INSTABILITY OF A LONG RIBBON HANGING IN AXIAL AIRFLOW

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ABSTRACT

A ribbon hanging in a vertical airstream experiences sudden vibrations by flutter when the flow velocity reaches a critical value. The experiments conducted here for strips made of different materials show two distinct behaviors depending on the length of the strip. For short strips, the critical flow velocity depends strongly on the length, whereas for longer strips the critical velocity is independent of the length. These behaviors are analysed using a model derived by Datta, based on slender body approximation and unsteady potential flow theory. This yields an equation similar to that pertaining to a hanging pipe conveying fluid. The corresponding critical velocities are in good agreement with those of the experiments on a set of twelve different ribbons

For each strip, an asymptotic velocity can be defined from high lengths results. The model predicts that this velocity only depends on the ratio between the fluid added mass and the ribbon mass. This is compared with experiments using strips of various width and materials.

1. INTRODUCTION

Vibration of paper due to axial airflow is an important issue for paper manufacturing and paper printing (Watanabe, 2002). In those industries long bands of paper are fed through machines at high speed. Paper is consequently swept by an airflow which is likely to cause instabilities. Such vibrations can provoke folding, wrinkling and even tearing of the paper bands, thereby limiting the production pace.

A strip hanging in a vertical airflow is observed to vibrate when the flow velocity is raised above a critical value. This system was first studied by Datta and Gottenberg (1975) who developed a model based on potential flow theory and conducted experiments with Mylar strips.

A similar problem is that of a hanging pipe conveying a water flow, see Païdoussis (1998) for an extensive review. Recently, Doaré and de Langre (2002) showed that an asymptotic régime exists for long pipes, where the characteristics of the instabilities do not depend on the pipe length. The corresponding transition length could be derived by considering the local stability of bending waves.

The aim of the present paper is to conduct for hanging ribbons an analysis similar to that of Doaré and de Langre (2002) for pipes. In section 2, the experimental results are given. They are analysed in section 3 using a potential flow model proposed by Datta. The particular case of long ribbons is addressed in section 4.

2. EXPERIMENTS

We test four different materials: paper, Mylar, fabric and silk. In order to investigate the effect of the width, three different ribbon widths B are selected for each material: 20 mm, 30 mm and 55 mm. Hence, a set of twelve different ribbons is tested. The paper is classical printer paper, the fabric is from cotton sheets, the Mylar is a polyester film used as dielectrics and the silk is taken from an advertising streamer. The flexural rigidity D of each ribbon is measured through the buckling height h_c (minimum height for which a vertical strip, clamped at its bottom, buckles under its own weight), Doubrère (2001)

$$D = \frac{h_c^3 mg}{7.83} \tag{1}$$

where g denotes gravity and m is the mass per unit area. The four materials have very different characteristics, as detailed in Table 1.

	m (g/m^2)	h_c (mm)	D (kg.m^2/s^2)
Paper	79,0	130	220.10^{-6}
Mylar	51,6	72	24.10^{-6}
Fabric	216,5	40	17.10^{-6}
Silk	63,0	26	1,4.10^{-6}

Table 1: *Ribbon characteristics: m, mass per unit area; h_c, buckling height; D, flexural rigidity.*

A strip of length L is hanged in a vertical wind tunnel. The flow velocity is progressively raised until a critical velocity U$_c$ is reached and steady vibrations are observed, Figure 2. To analyse the dependence of U_c on L, we vary the length of the strip by progressively shortening it.

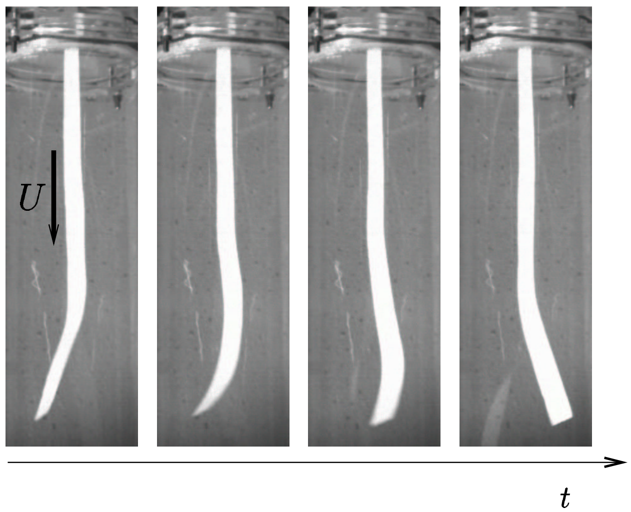

Table 2: Flutter of a hanging paper strip over a half period.

The critical flow velocity is plotted in Figures 1 and 2 as a function of the ribbon length for each material and for each width. In all cases, two regimes are observed. For short ribbons, the critical velocity depends strongly on the length, whereas for long ribbons the critical velocity is weakly dependent on the length. This is very pronounced in the case of paper and Mylar, Figure 1, but is less obvious in the case of fabric and silk, Figure 2. These evolutions are very similar to those observed for fluid-conveying pipes (Doaré and de Langre, 2002).

3. MODEL

The Reynolds number based on the wind tunnel diameter is $Re_d \approx 70000$ ($U = 5$ m/s, characteristic flow velocity and $d = 19.4$ cm, diameter of the test section). The critical Reynolds number for transition in Hagen-Poiseuille flow being 2100 (Bird et al, 1960), the flow is fully turbulent. Here the experimental set-up (fine grid, honeycomb and convergent at the inlet) provides a low turbulence level, of less than to 0.1% so that we assume the flow to be uniform and steady.

The ribbons have very small thickness, between 0.05 and 0.6 mm, compared to their length, 5 to 50 cm. Thus, they can be considered as slender bodies.

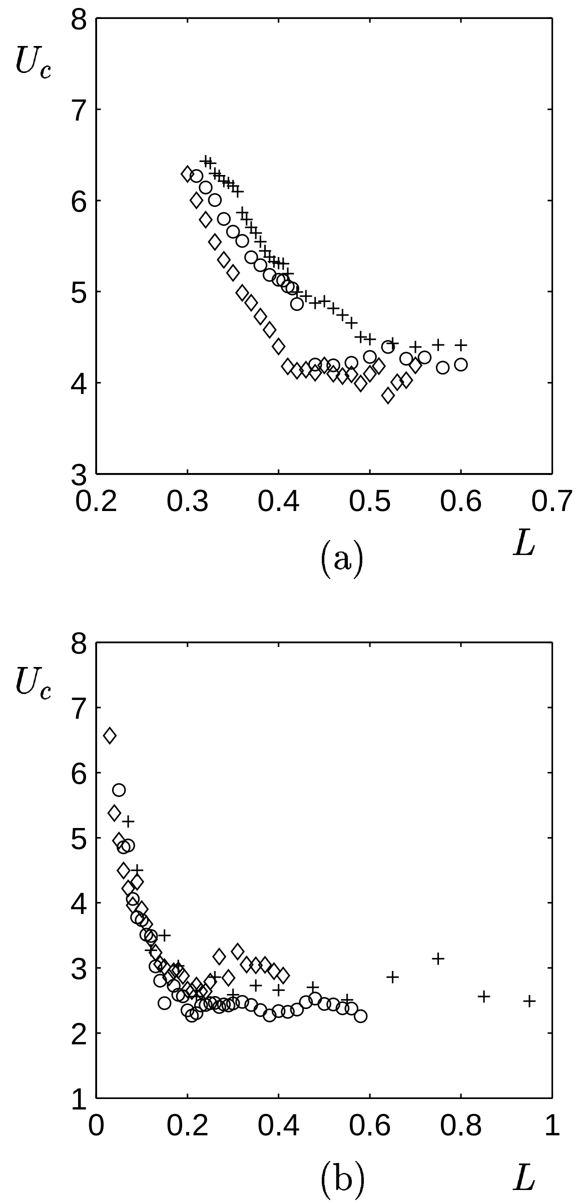

Figure 1: Effect of ribbon length on the critical velocity for flutter: (a) paper and (b) Mylar. Ribbon width: (◇) 20 mm, (+) 30 mm and (o) 55 mm.

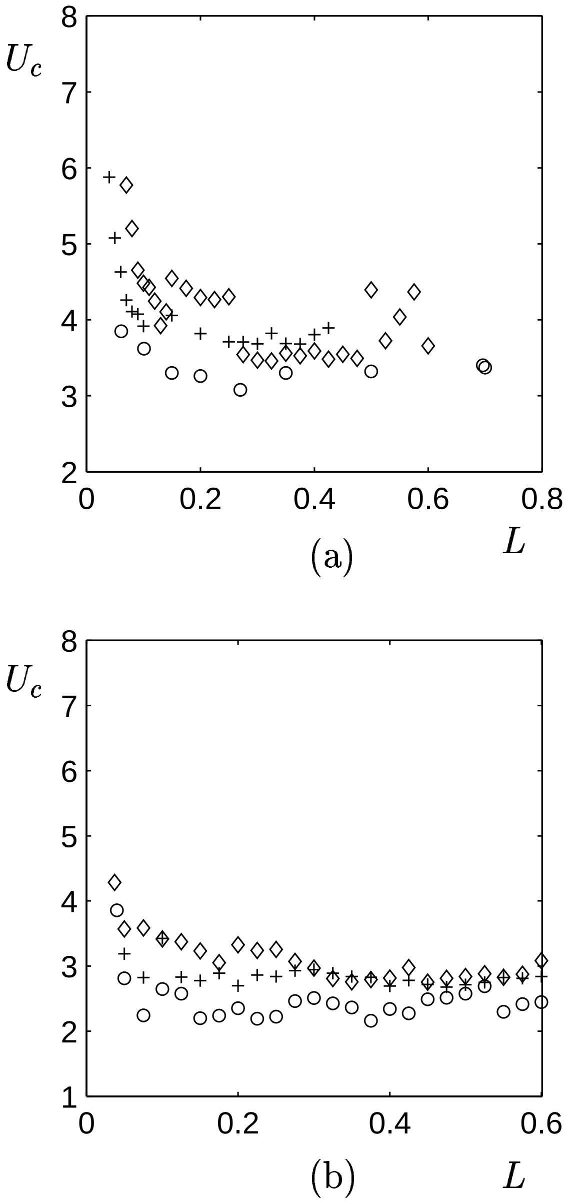

Figure 2: Effect of ribbon length on the critical velocity for flutter: (a) fabric and (b) silk. Ribbon width: (⋄) 20 mm, (+) 30 mm and (o) 55 mm.

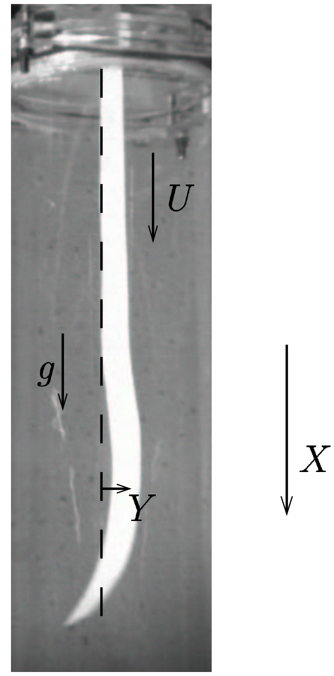

Figure 3: Ribbon hanging in a vertical airstream

We use an unsteady potential flow theory to derive fluid action (Païdoussis, 2003).

Under these conditions, the equation governing the lateral deflection of the strip, Figure 3, as proposed by Datta, reads

$$D\frac{\partial^4 Y}{\partial X^4} - \frac{\partial}{\partial X}\left[\left(mg(L-X) - MU^2\right)\frac{\partial Y}{\partial X}\right] + 2MU\frac{\partial^2 Y}{\partial X \partial \tau} + (m+M)\frac{\partial^2 Y}{\partial \tau^2} = 0, \quad (2)$$

where Y is the deflection of the strip, τ is time, g is gravity, L is the total length of the strip, U is the flow velocity and M is the added mass due to the presence of the fluid.

For a unit area of strip, the added mass is taken as $M = \pi\rho B/4$, which models an infinitely long rigid plate undergoing pure translation. The added mass in air is not negligible here, being of the same order as the ribbon mass.

Note that for typical values of the parameters, such as $U = 4$ m/s, $L = 0,3$ m and a flutter frequency of $f = 5$ Hz, the reduced velocity is $U_R = 2.7$. Hence, all the terms in equation (2) are of similar order of magnitude and none of them can be neglected.

The boundary conditions associated to equation (2) are

$$Y(0)=\frac{\partial Y}{\partial X}(0)=0 \quad \text{and}$$
$$\frac{\partial^2 Y}{\partial X^2}(L)=\frac{\partial^3 Y}{\partial X^3}(L)=0\ , \tag{3}$$

respectively for the clamped top and for the free extremity at the bottom.

Following Doaré and de Langre (2002), we define now dimensionless variables using $\eta = (D/mg)^{1/3}$ as the characteristic length

$$x=\frac{X}{\eta}\ ; \qquad y=\frac{Y}{\eta}\ ;$$
$$t=\left[\frac{mg}{\eta\,(M+m)}\right]^{1/2}\tau\ ; \qquad v=\left[\frac{M}{\eta mg}\right]^{1/2}U\ ;$$
$$\beta=\frac{M}{M+m}\ ; \qquad \ell=\frac{L}{\eta}\ .$$

Equation (2) then becomes

$$\frac{\partial^4 y}{\partial x^4}-\frac{\partial}{\partial x}\left[(\ell-x)\frac{\partial y}{\partial x}\right]+v^2\frac{\partial^2 y}{\partial x^2}$$
$$+2\sqrt{\beta}v\frac{\partial^2 y}{\partial x\partial t}+\frac{\partial^2 y}{\partial t^2}=0, \tag{4}$$

with clamped boundary conditions ($y'(0) = y''(0) = 0$) at the top and free conditions at the bottom end ($y''(\ell) = y'''(\ell) = 0$).

This equation is identical to the one used by Doaré in the case of a fluid-conveying pipe.

Material	Width B	Mass parameter β
Paper	20 mm	0.19
	30 mm	0.26
	54 mm	0.40
Mylar	20 mm	0.27
	30 mm	0.35
	54 mm	0.50
Fabric	20 mm	0.08
	30 mm	0.12
	55 mm	0.19
Silk	20 mm	0.23
	30 mm	0.31
	55 mm	0.45

Table 3: *Mass parameter varying according to the material and the ribbon width.*

For a given flow velocity v, the strip is unstable if one of the eigenmodes of this system has a negative damping. The governing parameters are ℓ, the reduced ribbon length, v, the dimensionless flow velocity and β, the mass ratio. The critical velocity v_c is the lowest value of v such that an unstable mode exists. This critical velocity depends on two parameter, ℓ and β. As in Doaré and de Langre (2002), we derive the characteristics of the eigenmodes of this equation using a Galerkin approximation based on the eigenmodes without flow nor gravity. This is done for the twelve different values of β, Table 3, as a function of the dimensionless length ℓ.

In Figures 4 and 5 we compare four typical results of the model with experimental data. On all cases the dependence of the critical velocity with length is similar to that observed in the experiments: (a) for short ribbons the dependence is strong (b) for long ribbons the dependence is weak.

The order of magnitude of the critical velocity is well recovered by the model.

4. THE LONG STRIP CASE

Both the experiments and the model show low length dependence of the critical velocity for long strips. We may therefore define for each ribbon an asymptotic critical velocity, v_c^∞.

In the model this velocity only depends on the mass ratio β. We compare the experimental values of this asymptotic velocity with those derived from the model, for all values of β given in Table 3. This is shown in Figure 6.

The monotonic evolution of v_c^∞ given by the model is seen to be a good approximation of the experimental critical velocities. Note that in the experiments β is varied either by changing the ribbon mass or by changing the added mass through the ribbon width. This results in non-monotonic evolutions of the experimental values of v_c^∞, indicating that β is not the only parameter of the system. This is clearly the limit of Datta's model.

5. CONCLUSION

We carried out experiments for twelve different ribbons hanging in axial airflow. The tests confirm the expected likenesses of behavior between the hanging ribbon and the hanging pipe: the critical velocity for short ribbons depends strongly on the length whereas the critical velocity depends weakly on the length for longer

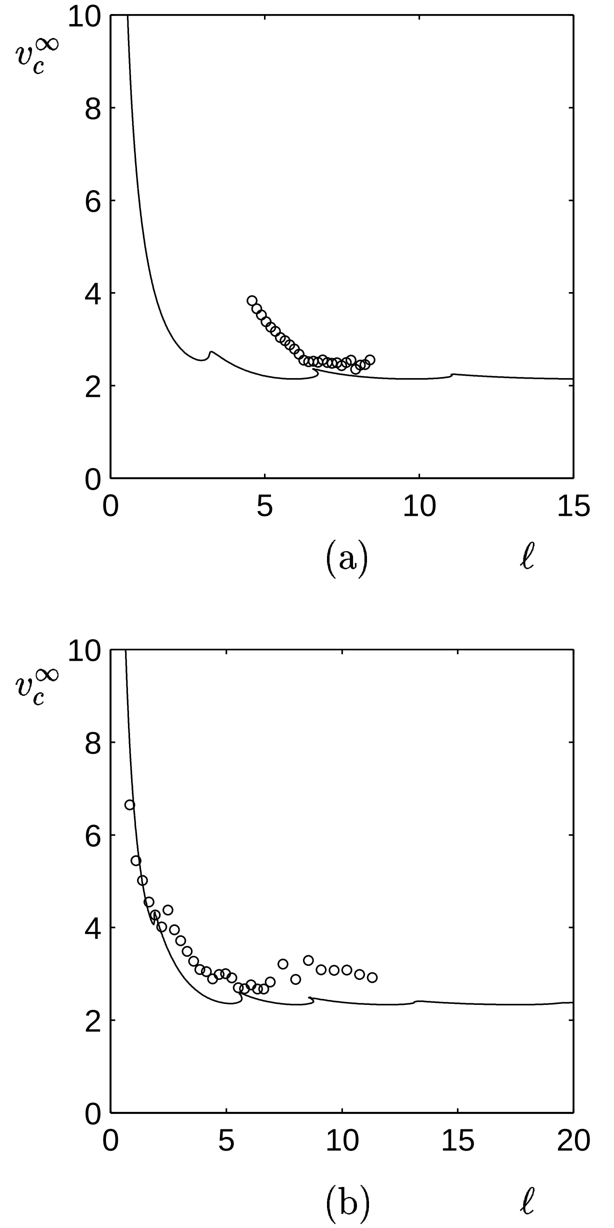

Figure 4: Effect of length on the critical velocity. (-) Computations, (o) Experiments; (a) Paper, B=20 mm; (b) Mylar, B=20 mm.

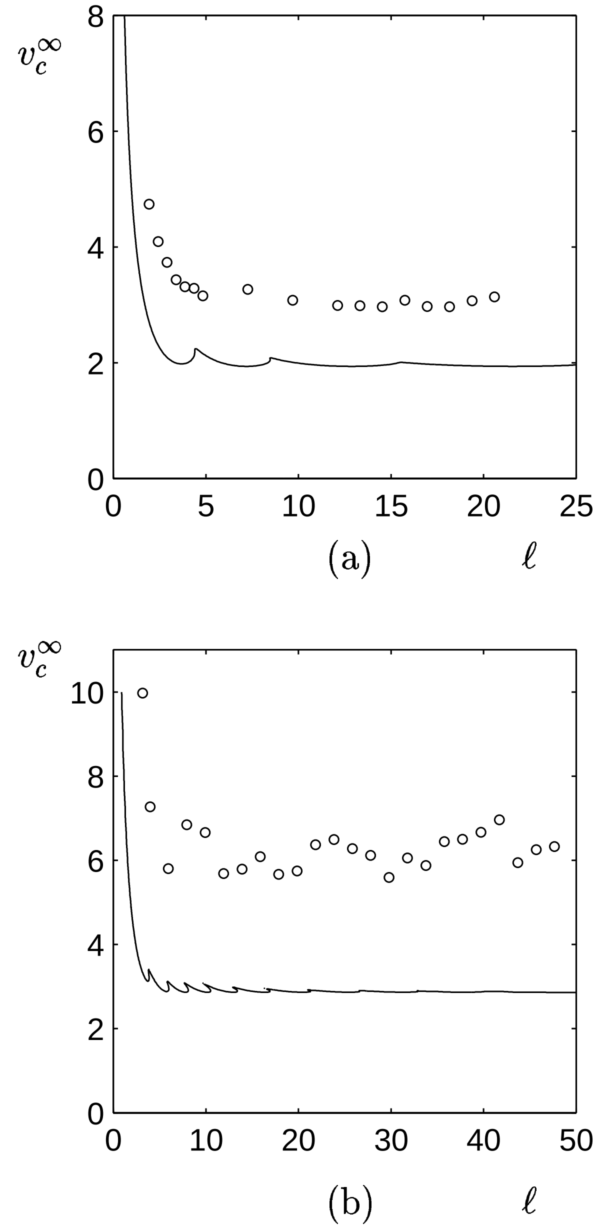

Figure 5: Effect of length on the critical velocity. (-) Computations, (o) Experiments; (a) Fabric, B=30 mm; (b) Silk, B=55 mm.

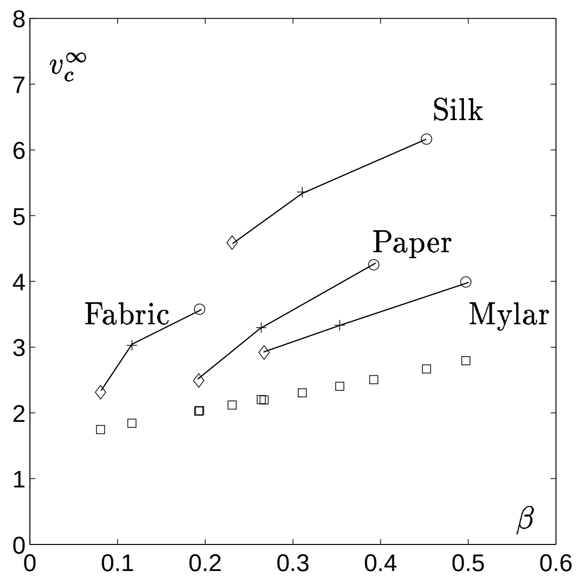

Figure 6: Asymptotic critical velocity dependence on the mass ratio. (□) Model, equation (4); (o) B=54 mm, (+) B=30 mm, (◇) B=20 mm.

ribbons.

The experimental conditions make Datta's model for airswept hanging strips relevant in our case. We proposed a non-dimensionalisation for these equations that leads to the same equation as Doaré's for a hanging fluid conveying pipe. The mass parameter β appears as the variable characteristic of the strip.

The model predictions are in good agreement with the experimental data. At low length, it predicts a critical velocity strongly dependent on the length; at high length, it predicts a critical velocity almost constant with the length.

The high length behavior allows to define an asymptotic velocity v_c^∞ for each ribbon. The model predicts that v_c^∞ is an increasing function of the mass parameter only. The experiments confirm this result as long as ribbons of the same material are concerned. The model is limited however to compare high length behavior of strips made of different materials. Another parameter, material-dependent like internal friction, might also be taken into account in the model. This is currently being done.

6. REFERENCES

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