# On the transient response of road vehicles to cross-wind gust

P. Hémon[1]

ABSTRACT: *We deal in this paper with the transient response of road vehicles when a sudden change arises in the wind velocity, in terms of yawing angle and modulus. The objective is to demonstrate that the coupling between lateral and yawing motion is of major importance in the transient response and hence cannot be neglected. This is due to a mechanism called transient growth of energy which we have demonstrated recently the existence. Starting from a simple linear quasi-steady model, we show that the yawing rate is responsible of a transient growth of the energy, hence generating a transient amplification of the vehicle response before the exponential decay. This short term instability is a consequence of the dissipative forces induced by the aerodynamic loads, which generate the coupling between the two degrees of freedom. We compare different kinds of vehicles in regards to this phenomenon.*

## 1 INTRODUCTION

The transient motion of passenger cars consecutive to lateral aerodynamic loads, such as during overtaking or because of a cross-wind gust, has probably been experienced by every driver. Although this is the cause of a very small number of accidents, perception of such transient phenomena is considered as an important point in the drivers' opinion of a car performance (Bourdassol, 1996).

In this paper, we present an investigation focused on the transient growth of the energy. Transient growth is an amplification of the energy of a stable system, before it ultimately decreases. This phenomenon was evidenced in recent years in the field of hydrodynamic stability (Schmid & Henningson, 2001). Moreover, if the energy growth is large enough, a non linear instability can be triggered by amplitude effect even when the system is linearly stable at small amplitude. This has been interpreted as a possible scenario for by-pass transition to the instable behaviour, before the linear critical velocity.

In the field of fluid-structure interactions, transient growth can occur even for linear systems in the subcritical range as a consequence of the interaction of non orthogonal modes. These modes, due to the non conservative forces interact in such a way that the energy of the stable system is transiently amplified before it exponentially decays at the rate of the least stable mode (Schmid & de Langre, 2003).

Recently this mechanism has been observed experimentally before the coupled flutter of an airfoil (Hémon, de Langre & Schmid, 2004). Comparisons with numerical simulation using linear modelling were satisfactory.

The objective of the present paper is to show that such phenomenon can occur for ground vehicles. First we establish a simple quasi-steady model which is used then in a series of numerical simulations for parametric studies.

## 2 THE DYNAMICAL SYSTEM

We present in this section the two degrees of freedom dynamical system which is applied to the lateral stability of a passenger car.

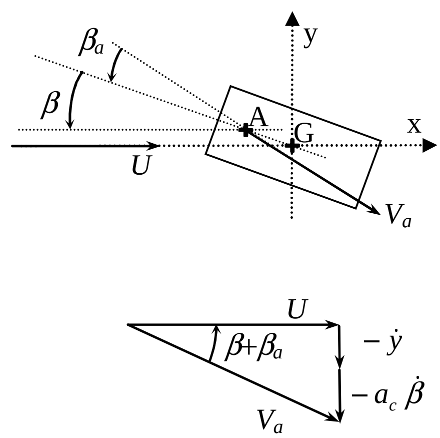


Figure 1. Definition of geometrical parameters and apparent velocity at point A

### *2.1 Governing equations*

We consider a vehicle which travels on a straight line at a constant velocity *U*. For any given reasons, it is submitted to lateral transient loads that will induce a transient response.

[1] Dr. Pascal Hémon, Department of Mechanics, LadHyX, Ecole Polytechnique - CNRS, F-91128 Palaiseau Cédex, France,
E-mail : pascal.hemon@ladhyx.polytechnique.fr

The equations of motions in lateral displacement $y$ and in yaw angle $\beta$ as shown in Figure 1, reads (Fung, 1993)

$$\begin{cases} m\,\ddot{y} + 2m\eta_y\, 2\pi\, f_y\, \dot{y} + m\left(2\pi\, f_y\right)^2\, y = F_y \\ J\,\ddot{\beta} + 2\,J\,\eta_\beta\, 2\pi\ f_\beta\, \dot{\beta} + J\left(2\pi\ f_\beta\right)^2\, \beta = N \end{cases} \tag{1}$$

where $m$ is the vehicle mass, $J$ the inertia momentum, $\eta_y$ and $\eta_\beta$ the damping ratios which are supposed to be small, $f_y$ and $f_\beta$ the two frequencies for each degree of freedom, $F_y$ the aerodynamic side force and $N$ the aerodynamic yawing moment. In practice for a road vehicle, the stiffness for the lateral stability is mainly provided by the deformable tires, which are supposed to perfectly grip the road.

The aerodynamic forces are expressed with the dimensionless coefficient of side force $C_y$ and yawing moment $C_n$, so as

$$\begin{aligned} F_y &= \frac{1}{2}\rho S V_a{}^2\, C_y\left(\beta_a\right) \\ N &= \frac{1}{2}\rho S\ell\, V_a{}^2\, C_n\left(\beta_a\right) \end{aligned}. \tag{2}$$

These coefficients depend on the relative yaw angle $\beta_a$ which is supposed here to be small. $S$ is the reference surface, $\ell$ the reference length, $\rho$ the air density.

Here $V_a$ is the relative velocity on the vehicle during its movement. It is estimated at the aerodynamic centre A at a distance $a_c$ ahead of the centre of mass G of the vehicle, with (Fung, 1993; Kermode 1982):

$$a_c = \ell\frac{\partial C_n/\partial\beta}{\partial C_y/\partial\beta}. \tag{3}$$

The relative velocity and relative yaw angle depend on $y$ and $\beta$, see Figure 1, through geometrical relations

$$\begin{aligned} V_a{}^2 &= U^2 + \left(\dot{y} + a_c\ \dot{\beta}\right)^2 \\ \beta_a &= \operatorname{arctg}\left[\left(\dot{y} + a_c\ \dot{\beta}\right)/U\right] - \beta \end{aligned}. \tag{4}$$

Introducing these expressions in equation (2) and after linearization, and assuming the mean yaw angle to be zero (*i.e.* $U$ is parallel to x), we obtain the linearized forces:

$$\begin{aligned} F_y &= \frac{1}{2}\rho U^2\, S\left(\frac{\dot{y}}{U} + \frac{a_c\,\dot{\beta}}{U} + \beta\right)\frac{\partial C_y}{\partial\beta} \\ N &= \frac{1}{2}\rho U^2\, S\,\ell\left(\frac{\dot{y}}{U} + \frac{a_c\,\dot{\beta}}{U} + \beta\right)\frac{\partial C_n}{\partial\beta} \end{aligned}. \tag{5}$$

This simple aeroelastic model shows that coupling between the two degrees of freedom occurs through the aerodynamic forces.

### *2.2 Stability of the system*

The instable behaviour of this system is results from the added stiffness and damping, for which pure and coupled terms appear.

A standard stability analysis can be performed in order to derive the smallest critical velocity $U_c$ that leads to amplified motions. The coupled system (1), (5) can yield

- a single degree of freedom instability, by negative damping such as galloping or, by negative stiffness in yaw such as a divergence;
- a coupled instability, by stiffness such as in classical flutter, or by coupled damping.

Due to the specific configuration and aerodynamic characteristics of a passenger car, as detailed in the next section, it can be shown here that the smallest critical velocity is due to the loss of the pure yawing stiffness and is given by

$$U_c = 2\pi\, f_\beta\sqrt{\frac{2J}{\rho S\ell\,\partial C_n/\partial\beta}}. \tag{6}$$

A normalized velocity $U^*$ is defined such that $U^* = U/U_c$. For the study of transient growth below the critical velocity, $U^* < 1$, temporal simulations of the governing equations (1), (5), are performed. Note that particular attention needs to be taken in the numerical integration of (1), to avoid spurious damping effects near the instability threshold. We used here a fourth order numerical scheme, having no numerical damping and a phase error of the $4^{th}$ order in time step. Initial conditions will be discussed in the next section.

To quantify transient growth, a post processing of the simulation results provides the mechanical energy of the system, which is the sum of kinetic and potential energies and expressed as

$$E(t)=\frac{1}{2}m\,\dot{y}^2+\frac{1}{2}J\,\dot{\beta}^2+\frac{1}{2}m\left(2\pi f_y\right)^2 y^2 + \frac{1}{2}J\left(2\pi f_\beta\right)^2\beta^2 \quad . \tag{7}$$

This energy is normalized hereafter by the initial energy *Eo* associated with the initial conditions. The model being linear, the dimensionless energy is independent of the initial conditions magnitude (Schmid & de Langre, 2003).

### *2.3 Vehicles characteristics*

The above dynamical model is applied to two typical ground vehicles, a two bodies shape (2B) and a three bodies shape (3B) as sketched in Figure 2. The structural parameters are taken the same for both vehicle and presented in Table 1. They are chosen to be typical of a passenger car (Bourdassol, 1996). The reference length is 2.70 m and the air density is 1.2 kg/m$^3$. Note that the structural dynamics of the vehicle is here very simplified since it is not the purpose of the study.


Figure 2. Sketch of the two typical vehicle shapes compared in this study

Similarly, the aeroelastic model remains simple and might be improved using for instance the admittance functions as proposed by Filippone (2003). But it is not considered here to be significant for the transient growth problem.

The two vehicles have their difference in their aerodynamic characteristics as shown in Table 2. In both cases the aerodynamic centre is located aft the middle, as for all ground vehicles, but it is much forward on the 3B vehicle.

As it has been said already, this set of coefficients, after introducing them in the model of the previous section, leads to an instability generated by loss of the yawing stiffness. This is a one degree of freedom instability. The corresponding critical velocity given in equation (6) depends only on the yawing moment derivative. After computation, we obtain 55.2 m/s and 45.1 m/s for vehicles 2B and 3B respectively.

Although the critical velocity is related to a simple one degree of freedom instability, it is important to notice that for the transient motion analysis, all terms in equation (5) have to be taken into account, since all terms may be shown here to be of similar magnitude.

Table 1 Parameters of the vehicle

| $m$ (kg) | $J$ (kg.m$^2$) | $\eta_y$ (%) | $\eta_\beta$ (%) | $f_y$ (Hz) | $f_\beta$ (Hz) |
|---|---|---|---|---|---|
| 1000. | 100. | 10 | 10 | 1.0 | 1.0 |

Table 2 Aerodynamic characteristics of the two vehicles

| Vehicle | $S\,\partial C_y/\partial\beta$ (m$^2$/rad) | $S\,\partial C_n/\partial\beta$ (m$^2$/rad) | $a_c$ (m) |
|---|---|---|---|
| 2B | -4.8 | +0.8 | -0.45 |
| 3B | -3.8 | +1.2 | -0.85 |

## 3 RESULTS OF SIMULATIONS

We present in this section the simulations results obtained with the previous model and data.

### *3.1 Influence of initial conditions*

The influence of the initial conditions on energy growth is displayed in Figure 3 for a normalized velocity $U^*$ = 0.89 and the vehicle 3B. The four elementary possibilities for each degree of freedom, are presented.

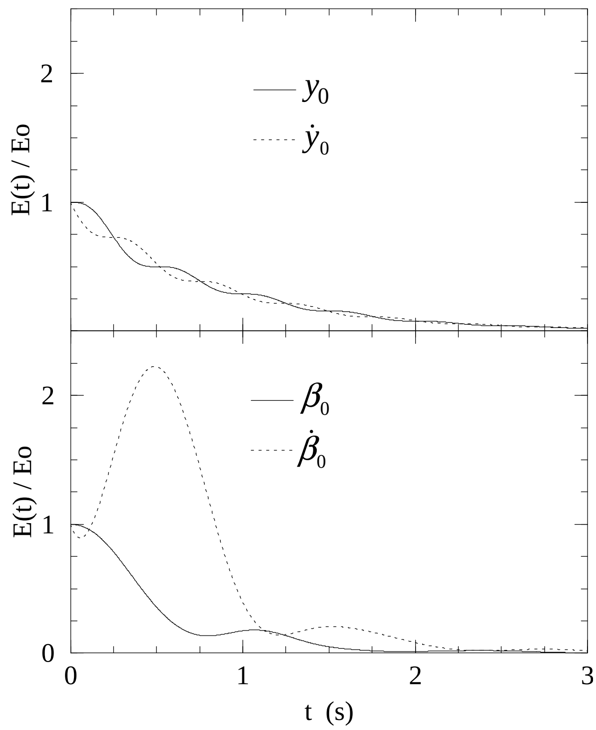


Figure 3. Transient energy amplification for various initial conditions, *U*=40 m/s (*U** =0.89), 3B vehicle.

The main result is that transient amplification is obtained for one case only, when an initial yawing rate $\dot{\beta}_0$ is applied. All other possibilities lead to decreasing energies.

This result is of major importance for the lateral response of car submitted to cross-wind gust because most of the wind tunnel studies focus their investigation on the evaluation of the unsteady forces without taking account the yawing rate, i.e. without performing a real dynamic test (Noger & Széchenyi, 2004).

Then transient growth of energy due to the yawing rate effect might be one of the reasons why some discrepancies are observed between wind tunnel tests and road tests with a real vehicle.

### *3.2 Comparison of vehicles*

Having identified the initial condition leading to transient growth, we can now compare the behaviour of the two vehicles.

There are two ways of comparison, the more obvious one being at a constant velocity *U*. This is done in Figure 4 where the three bodies vehicle is seen to be more sensitive to the phenomenon than the two bodies vehicle.

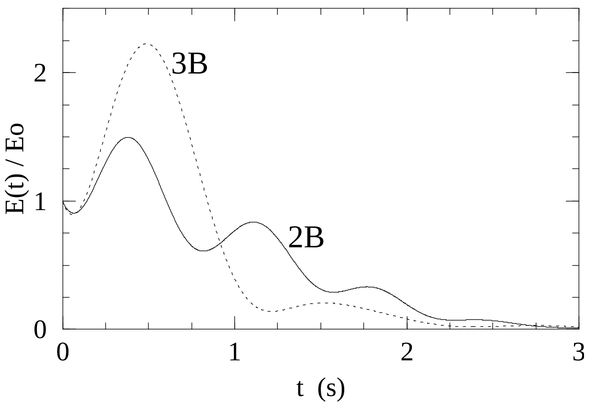


Figure 4. Comparison of the transient energy amplification between the two considered vehicles with initial condition in yawing rate, *U*=40 m/s.

But it has been seen that the two vehicles have a different critical velocity due to the loss of yawing stiffness. Another way of comparison is then at a constant reduced velocity *U**.

Schmid and de Langre (2003) have shown that the maximum energy amplification before coupled flutter follows an asymptotic law of the kind $1/1-U^{*4}$ versus the velocity parameter $1-U^*$. In that case, the critical velocity used in the reduced velocity definition is related to the coupled flutter instability, which is not our case here.

In Figure 5 we compare the evolution of the maximum energy amplification, extracted from the temporal numerical simulations for the two vehicles, with the aforementioned asymptotic law. The disagreement is indeed obvious. But the remarkable point is that the two bodies vehicle is, in such space of dimensionless parameters, more sensitive to the transient growth than the three bodies vehicle. This is explained because transient growth of energy is due to the aeroelastic coupling between the two degrees of freedom.

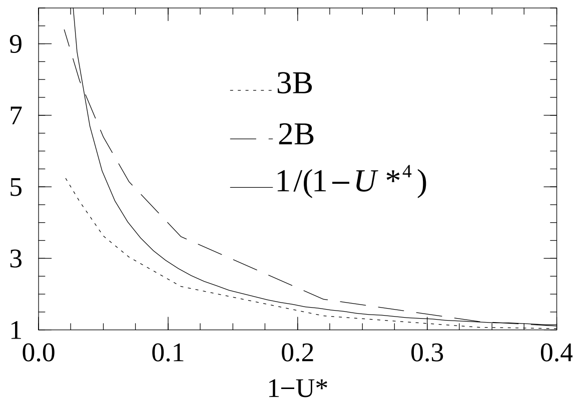


Figure 5. Maximum energy amplification versus velocity parameter for the two considered vehicles and comparison with the asymptotic law.

### *3.3 Further discussion*

Now we investigate more accurately the transient growth behaviour and its effects felt by the driver.

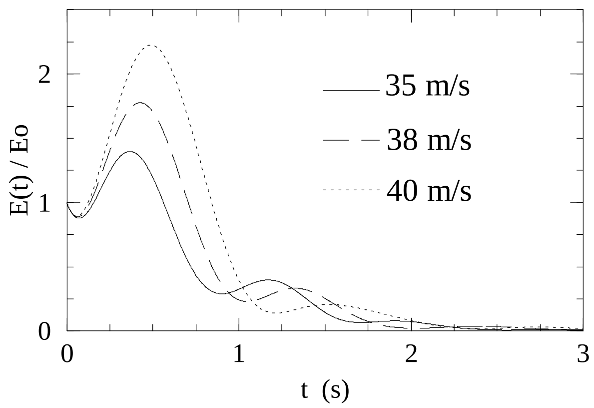


Figure 6. Influence of the free-stream velocity on the transient energy amplification with initial condition in yawing rate, 3B vehicle.

The first remark is related to the high sensitivity of the phenomenon to the free-stream velocity, when approaching the critical velocity, which is illustrated in Figure 6, at 35 m/s (*U**=0.78), 38 m/s (*U**=0.84) and 40 m/s (*U**=0.89). This behaviour is obviously shown in the previous Figure 5. However the time histories of the energy with dimensional velocities in m/s as the variable parameter reinforce the remark.

Another significant feature lies in the duration of the phenomenon which is of the order of the eigenperiod of the individual degrees of freedom, i.e. one second. For other applications in fluid-structure interactions domain, it was observed indeed that the duration was much larger than the period of the motions (Schmid & de Langre, 2003).

It means that a driver, and his passengers, will be here very sensitive to the transient growth effects because such a characteristic time is exactly in the main receptivity range of the human body, as stated by the standards (ISO 2631-1, 1997).

We present in Figure 7 the time histories of the two motions during transient growth of energy, and in Figure 8 the corresponding aeroelastic forces. The lateral displacement *y* reaches approximately the amplitude of almost 6 mm in one second, which in practice produces a relatively low acceleration. Therefore the driver will not be influenced by the lateral displacement.

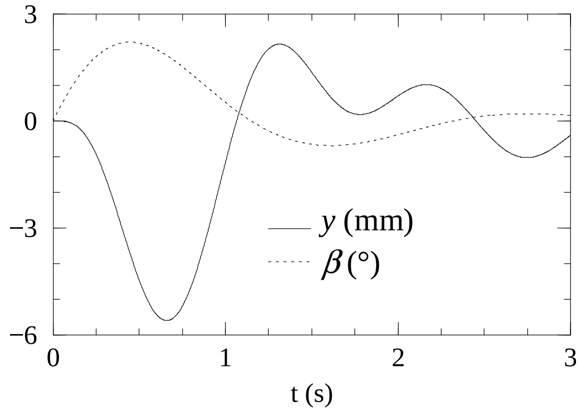


Figure 7. Transient evolution of the lateral displacement and yaw angle generated by an initial yawing rate, *U*=40 m/s (*U** =0.89), 3B vehicle.

However, the yaw angle reaches a peak value around 2°: although it could be seen as a small value, in terms of vehicle dynamics, the sensitivity of the driver's vision, on highways for instance, makes these few degrees in practice very important in his opinion.

Indeed, on highways where the phenomenon is mainly reported, the driver's vision is usually set far forward due to the vehicle velocity. Then, by reference with the lines drawn on the road, a very small change in yaw angle of the vehicle can be detected by the driver and eventually subjectively amplified.

This extreme sensitivity to a small change in yaw angle, when it is associated with a characteristic time right in the receptivity of the human body, might be one of the reasons why this lateral stability problem is so important in the drivers' opinion.

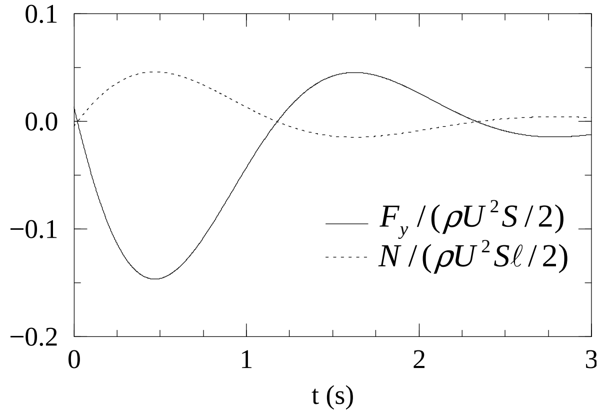


Figure 8. Transient evolution of the lateral force and yawing moment generated by an initial yawing rate, *U*=40 m/s (*U** =0.89), 3B vehicle.

## 4 CONCLUSIONS

Transient growth of energy has been shown to be possible in realistic situation of passenger car response when submitted to lateral aerodynamic loads. The energy amplification occurs when the vehicle is submitted to an initial yawing rate.

This can be the consequence of an overtaking or a sharp turn on the wheel. In a simplified scenario indeed, we can assume that the vehicle under consideration is first submitted to a steep change of wind direction, as the one caused by an overtaking, which produces transient aerodynamic forces. These forces generate a response of the vehicle, eventually modified by the driver's response. Then, when the vehicle has passed through the perturbed aerodynamic field caused by the wake of the overtaken vehicle, it comes back in a quiet region with initial conditions that are the consequences of the previous motions.

The analysis of the resulting motion parameters shows that the driver opinion might be influenced by the yawing motion, due to the extreme sensitivity of the human vision to a small change of yaw angle.

It implies that the correct simulation of lateral stability of cars has to include a study of the vehicle transient dynamics, especially with inclusion of the yaw angle.

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