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The transient temporal response of a flexible bridge deck subjected to a single gust

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ABSTRACT

Temporal simulations are increasingly performed in wind effects analysis of flexible structures. By comparison with classical techniques such as spectral methods, temporal simulations provide the advantage of easily combining different kinds of loads, can take nonlinearities into account and also provide the only way to reproduce transient behaviors. In that context this study deals with the transient response of a two-degrees-of-freedom streamlined bridge deck section subjected to a single gust. Experimental evidence of the potentially high level of transient energy amplification due to that kind of extraneous excitation have been recently demonstrated for an airfoil section and for a streamlined bridge deck section, below the critical coupled-mode flutter wind speed. The present study then focuses on the validation of a time-dependent model, based on a simple formulation of both the motion-dependent and the buffeting forces, for catching that kind of transient behavior. A parametric study is also made in order to highlight the impact of the pitch–plunge frequency ratio on the energy amplification below the critical condition.
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1. Introduction

Wind-induced responses of flexible structures encountered in civil engineering are traditionally studied using frequency domain approaches. Based on linear formulations of motion-dependent and buffeting loadings, spectral methods are generally sufficient for catching the critical parameters for the onset of flutter or calculating the variance associated with the dynamical response to a stationary turbulent wind [1,2].

Meanwhile, time domain analysis of wind effects on structures has been increasingly performed in recent years. By comparison with spectral methods, temporal simulations provide the advantage of easily combining different kinds of loads, can take structural and/or aerodynamic nonlinearities into account and also provide the only way to reproduce transient behaviors. A time domain approach has been successfully used in [3] for the analysis of flutter and buffeting responses of bridges. Effects of turbulence and aerodynamic nonlinearity have been pointed out in [4]. More recently, Costa et al. [5] proposed a wind–bridge interaction study using a time domain approach.

In the present paper we focus on a new type of short-term instability that has been recently highlighted in the field of fluid-structure interactions: the transient growth response to an initial perturbation before coupled-mode flutter. Theoretically studied in [6], this short-term energy growth can lead to substantial amplitude motion, even in a stable dynamical system, due to the non-orthogonal modes involved in the system. It is strongly dependent on the initial conditions.

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Table 1
Structural parameters of the two different bridge deck sections studied.

	f_z/f_α	f_α (Hz)	f_z (Hz)	k_α (N m/rad)	k_z (N/m)	J_O (kg m^2)	m (kg)	η_α (%)	η_z (%)
Case A	0.62	7.12	4.43	1.33	519.36	6.64 e^{-4}	0.66	0.3	0.08
Case B	0.44	8.00	3.56	1.67	309.16	6.61 e^{-4}	0.62	0.24	0.07

Experimental evidence of transient growth of the energy before coupled-mode flutter has been recently demonstrated for an airfoil section in [7] and a streamlined bridge deck section [8,9] subjected to mechanical or gust perturbation. Those results have shown that short-term energy amplification of the initial energy can reach a factor of more than 5 and can even trigger flutter instability in the case of nonlinear structures [10].

In that context, the present study deals with the transient response of a two-degrees-of-freedom bridge deck section subjected to a single gust. A rigid bridge deck section is flexibly mounted in heave and pitch in a steady air flow. The velocity is maintained under the critical condition, i.e. below the coupled-mode flutter critical wind speed. A superimposed single gust produces an initial excitation. The experimental setup and parameters are first detailed. Then a time-dependent model, based on a simple formulation of both the motion-dependent and the buffeting forces is presented. Computed results are compared with the experimental results. Finally, a numerical parametric study is done in order to highlight the impact of the pitch–plunge frequency ratio on the energy amplification below the critical flutter wind speed.

2. Transient wind tunnel tests

2.1. The experimental setup and identification of parameters

The main points of the experimental setup are recalled here. Further details concerning the experimental procedure can be found in previous work [8,9]. The bridge deck section is flexibly mounted in heave and pitch in a closed wind tunnel with the setup shown in Fig. 1. The two degrees of freedom $z(t)$ and $\alpha(t)$ are measured using laser displacement sensors connected to an acquisition system.

The elastic center of the deck section model is located at its gravity center, i.e. the mid-chord. The equations of motion for this structurally non-coupled two-degrees-of-freedom system can then be expressed as follows [11]:

$$\begin{aligned} & m\ddot{z} + 2m\eta_z\omega_z\dot{z} + k_z z = F_z, \\ & J_O\ddot{\alpha} + 2J_O\eta_\alpha\omega_\alpha\dot{\alpha} + k_\alpha\alpha = M_O. \end{aligned} \tag{1}$$

Assuming that the structural damping is small, the eigenvalues can be written in the form

$$\lambda_\alpha = \omega_\alpha^2 = (2\pi f_\alpha)^2 = k_\alpha/J_O; \qquad \lambda_z = \omega_z^2 = (2\pi f_z)^2 = k_z/m. \tag{2}$$

Structural parameters are identified for each degree of freedom under zero wind velocity. Both the natural frequencies f_z and f_α are obtained by spectral analysis. A static weight calibration technique is used to assess the stiffnesses k_z and k_α. The inertia J_O and mass m are then deduced, using

$$m = k_z/\lambda_z; \qquad J_O = k_\alpha/\lambda_\alpha. \tag{3}$$

Pure structural damping values η_z and η_α are determined using a standard decrement technique in free-decay tests under the zero-wind condition.

In the present study two different cases characterized by two different frequency ratios f_z/f_α between the heaving and pitching motions are tested. The structural parameters are summarized in Table 1.

The gust is produced by a flap mounted upstream from the test section. It is pre-tensioned with a spring and suddenly released. A typical time history of the perturbation of the flow velocity is plotted in Fig. 2, where $\bar{U}$ is the mean velocity, $u(t)$ and $w(t)$ being the longitudinal and vertical perturbations respectively. The flap generates a transient short impulse in the wind velocity, leading to a unique peak in the longitudinal component u, and two opposite peaks in the vertical component w. The time duration of this perturbation is about 0.05 s, which is three times below the typical period of the system.

2.2. Transient results

Because of the aerodynamic loading, a two-degrees-of-freedom bridge deck section can experience coupled-mode flutter instability above a critical velocity U_c. For both Case A and Case B the critical wind speed has been experimentally measured, at respectively 16.1 and 21.3 m/s. The responses of both the section model subjected to mechanical excitation (a sudden release of an initial pitch angle) or gust excitation (produced by the upwind flap) are then studied for different mean velocities below the critical condition.

For each test, the mechanical energy of the system, defined as the sum of the kinetic energy and the potential energy, is computed from the measurements of z and α such that

$$E(t) = \frac{1}{2}m\dot{z}^2(t) + \frac{1}{2}J_O\dot{\alpha}^2(t) + \frac{1}{2}k_z z^2(t) + \frac{1}{2}k_\alpha\alpha^2(t). \tag{4}$$

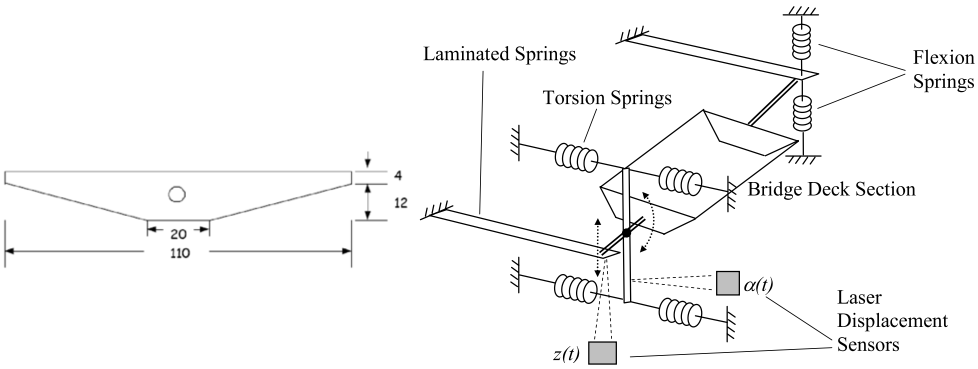

Fig. 1. Bridge deck cross section and experimental setup schematics, with dimensions in mm.

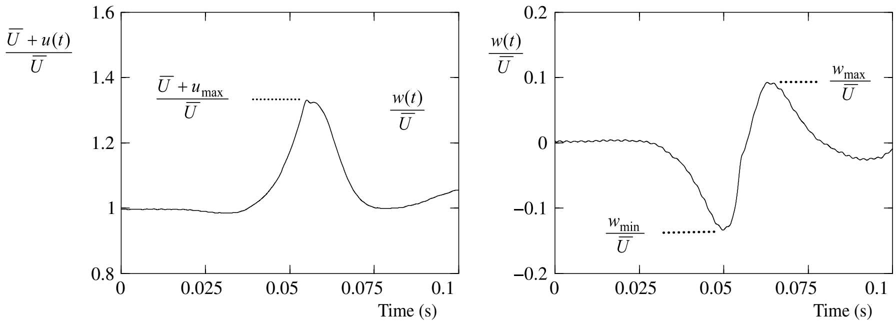

Fig. 2. Sample of upstream velocity perturbation measured with a 2D hot wires probe.

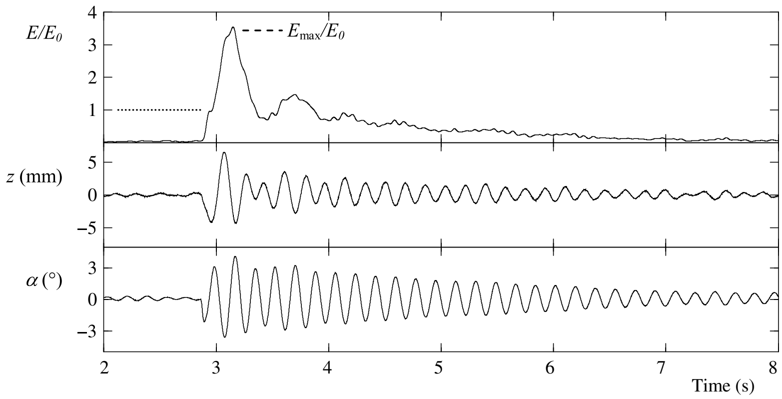

Fig. 3. Typical transient response of the deck to a gust excitation, $\bar{U}/U_c = 0.91$; Case A.

A typical result of the transient response is given in Fig. 3 for the Case A subjected to gust excitation. Before the critical condition, the dynamical response is characterized by initial amplification of energy followed by a monotonic decay due to the asymptotic stability of the system. In Fig. 3, the mechanical energy is normalized using E_0, the initial energy of the system. For initial mechanical excitation, E_0 is simply computed as the potential energy that is produced by the imposed initial pitch angle. For gust excitation, the initial energy E_0 transmitted to the system can be measured at the distinct local peak on the total energy response occurring before the transient growth amplification due to the dynamics of the system. This gust signature which can be viewed as the gust energy that is immediately transmitted to the deck is clearly noticeable in Fig. 3 at a time close to 2.9 s. One can also notice that this local peak occurs when the deck reaches approximately the first minimum in the plunge, while it reaches approximately the maximum velocity in the pitch. Experimental values of this initial energy E_0 are reported in Fig. 4 for both Case A and Case B and various wind velocities. These results show that the evolution of E_0 overlaps in the common velocity region between Case A and Case B, indicating that this reference energy of the gust impulse is independent of the structure. More details about the experimental procedure for the evaluation of E_0 can be found in [9].

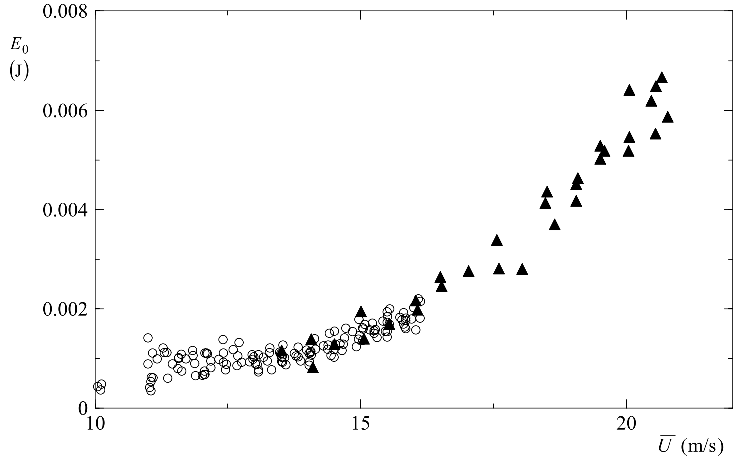

Fig. 4. Initial energy transmitted by the gust versus velocity. Case A: ◯; Case B: ▲.

3. Numerical simulations with time-dependent models

3.1. Time-dependent models

Assuming the validity of linear superposition, the time-dependent model is expressed as the sum of motion-dependent (m) and buffeting (b) loads:

$$F_z(t) = F_z^m(t) + F_z^b(t) \tag{5}$$

$$M_O(t) = M_O^m(t) + M_O^b(t). \tag{6}$$

The motion-dependent sectional forces, per unit span, can be expressed as

$$F_z^m(t) = \frac{1}{2}\rho B\bar{U}^2\left[H_1\frac{\dot{z}(t)}{\overline{U}} + H_2\frac{B\dot{\alpha}(t)}{\overline{U}} + H_3\alpha(t) + H_4\frac{z(t)}{B}\right] \tag{7}$$

$$M_O^m(t) = \frac{1}{2}\rho B^2\bar{U}^2\left[A_1\frac{\dot{z}(t)}{\overline{U}} + A_2\frac{B\dot{\alpha}(t)}{\overline{U}} + A_3\alpha(t) + A_4\frac{z(t)}{B}\right]. \tag{8}$$

This motion-dependent linear formulation, originally proposed in [12] for a bridge deck undergoing small amplitude harmonic motions, introduces B, the deck section length, ρ, the air density, $\overline{U}$, the mean velocity, and flutter derivative coefficients H_i and A_i, which depend on the reduced angular frequency $K = 2\pi fB/\bar{U}$.

The flutter derivatives are written using the unsteady airfoil theory [11] adapted to the case of a bridge deck which has its rotation center and its gravity center both located at mid-chord [13]:

$$H_1 = -C_z'F, \qquad H_2 = C_z'\left[\frac{F}{4} + \frac{G}{K}\right] + \frac{C_z'}{4}, \qquad H_3 = C_z'\left[F - \frac{KG}{4}\right], \qquad H_4 = C_z'\left[KG + \frac{K^2}{4}\right]. \tag{9}$$

$$A_1 = -C_M'F, \qquad A_2 = C_M'\left[\frac{F}{4} + \frac{G}{K}\right] - \frac{C_z'}{16}, \qquad A_3 = C_M'\left[F - \frac{KG}{4}\right] + C_z'\frac{K^2}{128}, \qquad A_4 = -C_M'KG. \tag{10}$$

F and G which also depend on K are respectively the real and imaginary parts of Theodorsen's function [14]. Those expressions also introduce the static lift slope C_z' and pitching moment slope C_M' of the deck section at small angles of attack. Both have been experimentally identified in wind tunnels in the appropriate Reynolds number range. With $C_z' \approx 5.65$ and $C_M' \approx 1.8$ the aerodynamic center of the deck section is found to be at 32% of the chord.

A transient formulation based on Küssner's function is used for the buffeting terms [11]:

$$F_z^b(t) = \frac{1}{2}\rho B\bar{U}^2 C_z'\Psi(\tau) \tag{11}$$

$$M_O^b(t) = \frac{1}{2}\rho B^2\bar{U}^2 C_M'\Psi(\tau) \tag{12}$$

where the transient function Ψ of the non-dimensional time $\tau = 2\,\bar{U}t/B$ is calculated using Duhamel's integral such that

$$\Psi(\tau) = \varphi(0)\frac{w(\tau)}{\bar{U}} + \int_0^\tau \frac{w(\sigma)}{\bar{U}}\varphi'(\tau - \sigma)\,d\sigma \tag{13}$$

$$\varphi(\tau) = 1 - 0.5\exp(-0.13\tau) - 0.5\exp(-\tau). \tag{14}$$

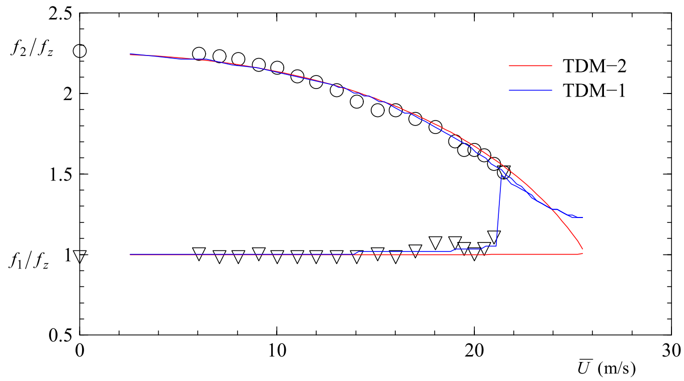

Fig. 5. Evolution of frequencies versus wind velocity. Case B, $U_c = 21.5$ m/s.

The gust vertical component $w(t)$ is fitted from the experiments with two Gaussian distribution functions, with $w_{\min}$, $w_{\max}$ and the time scale of the gust as parameters. Küssner's function φ is approximated with the expression of Jones [11] for an elliptic airfoil. In this model the effect of the longitudinal component of the gust $u(t)$ is neglected. Indeed it induces an additional term proportional to the mean lift coefficient (or pitching moment) at zero angle of attack, which is relatively small.

In the next section, computed solutions will be compared to experimental results using two alternative versions of the time-dependent formulation of the motion-dependent forces. In the first one, named TDM-1, the full model expressed in Eqs. (7)–(8) will be used. The second one is a simplified version, named TDM-2, which neglects aerodynamic damping: coefficients H_1, H_2, A_1 and A_2 are then set to zero.

Temporal simulations of the problem are performed with a Newmark algorithm which has no numerical damping that could corrupt the solution. Computed time histories of z and α are then processed for calculating the total mechanical energy following Eq. (4) as in the experimental procedure.

3.2. Results and comparison with experiments

Due to the aerodynamic forces, the frequency of both the modes of the system might be dependent on the wind velocity. For a bridge deck section characterized by an aerodynamic center located among the elastic centers and a frequency ratio f_z/f_α less than 1, a coupled-mode flutter involving a frequency merging can occur. The frequency evolution of the two modes versus the wind velocity has been experimentally measured using Fourier analysis of the heaving and pitching response of the system undergoing free-decay tests. Results obtained for Case B are compared in Fig. 5 with computations using full (TDM-1) and no-damping (TDM-2) time-dependent formulations of the motion-dependent forces.

At low velocities, both models give a good evolution of the system dynamics with a decrease of the pitching frequency, while the heaving one remains almost constant. However when the velocity approaches the critical flutter velocity, the model neglecting the aerodynamic damping fails to predict the sudden merging of frequencies due to a sharp increase of the heaving mode. On the other hand, the full aeroelastic model TDM-1 fits the experimental results very well and predicts a critical velocity in accordance with the experiments. It is well known that damping terms can exert a substantial effect on the long-term stability of a dynamical system. In that context those results clearly demonstrate that, for the prediction of the onset of coupled-mode flutter, a full model of the aeroelastic effects is indispensable.

Using the two motion-dependent models alternatively, computed responses to initial mechanical excitation are now compared against the experiments. The test procedure used is as follows: for a wind velocity under the critical condition a negative pitch angle $\alpha_0 = -2.2°$ is imposed and the deck is suddenly released. Experimental and computed maximum energy amplifications are plotted in Fig. 6 versus the wind velocity ratio $\overline{U}/U_c$. The model that catches the dynamics of this short-time instability is now TDM-2, which neglects aerodynamic damping. Indeed the full TDM-1 model underpredicts the energy amplification while the TDM-2 model fits very well with the experiments.

Motion-dependent models are now used to predict the transient behavior due to a single gust excitation, buffeting loads induced by the gust being introduced in our time-dependent formulation using Eqs. (11)–(14). Examples of the time histories of the energy, pitch angle and heaving position are shown in Fig. 7. One can clearly observe that the full model TDM-1 still fails to reproduce the transient dynamics of the system. Looking at the long-time dynamical response of the system one can notice that this full aeroelastic model follows the energy profile but overdamped it, as for the response in both the heaving and pitching motions. This demonstrates that the aerodynamic damping terms based on Theodorsen's airfoil formulation are overestimated. Among the four aerodynamic damping terms, the pure terms H_1 and A_2 are those which most strongly affect the damping in heave and pitch. They can be used to calculate added aerodynamic damping coefficients that can be

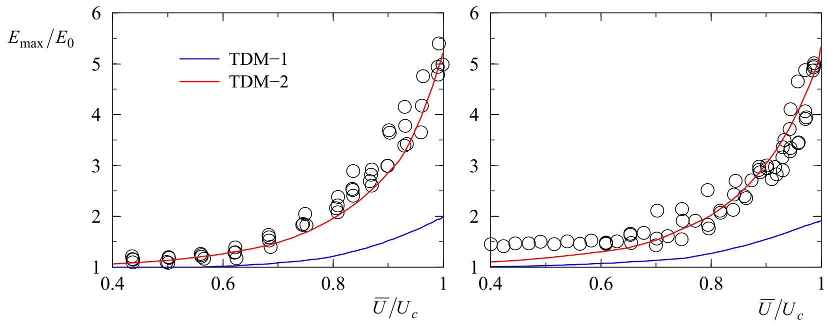

Fig. 6. Maximum energy amplification versus velocity ratio. Mechanical excitation $\alpha_0 = -2.2°$. Case A: left; Case B: right.

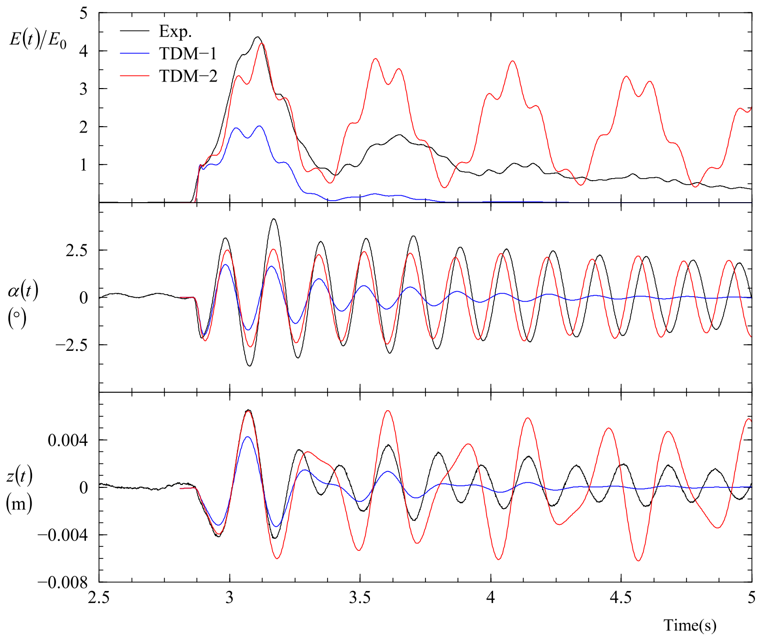

Fig. 7. Time evolution of the energy, angular displacement and vertical displacement. Excitation by gust, $\bar{U}/U_c = 0.96$; Case B.

compared to the pure structural damping of the system using the following expressions:

$$\eta_z^{ae} = \frac{1}{4m\omega_z}\rho\overline{U}BH_1 \qquad \eta_\alpha^{ae} = \frac{1}{4J_0\omega_\alpha}\rho\overline{U}B^3A_2. \tag{15}$$

Using the flutter derivative formulation expressed in Eqs. (9) and (10) one finds, close to the critical velocity, $\eta_z^{ae} \approx 2.5\%$ and $\eta_\alpha^{ae} \approx 10\%$. Those values, forty times higher that the pure structural damping coefficients η_z and η_α, explain the strong effect of the aerodynamic damping terms, between the short-term and long-term behaviors. But while an inaccurate evaluation of those aerodynamic damping coefficients seems to have an insignificant effect on the frequency evolution of the system, as with TDM-1, this is not the case for the transient growth behavior. Indeed, as shown in Fig. 7, the early-time dynamics due to the transient growth of the energy is well simulated by the simple model TDM-2, leading to a correct evaluation of the maximum energy amplification, while TDM-1 overdamps the response.

These results demonstrate that the motion-induced flow mechanisms generating aerodynamic damping are either not effective or have an insignificant effect on the transient regime. On the other hand the TDM-2 model overestimates the long-time dynamical response. In that context and in order to accurately predict the long-time dynamics of the system, a better evaluation of the flow-induced damping terms seems then necessary. A bridge deck section, even streamlined, has a negative mean lift force which is induced by a flow which is indeed different from the one around a zero-lift thin airfoil which is used in Theodorsen theory.

In the present paper we are mainly focused on the transient behavior of the system, seeking the simplest model capable of reproducing the physics. The efficiency of the TDM-2 model for predicting the short-time amplification due to the transient growth of the energy is confirmed in Fig. 8, which presents the experimental and computed maximum energy amplification versus wind velocity ratio for Case A and Case B.

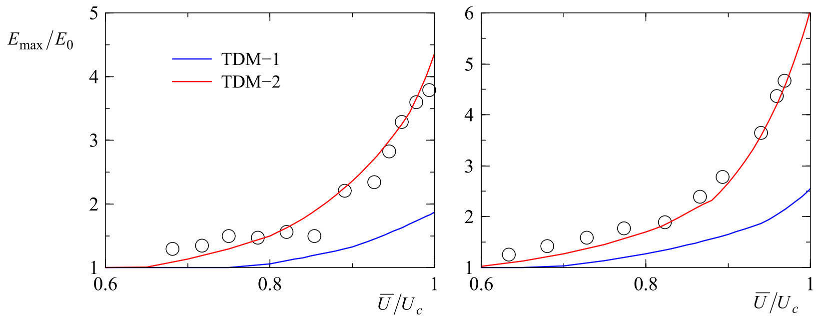

Fig. 8. Maximum energy amplification versus velocity ratio; excitation by gust. Case A: left; Case B: right.

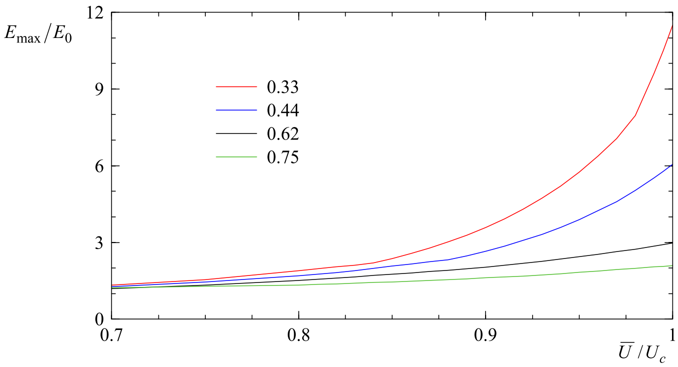

Fig. 9. Effect of the frequency ratio on the maximum energy amplification versus velocity ratio.

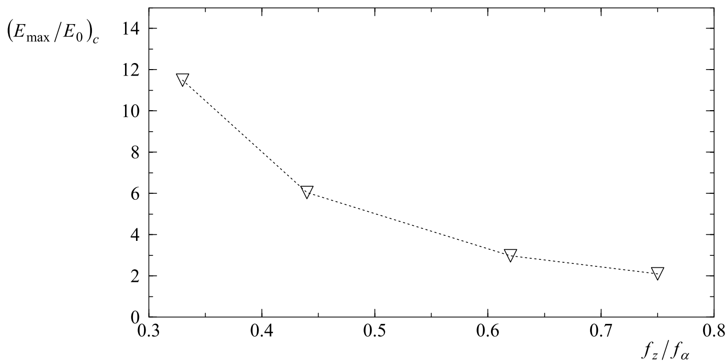

Fig. 10. Influence of the frequency ratio on the maximum energy amplification at critical velocity.

3.3. A numerical study of the impact of the frequency ratio

In this section a short parametric study is performed to highlight the impact of the frequency ratio between heaving and pitching modes f_z/f_α. With Case B as a reference, three other ratios have been built, changing the structural stiffness and thus the heaving natural frequency only. For all those frequency ratios the coupled-mode flutter velocity is first computed using the full motion-dependent model (TDM-1) validated in Section 3.2. The numerical model TDM-2, which provides the best results in the transient regime, is then used to study the transient response to a single gust excitation.

The maximum energy amplification versus velocity ratio is given in Fig. 9 for $f_z/f_\alpha = 0.33$, 0.44 (Case B), 0.62 ($\approx$ Case A) and 0.75. The associated critical velocities are $U_c = 22.6$, 21.5, 18.65 and 15.6 m/s respectively. It is then interesting to notice that the lower frequency ratio, which is *a priori* the most "secure" one in terms of coupled-mode flutter sensitivity, leads to the highest amplification of energy. This trend is shown more clearly in Fig. 10 where the maximum energy amplification just below the critical velocity, denoted as $(E_{max}/E_0)_c$, is plotted versus the frequency ratio. On the other hand one cannot ignore the fact that the critical velocity decreases with the frequency ratio and, as reported in Fig. 4, the initial energy E_0 transmitted by the gust also increases with the velocity. In that context, as seen in Fig. 11, whatever the frequency ratio, the maximum energy which is reached during the transient response remains almost the same for a given velocity $\bar{U}$.

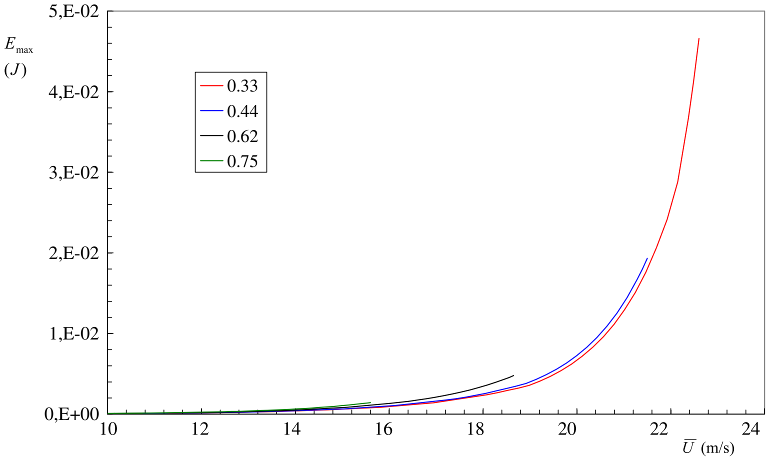

Fig. 11. Effect of the frequency ratio on the maximum energy versus the velocity.

4. Conclusions

Wind tunnel experiments have shown that a two-degrees-of-freedom bridge deck section subjected to a single gust can exhibit transient growth of energy before the coupled-mode flutter condition. Time-dependent models based on classical formulations of both the motion-dependent and the buffeting forces have been tried to reproduce this transient behavior. The buffeting term, expressed in terms of a convolution integral using a Küssner function based indicial function has been successfully used to compute the gust excitation.

For transient response analysis, numerical results were found to be in good agreement with the experiments using a motion-dependent formulation of the aerodynamic forces neglecting the aerodynamic damping terms. On the other hand, a full aeroelastic model remains indispensable for correctly evaluating the coupled-mode flutter critical velocity.

A short numerical parametric study has been made in order to highlight the impact of the pitch–plunge frequency ratio on the energy amplification below the critical flutter wind speed. Although the lower frequency ratio leads to the highest amplification of energy, before the critical conditions, the maximum energy which is reached during the transient response was found to remain quasi-constant for a given mean velocity $\bar{U}$.

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