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Transient growth of energy and aeroelastic stability of ground vehicles

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Abstract

Transient growth of energy is shown to be possible in the lateral dynamics of passenger cars. This mechanism might be generated during one vehicle overtaking another. Starting from a simple linearized quasi-steady model, which couples the lateral displacement and the yaw angle of the vehicle, the transient growth appears when an initial condition in the yawing rate is applied. ***To cite this article: P. Hémon, C. Noger, C. R. Mecanique 332 (2004).***

Résumé

Croissance transitoire de l'énergie et stabilité aéroélastique des véhicules terrestres. On montre la possibilité d'une croissance transitoire de l'énergie dans la dynamique latérale de véhicules automobiles. Ce mécanisme peut être engendré par exemple à la suite d'un dépassement entre deux véhicules. A partir d'un modèle simple, quasi-stationnaire, linéarisé et couplant le mouvement de dérive et de lacet du véhicule, une croissance transitoire apparaît lorsqu'une condition initiale en vitesse de dérapage est appliquée. ***Pour citer cet article : P. Hémon, C. Noger, C. R. Mecanique 332 (2004).***

Keywords: Fluid mechanics; Vibrations; Transient growth; Aeroelastic stability; Ground vehicle

Mots-clés : Mécanique des fluides ; Vibrations ; Croissance transitoire ; Stabilité aéroélastique ; Véhicule terrestre

Version française abrégée

On présente dans ce papier une étude de croissance transitoire de l'énergie appliquée aux véhicules automobiles. Ce phénomène est classiquement étudié dans le domaine des instabilités hydrodynamiques [2] mais il reste un concept relativement nouveau en interactions fluide-structure [3]. Pour un système stable, la croissance transitoire est due à la présence de modes non normaux produits par les forces non conservatives. Elle se traduit par une amplification temporaire de l'énergie qui décroît ensuite.

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L'objectif de ce papier est de montrer la présence de croissance transitoire de l'énergie dans la dynamique latérale de véhicules terrestres, notamment à la suite d'un dépassement entre deux véhicules.

On considère un véhicule avançant à la vitesse U dont on étudie le mouvement de translation latérale y et de lacet β, comme indiqué Fig. 1. Les équations du mouvement sont données par (1), où m est la masse, J le moment d'inertie, η_y et η_β les amortissements réduits et f_y et f_β les fréquences de chaque degré de liberté. Les efforts aérodynamiques sont la dérive F_y et le moment de lacet N qui s'expriment à l'aide des coefficients sans dimension C_y et C_n suivant la relation (2). Ces coefficients dépendent de l'angle apparent β_a et de la vitesse apparente V_a appliqués au véhicule au cours du mouvement. Ils sont estimés au foyer aérodynamique A [4,5] dont la position est donnée par (3). La vitesse apparente et le lacet apparent dépendent des mouvements y et β, cf. Fig. 1 et s'expriment par (4). Après linéarisation on obtient les expressions (5) dans lesquelles l'angle de dérapage moyen est supposé nul.

Le système constitué de (1) et (5) est couplé par amortissement et par raideur. L'analyse de stabilité montre que la limite d'instabilité est atteinte par perte de raideur pure en lacet, ce qui conduit à une vitesse critique U_c exprimée dans (6). Par la suite, l'étude de croissance transitoire est réalisée par simulation numérique du problème (1), (5), en gardant une vitesse stable, $U < U_c$. L'énergie (7) est utilisée pour quantifier la croissance transitoire.

Le modèle est appliqué à un véhicule dont les paramètres sont donnés dans le Tableau 1. L'influence du type de condition initiale est illustrée Fig. 2. L'énergie est normalisée à l'aide de l'énergie initiale E_0 associée aux conditions initiales. On constate qu'une croissance transitoire apparaît pour une condition initiale en vitesse de dérapage $\dot{\beta}_0$. Dans ce cas, l'influence de la vitesse est présentée Fig. 3, où l'on constate que le taux d'amplification de l'énergie varie fortement près du seuil.

Un résultat intéressant est le fait que la durée pendant laquelle il y a amplification est du même ordre de grandeur que la période fondamentale du système. Le conducteur d'un véhicule dans cette situation pourrait ainsi être très sensible à la croissance transitoire de l'énergie, car cette durée coïncide effectivement avec la zone de perception humaine maximale [6].

L'étude de la stabilité latérale de véhicule terrestre doit donc correctement prendre en compte les aspects transitoires du problème, en incluant les mouvements latéraux de dérive et de lacet, en particulier à la suite d'un dépassement entre deux véhicules, voire d'un coup de volant produit par le conducteur.

1. Introduction

The transient motion of passenger cars due to lateral aerodynamic loads, such as in overtaking process or because of a cross-wind gust, has probably been experienced by every driver. Although this is the cause of a very small number of accidents, the perception of such transient phenomena is considered as an important point in the drivers' opinion of a car performance [1].

In this paper, we present an investigation focused on the transient growth of the energy. Transient growth is an amplification of the energy of a stable system, before it ultimately decreases. This phenomenon was displayed in recent years in the field of hydrodynamic stability [2]. It can occur for linear systems in the subcritical range as a consequence of the interaction of nonorthogonal modes. These modes, due to non conservative forces interact in such a way that the energy of the stable system is transiently amplified before it exponentially decays at the rate of the least stable mode [3].

The objective of this paper is to show that such phenomenon can occur for ground vehicles.

2. Dynamical system modelling

2.1. Governing equations

We consider a vehicle which travels on a straight line at a constant velocity U. For any given reason, it experiences lateral transient loads that will induce a transient response. The equations of motions in lateral displacement y and in yaw angle β, as shown in Fig. 1, can be written [4]:

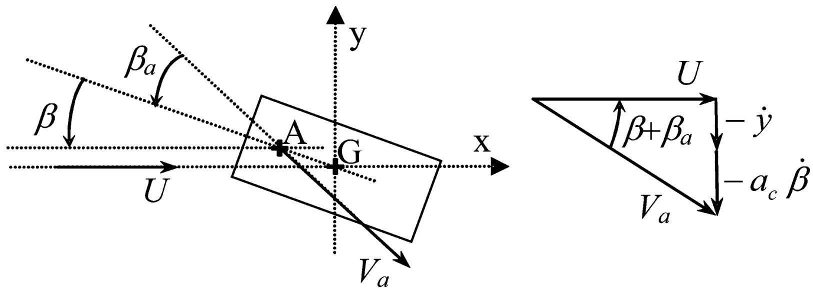

Fig. 1. Definition of geometrical parameters and apparent velocity at point A.

Fig. 1. Définition des paramètres géométriques et vitesse apparente au point A.

$$\begin{cases} m\ddot{y} + 2m\eta_y 2\pi f_y \dot{y} + m(2\pi f_y)^2 y = F_y \\ J\ddot{\beta} + 2J\eta_\beta 2\pi f_\beta \dot{\beta} + J(2\pi f_\beta)^2\beta = N \end{cases} \tag{1}$$

where m is the vehicle mass, J the inertia momentum, η_y and η_β the damping ratios which are supposed to be small, f_y and f_β the two frequencies for each degree of freedom, F_y the aerodynamic side force and N the aerodynamic yawing moment. In practice, for a road vehicle, the stiffness for the lateral stability is mainly provided by the deformable tyres, which are supposed to perfectly grip the road.

The aerodynamic forces are expressed with the dimensionless coefficient of side force C_y and yawing moment C_n, so that:

$$F_y = \frac{1}{2}\rho S V_a^2 C_y(\beta_a), \qquad N = \frac{1}{2}\rho S\ell V_a^2 C_n(\beta_a) \tag{2}$$

These coefficients depend on the relative yaw angle β_a which is supposed here to be small. S is the reference surface, ℓ the reference length, ρ the air density. Here V_a is the relative velocity on the vehicle during its movement. It is estimated at the aerodynamic centre A at a distance a_c ahead of the centre of mass G of the vehicle, with [4,5]:

$$a_c = \ell\frac{\partial C_n/\partial\beta}{\partial C_y/\partial\beta} \tag{3}$$

The relative velocity and relative yaw angle depend on y and β, see Fig. 1, through geometrical relations:

$$V_a^2 = U^2 + (\dot{y} + a_c\dot{\beta})^2, \qquad \beta_a = \mathrm{arctg}\big[(\dot{y} + a_c\dot{\beta})/U\big] - \beta \tag{4}$$

Introducing these expressions in (2) and after linearization, and assuming the mean yaw angle to be zero (i.e., U is parallel to x), we obtain the linearized forces:

$$F_y = \frac{1}{2}\rho U^2 S\left(\frac{\dot{y}}{U} + \frac{a_c\dot{\beta}}{U} + \beta\right)\frac{\partial C_y}{\partial\beta}, \qquad N = \frac{1}{2}\rho U^2 S\ell\left(\frac{\dot{y}}{U} + \frac{a_c\dot{\beta}}{U} + \beta\right)\frac{\partial C_n}{\partial\beta} \tag{5}$$

This simple aeroelastic model shows that coupling between the two degrees of freedom occurs through the aerodynamic forces. The unstable behaviour of this system results from the added stiffness and damping, for which pure and coupled terms appear.

A standard stability analysis can be performed in order to derive the critical velocity U_c that leads to amplified motions. It can be shown that here the smallest critical velocity is due to the loss of the pure yawing stiffness and is given by

$$U_c = 2\pi f_\beta\sqrt{\frac{2J}{\rho S\ell\partial C_n/\partial\beta}} \tag{6}$$

A normalized velocity U^* is defined such that $U^* = U/U_c$. For the study of transient growth below the critical velocity, $U^* < 1$, temporal simulations of the governing equations (1), (5), are performed now. Note that particular attention needs to be taken in the numerical integration of (1), to avoid spurious damping effects near the instability threshold. We used here a fourth order numerical scheme, having no numerical damping and a phase error of the 4th order in time step.

Initial conditions will be discussed in the next section. To quantify transient growth, a post processing of the simulation provides the mechanical energy of the system, which is the sum of kinetic and potential energies and expressed as

$$E(t) = \frac{1}{2}m\dot{y}^2 + \frac{1}{2}J\dot{\beta}^2 + \frac{1}{2}m(2\pi f_y)^2 y^2 + \frac{1}{2}J(2\pi f_\beta)^2\beta^2 \tag{7}$$

2.2. Application on ground vehicles

The above dynamical model is applied to a typical ground vehicle. Numerical values are given in Table 1: they are chosen to be typical of a passenger car [1]. The reference length is 2.70 m and air density is 1.2 kg/m^3. From Eq. (3), it is seen that the aerodynamic centre is located aft of the gravity centre of the vehicle.

The critical velocity computed with (6) is 45.1 m/s. It is important to notice that for the transient motion analysis, all terms in Eq. (5) have to be taken into account, since all terms may be shown here to be of similar magnitude.

The influence of the initial conditions on energy growth is displayed in Fig. 2 for a normalized velocity $U^* = 0.89$. The energy computed using Eq. (7) is normalized by the initial energy E_0 associated with the initial conditions. The model being linear, the dimensionless energy is independent of the initial conditions magnitude [3]. The four elementary possibilities, for each single degree of freedom, are presented. The main result is that transient amplification is obtained for one case only, when an initial yawing rate $\dot{\beta}_0$ is applied. All other possibilities lead to monotonically decreasing energies.

Table 1
Vehicle parameters

Tableau 1
Paramètres du véhicule

m (kg)	J (kg·m^2)	η_y (%)	η_β (%)	f_y (Hz)	f_β (Hz)	$S\partial C_y/\partial\beta$ (m^2/rad)	$S\partial C_n/\partial\beta$ (m^2/rad)	a_c (m)
1000.	100.	10	10	1.0	1.0	−3.8	+1.2	−0.85

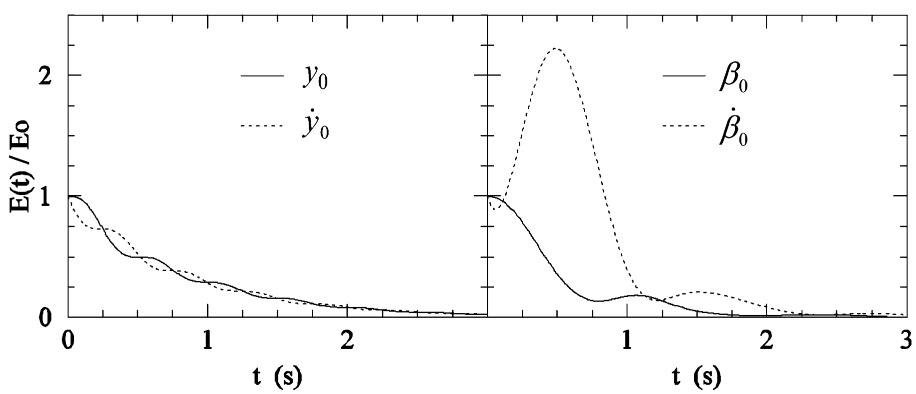

Fig. 2. Transient energy amplification for different initial conditions, $U = 40$ m/s ($U^* = 0.89$).

Fig. 2. Amplification transitoire de l'énergie pour différentes conditions initiales, $U = 40$ m/s ($U^* = 0.89$).

3. Discussion and conclusion

The influence of the car velocity, normalized by the critical velocity is displayed in Fig. 3. The energy amplification rate increases with velocity. This behaviour was theoretically studied in [3] where it was shown that the maximum transient amplification asymptotically scales with $1/1 - U^{*4}$. The difference between the present case and this asymptotic scaling is displayed Fig. 4, where the maximum amplification rate of energy is plotted versus normalized velocity. In fact, one must remember here that the critical velocity is due to the loss of pure yawing stiffness, although transient growth of energy is generated by the coupling stiffness.

Another important new feature is the fact that the time during which there is amplification is of the same order as the natural period of the vehicle. In previous studies of transient growth of energy [3], it was found instead that the characteristic time of the energy amplification was much larger than the natural period of the system. In the context of ground vehicles, the driver may then be sensitive to the transient growth of energy because the characteristic duration falls into the range of maximum human perception [6].

The fact that only an initial yawing rate leads to transient growth is of great importance in practice: it means that the physics involved in the lateral stability of vehicles cannot be simulated statically, i.e., without accounting for

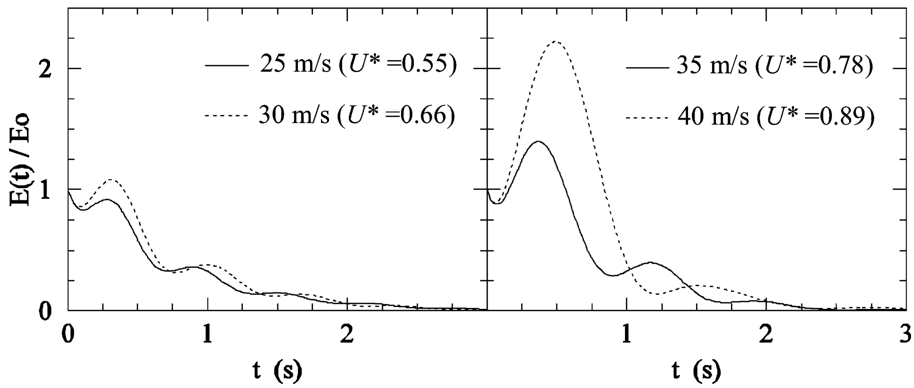

Fig. 3. Transient energy amplification for various wind velocities, initial condition on $\dot{\beta}_0$.

Fig. 3. Amplification transitoire de l'énergie pour différentes vitesses, condition initiale en $\dot{\beta}_0$.

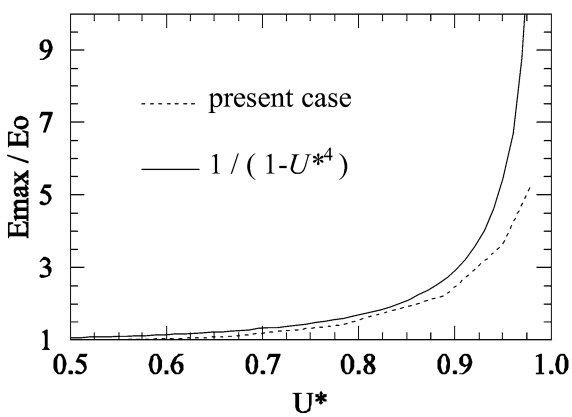

Fig. 4. Maximum energy amplification versus normalized velocity.

Fig. 4. Amplification maximale de l'énergie en fonction de la vitesse normalizée.

the yawing rate and side motion of the vehicle. One needs necessarily to include the complete vehicle dynamics to simulate real road dynamic behaviour.

Transient growth of energy has been shown to be possible in realistic situation of passenger car response when submitted to lateral aerodynamic loads. Energy amplification occurs when the vehicle is submitted to an initial yawing rate. This can be the consequence of an overtaking or a sharp turn on the wheel. In a simplified scenario indeed, we can assume that the vehicle under consideration is first submitted to a steep change of wind direction, such as the one caused by an overtaking, which produces transient aerodynamic forces. These forces generate a response of the vehicle, eventually modified by the driver's response. Then, when the vehicle has passed through the perturbed aerodynamic field caused by the wake of the overtaken vehicle, it comes back into a quiet region with initial conditions that are the consequences of the previous motions.

It implies that the correct simulation of lateral stability of cars has to include a study of the vehicle transient dynamics. It also might partially explain why static approaches of the overtaking, without lateral motions and yawing angles, lead to incorrect agreement with results of road behaviour tests.

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