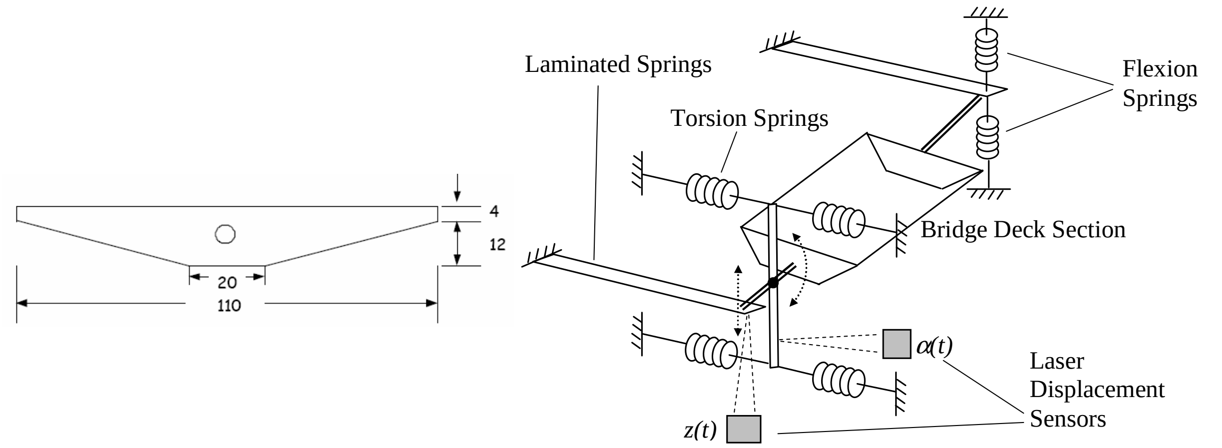

Figure 1: Bridge Deck Cross Section and Experimental Setup Schematics, dimensions in mm.

The equations of motion for the two degrees of freedom are provided for instance in [9]:

$$\begin{aligned} m\,\ddot{z} + 2m\eta_z\,\omega_z\,\dot{z} + k_z\,z &= F_z\,, \\ J_O\,\ddot{\alpha} + 2J_O\,\eta_\alpha\,\omega_\alpha\,\dot{\alpha} + k_\alpha\alpha &= M_O\,. \end{aligned} \tag{1}$$

Assuming that the structural damping is small, the eigenvalues can be written in the form

$$\lambda_\alpha = \omega_\alpha{}^2 = (2\pi\, f_\alpha)^2 = k_\alpha / J_O\,; \quad \lambda_z = \omega_z{}^2 = (2\pi\, f_z)^2 = k_z / m\,. \tag{2}$$

Structural parameters are identified for each degree-of-freedom taken independently under zero wind velocity. Both the natural frequencies f_z and f_α are obtained by spectral analysis. A static weight calibration technique is used to calculate the stiffness k_z and k_α. The inertia J_O and mass m are then deduced, using

$$m = k_z / \lambda_z\ ; \quad J_O = k_\alpha / \lambda_\alpha\ . \tag{3}$$

Pure structural damping values η_z and η_α are also determined using a standard decrement technique in free-decay tests. Two different cases are tested with different frequency ratio f_z / f_α between the heaving and pitching motions. Structural parameters are summarized in Table 1.

	f_z / f_α	f_α (Hz)	f_z (Hz)	k_α (Nm/rad)	k_z (N/m)	J_O (kg m^2)	m (kg)	η_α%	η_z%
Case A	*0.62*	*7.12*	*4.43*	*1.33*	*519.36*	*6.64 e^{-4}*	*0.66*	*0.3*	*0.08*
Case B	*0.44*	*8.00*	*3.56*	*1.67*	*309.16*	*6.61 e^{-4}*	*0.62*	*0.24*	*0.07*

Table 1: Structural parameters of the two different bridge deck sections studied.

The gust is produced by a flap mounted upstream the test section. It is pre-tensioned with a spring and suddenly released. A typical time history of the perturbation of the flow velocity is plotted Figure 2, where $\overline{U}$ is the mean velocity, $u(t)$ and $w(t)$ being the longitudinal and vertical perturbations respectively. After the perturbation, the velocity comes back to its mean value.

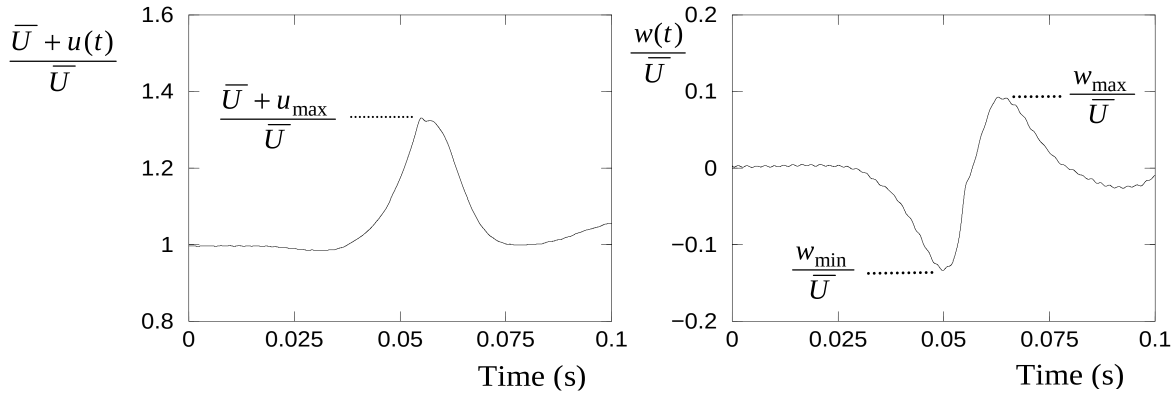

Figure 2: Sample of upstream velocity perturbation measured with a 2D hot wires probe.

2.2 Transient results

Response of the deck is measured for different mean velocities below the flutter critical velocity U_c (respectively 16.1 and 21.3 m/s for cases A and B). In this condition the system is stable and the deck motion is damped. However during the transient period the mechanical energy E is temporarily amplified. The mechanical energy is the sum of the kinetic energy and the potential energy of the 2 degrees of freedom, computed from the measurements of z and α such that

$$E(t)=\frac{1}{2}m\,\dot{z}^2(t)+\frac{1}{2}J_O\,\dot{\alpha}^2(t)+\frac{1}{2}k_z\,z^2(t)+\frac{1}{2}k_\alpha\,\alpha^2(t). \quad (4)$$

A typical result is given Figure 3 where E_0 is the initial energy produced by the gust on the deck. E_0 is measured at a given time after the flap release and does not depend on the structural parameters of the deck. This initial energy value is used to reduce the maximum energy $E_{\max}$ reached during the transient period (see Fig. 3). These results show that a sudden gust can generate temporarily large amplitude which can be detected only when one observe the transient period. In the following chapter we present the numerical model which is used to reproduce this transient mechanism.

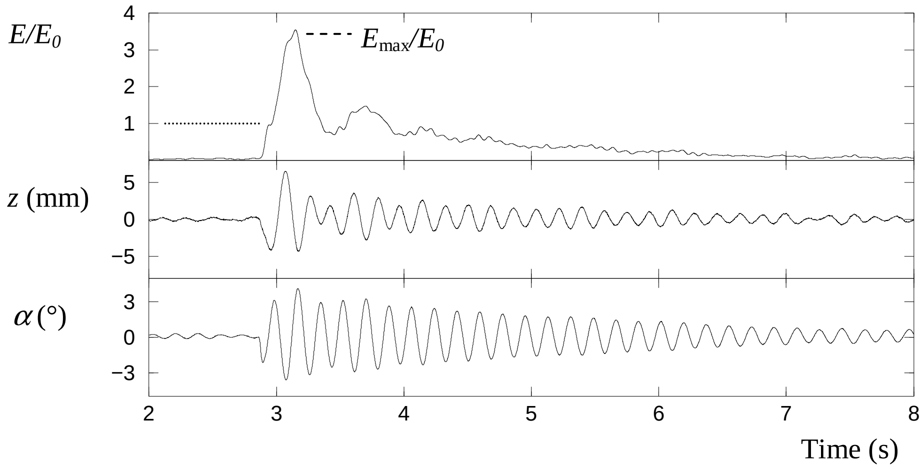

Figure 3: Typical transient response of the deck, $\overline{U}/U_c = 0.91$, Case A.

3 Numerical simulations with time-dependant models

The goal of the present study is to determine the simplest model capable of catching the transient growth of energy observed in the experiments. As already shown with an airfoil [3] the unsteady airfoil theory, as it is described in [9], seems a good starting point to achieve this objective.

3.1 Time-dependant models

The classical flutter derivatives formulation [10] is used for the motion-dependant forces and a transient formulation based on the Küssner's function is used for the buffeting terms [9]. In that context, the first time-dependant model provides the lift sectional force and the pitching moment following Eqs. (5-6). This model, which includes all flutter derivatives, is named TDM-1 in the following.

$$F_z(t) = \frac{1}{2}\rho B\overline{U}^2\left[H_1\frac{\dot{z}(t)}{\overline{U}} + H_2\frac{B\dot{\alpha}(t)}{\overline{U}} + H_3\,\alpha(t) + H_4\frac{z(t)}{B}\right] + \frac{1}{2}\rho B\overline{U}^2 C_z'\,\Psi(\tau) \tag{5}$$

$$M_O(t) = \frac{1}{2}\rho B^2\overline{U}^2\left[A_1\frac{\dot{z}(t)}{\overline{U}} + A_2\frac{B\dot{\alpha}(t)}{\overline{U}} + A_3\,\alpha(t) + A_4\frac{z(t)}{B}\right] + \frac{1}{2}\rho B^2\overline{U}^2 C_M'\,\Psi(\tau) \tag{6}$$

Terms H_i and A_i are flutter derivatives, B the deck length, ρ the air density, C_z' and C_M' the static lift and moment coefficient slope. The non-dimensional time is $\tau = 2\overline{U}\,t/B$ and Ψ is a transient function calculated using the Duhamel's integral such that:

$$\Psi(\tau) = \varphi(0)\frac{w(\tau)}{\overline{U}} + \int_0^\tau \frac{w(\sigma)}{\overline{U}}\varphi'(\tau-\sigma)\,d\sigma \tag{7}$$

$$\varphi(\tau) = 1 - 0.5\exp(-0.13\tau) - 0.5\exp(-\tau) \tag{8}$$

The gust vertical component $w(t)$ is fitted from the experiments with two Gaussian distribution functions, see Figure 2, with $w_{\min}$ and $w_{\max}$ and duration of the gust as the parameters. The Küssner's function φ is approximated with the expression of Jones [9] for elliptic airfoil as in Eq. (8). Note that in this model the effect of the longitudinal component of the gust $u(t)$ is neglected.

The flutter derivatives are written using the unsteady airfoil theory [9] adapted to the case of a bridge deck which has its rotation centre and its gravity centre both located at mid-chord [11]. Resulting expressions are given in Eqs. (9-10).

$$H_1 = -C_z' \; F, \quad H_2 = C_z' \left[\frac{F}{4} + \frac{G}{K}\right] + \frac{C_z'}{4}, \quad H_3 = C_z' \left[F - \frac{KG}{4}\right], \quad H_4 = C_z' \left[KG + \frac{K^2}{4}\right]. \tag{9}$$

$$A_1 = -C_M' \; F, \quad A_2 = C_M' \left[\frac{F}{4} + \frac{G}{K}\right] - \frac{C_z'}{16}, \quad A_3 = C_M' \left[F - \frac{KG}{4}\right] + C_z' \frac{K^2}{128}, \quad A_4 = -C_M' \; KG. \tag{10}$$

The reduced angular frequency is $K = 2\pi f B/\overline{U}$; F and G are the real part and the imaginary part of the Theodorsen function respectively, depending on K . This function is normally valid for thin airfoil but its extension to the case of streamlined bridge deck section is assumed here to remain valid.

The static lift slope C_z' and pitching moment slope C_M' are identified in wind tunnel for the tested deck model in the same Reynolds number range than in the transient tests, and found to be equal to 5.65 and 1.8 respectively. The aerodynamic centre is located at 32 % of the chord. Although the studied bridge deck shape is streamlined, it is however different than an airfoil which has a lift slope close to 2π and an aerodynamic centre at its first quarter of the chord.

The complete model TDM-1 is compared in the rest of the paper with a simplified version, named TDM-2, which neglects the aerodynamic dampings terms H_1, H_2, A_1 and A_2. The lifting force and the pitching moment then reads:

$$F_z(t) = \frac{1}{2}\rho B \overline{U}^2 \left[H_3 \alpha(t) + H_4 \frac{z(t)}{B}\right] + \frac{1}{2}\rho B \overline{U}^2 C_z' \; \Psi(\tau) \tag{11}$$

$$M_O(t) = \frac{1}{2}\rho B^2 \overline{U}^2 \left[A_3 \alpha(t) + A_4 \frac{z(t)}{B}\right] + \frac{1}{2}\rho B^2 \overline{U}^2 C_M' \; \Psi(\tau). \tag{12}$$

It is interesting to mention that the standard quasi-steady model in which the Theodorsen function becomes simply $F = 1$ and $G = 0$ has been tested first. But the results obtained didn't match at all the experimental behaviour and the quasi-steady model was abandoned rapidly. In the Table 2 a short synthesis of the main characteristics of model TDM-1 and TDM-2 is reported.

Model	Flutter derivatives		Buffeting terms	
	Damping	Stiffness	Admittance	Küssner function
TDM-1 – Eqs. (5-6)	Eqs. (9-10)	Eqs. (9-10)	Eq. (7)	Jones – Eq. (8)
TDM-2 – Eqs. (11-12)	No	Eqs. (9-10)	Eq. (7)	Jones – Eq. (8)

Table 2: Main characteristics of the time-dependant models of aeroelastics forces

Temporal simulations of the problem are performed with an improved Newmark scheme which has no numerical damping that could corrupt the solution. Then resulting time histories of z and α are processed for calculating the different quantities such as energy following Eq. (4), as if they were obtained by experiments.

3.2 Results and comparison with experiments

The first quantity to compare is the frequencies of the two motions versus the wind velocity. It is well known that the flutter occurs when these frequencies become equal at the critical velocity U_c, due to the decrease of the pitching frequency while heaving frequency remains almost constant. This evolution is given Figure 2. The frequencies have been measured using Fourier analysis of the two signals.

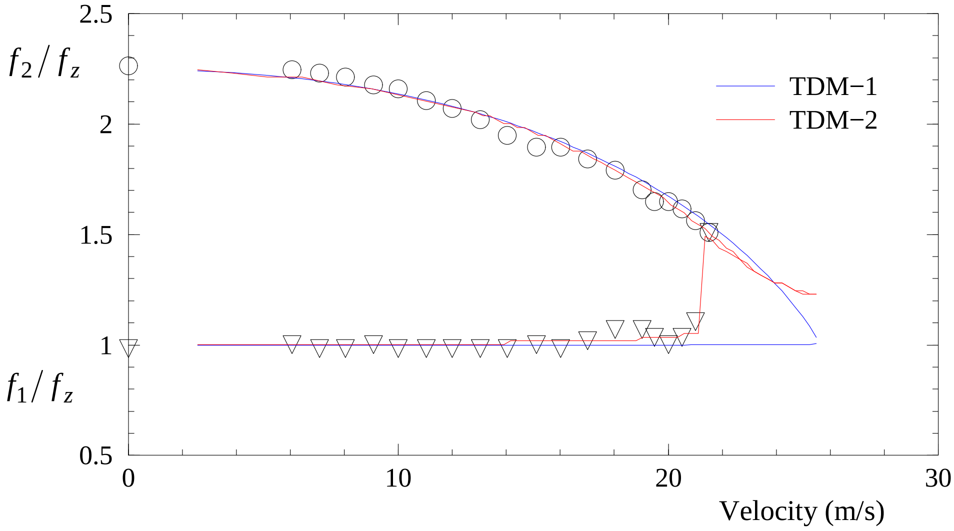

Figure 2: Evolution of frequencies versus wind velocity. Case B, U_c = 21.5 m/s

At low velocities, both models give a good evolution of the system dynamics. However when velocity approaches the critical flutter velocity, the model TDM-1 fails to predict the frequency coalescence and by consequence the critical velocity, while the model TDM-2 agrees very well the experimental results. This trend is confirmed in the following transient tests.

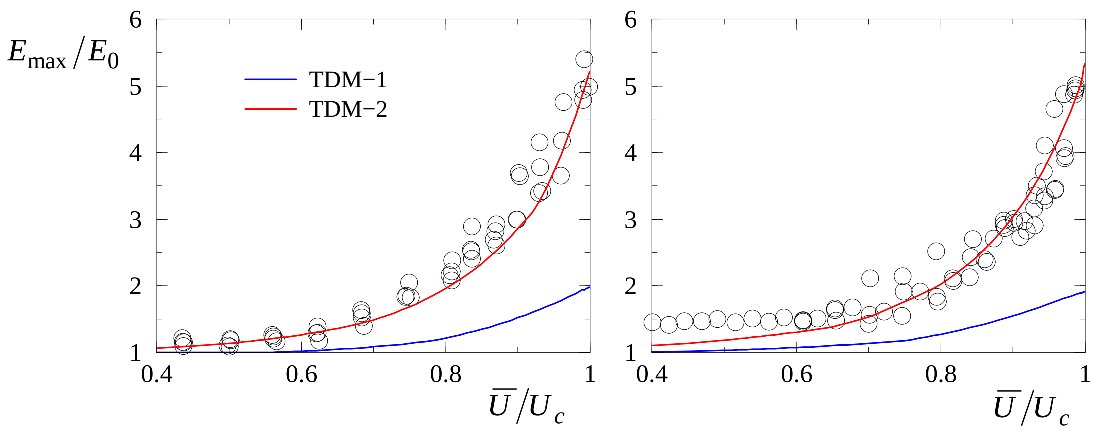

Figure 3: Maximum energy amplification versus velocity.
Mechanical excitation $\alpha_0 = -2.2°$. Case A: left; Case B: right

First we apply a mechanical excitation on the bridge deck without using the gust generated by the flap. The initial excitation is a negative pitch angle $\alpha_0 = -2.2°$ and we suddenly release the deck while recording the motion. The maximum energy amplification is plotted in Figure 3 versus wind velocity. Once again, the model TDM-2 catches very well the dynamics.

Note that in this case, the numerical simulations take into account the flutter derivatives terms only because of the initial excitation performed mechanically. But the real challenge is to replace now this mechanical excitation by the excitation with the gust. In this case the initial energy E_0 is more complex to evaluate than with the previous mechanical excitation where the energy was the potential energy induced by the initial pitch angle α_0. With the flap, the initial energy is the energy transferred to the deck by the gust. More detailed on the experimental procedure for its evaluation can be found in [7]. The experimental initial energy is plotted Figure 4 for cases A and B where we see that although the two cases are structurally different there is mixing of results in a given range of velocity, showing that this initial energy is independent of the bridge dynamics.

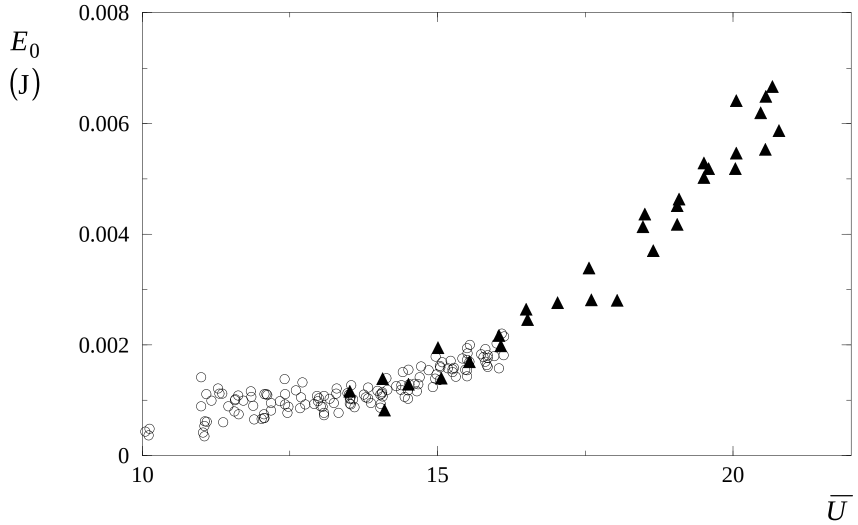

Figure 4: Initial energy of the excitation by the gust versus velocity. Case A: ○ ; Case B: ▲

We apply the same procedure with numerical results. An example of the time histories of energy, pitch angle and heaving position is shown in Figure 5. We clearly observe that the model TDM-1 still fails to reproduce the dynamics even with this new kind of aerodynamic excitation. Surprisingly the early time dynamics is well simulated by model TDM-2, especially the maximum energy amplification. This good agreement is confirmed in Figure 6 which presents the results for all range of velocities. However the long term behaviour is not well reproduced by the models. While TDM-1 generates too much damped signals, the damping of model TDM-2 is too weak by comparison with the experiments.

But in the point of view of structural engineering, the good agreement obtained with TDM-2 on the maximum energy amplification is considered satisfactorily. Indeed it is a good surprise that the Küssner's function of Eq. (8), combined with flutter derivatives using the Theordorsen's function, which are both valid for thin airfoil, can reproduce the transient behaviour of a streamlined bridge deck submitted to a gust. Moreover it is the simplest model, TDM-2, neglecting the aerodynamic dampings and the longitudinal component of the gust, taking the same buffeting function in pitch and lift, which furnishes the best results. This interesting feature certainly should not be as good as here, if the bridge deck were not of a streamlined kind.

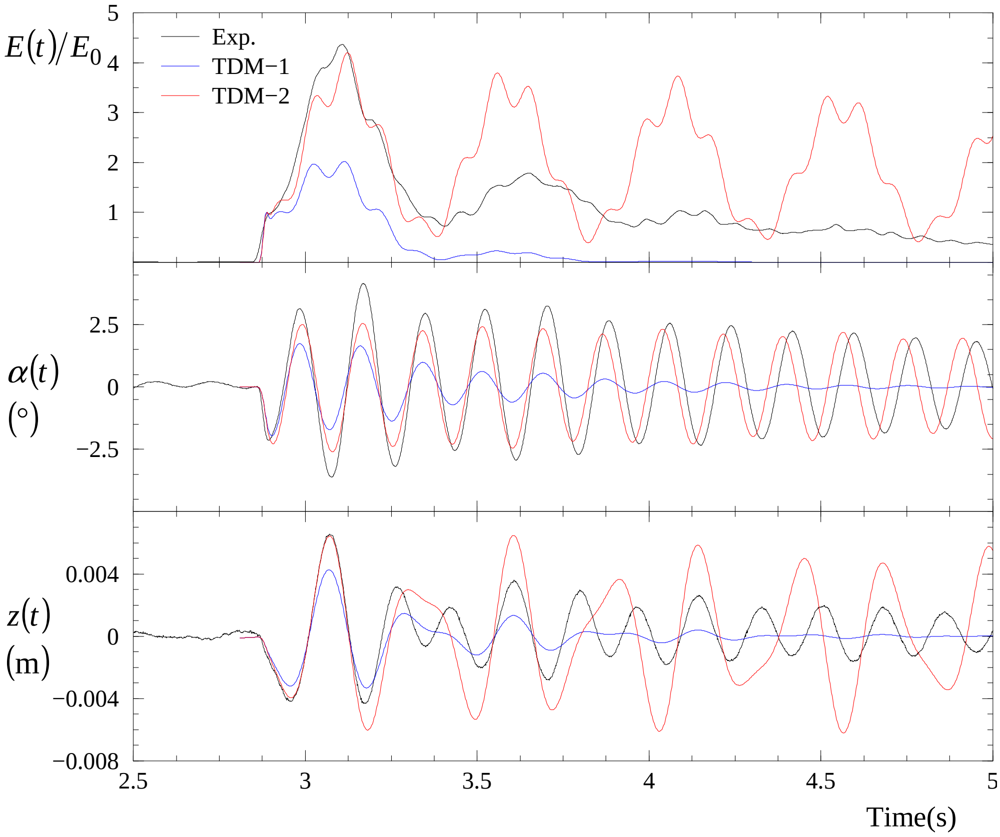

Figure 5: Time evolution of energy, angular displacement and vertical displacement. Excitation by flap, $\overline{U}/U_c = 0.96$, Case B.

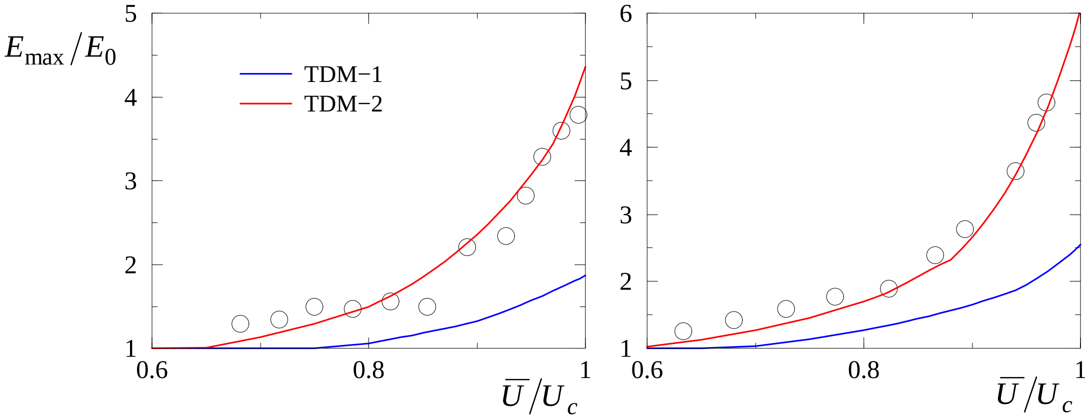

Figure 6: Maximum energy amplification versus velocity. Excitation by gust. Case A: left ; Case B: right

3.3 Numerical study of the frequency ratio

In this section we use the numerical model TDM-2, which provides validated results in terms of energy amplification, in a short parametric study in function of the frequency ratio between torsion and heaving modes frequencies f_z/f_α. The main results are the maximum energy amplification versus velocity, which are given in Figure 7 for the 4 ratios studied, $f_z / f_\alpha = 0.33$, 0.44 (Case B), 0.62 (Case A) and 0.75. For these cases the critical velocities are determined numerically and are $U_c = 22.6$, 21.5, 18.65 and 15.6 m/s respectively.

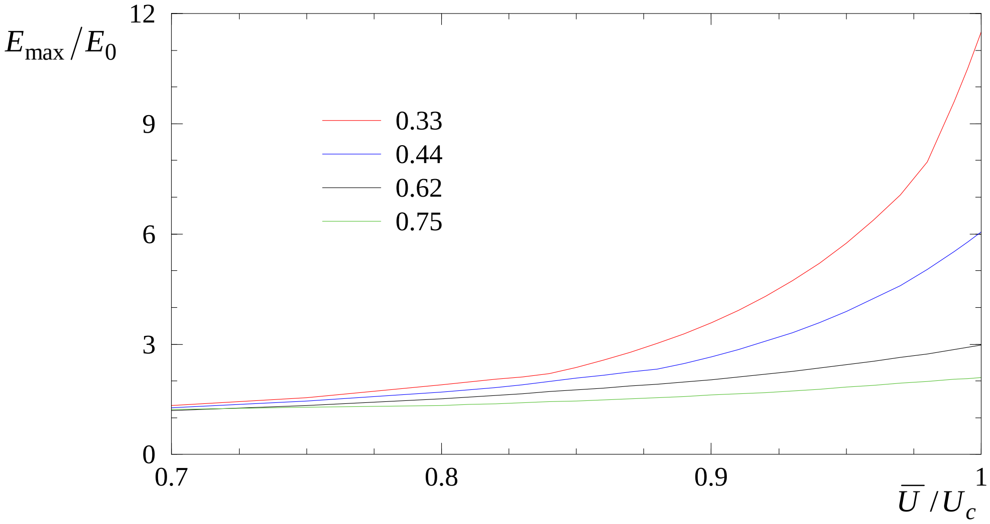

Figure 7: Effect of the frequency ratio on the maximum energy amplification versus velocity.

The interesting result is certainly the fact that the lower velocity ratio, which is *a priori* the most "secured" one in relation to coupled mode flutter, leads to the highest amplification of energy. This trend is shown more clearly in Figure 8 where we have plotted the maximum energy amplification just below the critical velocity, which is noted $(E_{max} / E_0)_c$ versus the frequency ratio. But attention must be paid to the fact that we speak here of dimensionless velocities $\overline{U}/U_c$. It is different when one argue with the velocity $\overline{U}$ because the critical velocity is much higher for the cases with low frequency ratios; hence the energy amplification is found higher when the velocity $\overline{U}$ is higher. Remember also that the initial energy E_0 is a function of the velocity as shown Figure 4.

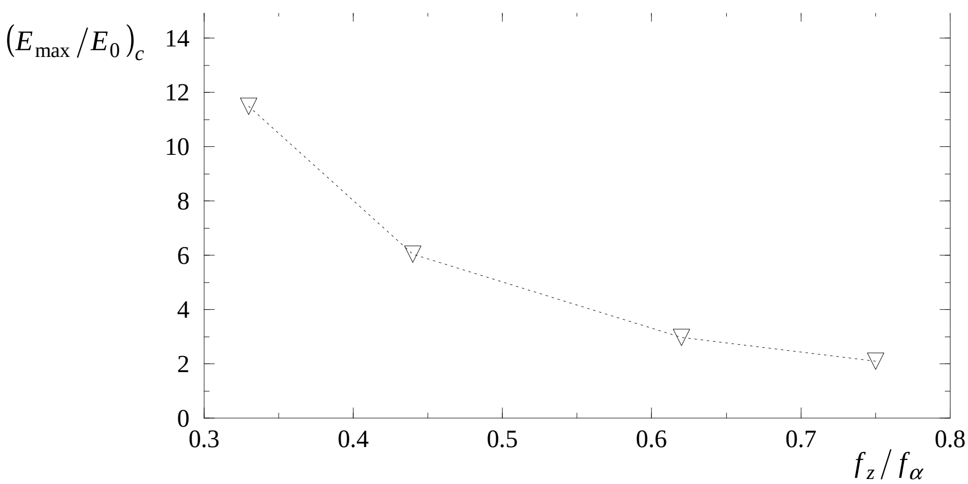

Figure 8: Influence of the frequency ratio on the maximum energy amplification at critical velocity.

4 Conclusion

Wind tunnel experiments have shown that a two degrees-of-freedom bridge deck section submitted to a single gust exhibits transient growth of energy. A time-dependant model based on a classical formulation of both the motion-dependant and buffeting forces has been used to reproduce this transient behaviour. Numerical results are in good agreement with the experiments when the flutter derivatives components comprise the four stiffness terms and when the four damping terms are neglected. The buffeting term, estimated with a Küssner's function was found able to reproduce the excitation by a gust. This study, showing that the transient growth of energy occurs on a streamlined bridge deck, reinforces the interest of using temporal simulations for wind-induced vibrations of flexible structures.

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