

# Wind induced vibrations of chimneys using an improved quasi-steady theory for galloping

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## Abstract

Aeroelastic phenomena play a very significant part in the design of large civil engineering structures. This paper deals with the interference galloping occurring on in-line low rise 3D-shaped chimneys. The quasi-steady and 2D assumptions are not well adapted to this problem. A weak quasi-steady model is introduced to improve the reliability of predictions. The originality of this work lies principally in its physical interpretations and in the calculation of the time lag between aeroelastic forces and structural deflection. The 3D nature of the problem is taken into account by using local pressure coefficients from wind tunnel tests. Some typical results are presented for cases where a critical wind velocity appears. © 1998 Elsevier Science Ltd. All rights reserved.

*Keywords:* Aeroelasticity; Galloping; Chimneys; Quasi-steady; Non-linear vibrations; Aerodynamics

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## 1. Introduction

It is now well known that aeroelastic phenomena need to be studied very early in the design of large civil engineering structures. In-line chimneys are typically a good example of this due to their high sensitivity to interference galloping.

In such cases, oscillations may reach relatively large amplitudes and a non-linear analysis is required to obtain reliable predictions. Moreover, the quasi-steady assumption does not correspond to a realistic energy exchange between the fluid and the structure so that predictive calculations need to be improved for industrial design [1]. In addition, low rise shapes with varying sections cannot be studied correctly with a pure 2D approach such as the classical Den Hartog criterion [2]. In the past, such

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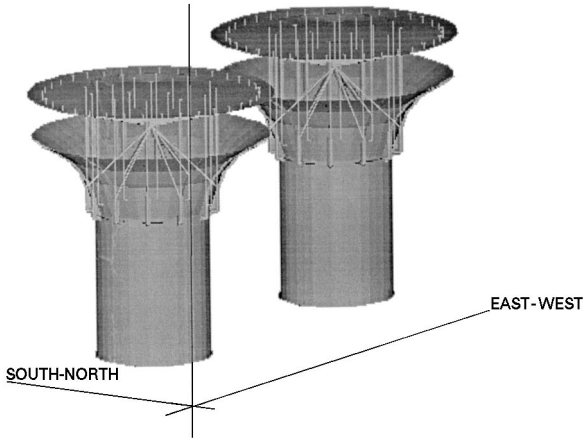


Fig. 1. In-line low rise and 3D-shaped chimneys.

problems have led to many different types of models as in Ref. [1,3], or more recently the quasi-unsteady model in Ref. [4]. The approach presented in this paper is different: it is based on a time-delayed model of the aeroelastic forces in the same manner as in Ref. [1], with an improved estimation of the so-called time lag.

This paper deals with the case of two chimneys. Fig. 1 shows the part of these chimneys which is subjected to wind above the plant roof. The chimneys are cantilevered at their base and their lower part is protected from the wind by the plant walls. Their total height is 45 m. Near the top, their diameter ( $D$ ) increases from 4 to 8 m. Due to the varying diameter, the distance ratio between the chimneys ( $L/D$ ) is 2.6 at the lower part decreasing to 1.3 at the top. A weighted mean value is 2.1. From Ref. [5], an aeroelastic instability could be expected due to the so-called bistable flow occurring for such values of the distance ratio.

As a result of the stiffeners mounted on the chimneys the bending modes in the north–south and east–west directions do not have exactly the same frequencies, the ratio being 1.071. These frequencies are about 3 Hz and the structural damping is very low. For the present study the damping ratio was taken to be 0.1% of critical.

The seaside location of the chimneys is subject to relatively strong winds, mainly from the west. Because of the long life span required of the structures, the problem is to predict the fatigue effects in which the unsteady wind loads on the downstream chimney play an important part. The methodology employed is based on steady wind tunnel measurements on a scale model. The resulting data are used in an aeroelastic computation.

### 1.1. The dynamic system

The dynamic behaviour of the downstream chimney can be described by

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{F}(\mathbf{X}(t - \tau), \dot{\mathbf{X}}(t)) \quad (1)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  are the mass, damping and stiffness matrices.  $\mathbf{X}$  is the deflection which is a function of time  $t$ . The aerodynamic force vector  $\mathbf{F}$  depends on the structural deflection at a delayed time and on the instantaneous vibration velocity. The time lag  $\tau$  will be dealt with later. The upstream chimney is assumed rigid and fixed and the motion of the downstream chimney is assumed to be a linear combination of the two first bending modes  $\mathbf{W}_1$  and  $\mathbf{W}_2$ ,

$$\mathbf{X}(t) = \alpha_1(t)\mathbf{W}_1 + \alpha_2(t)\mathbf{W}_2 \quad (2)$$

where  $\alpha_i$  are scalar functions of time. Then the equation of motion (1) can be simplified by a system of two scalar differential equations:

$$\ddot{\alpha}_i + 2\eta\omega_i\dot{\alpha}_i + \omega_i^2\alpha_i = \mathbf{W}_i^T\mathbf{F}(\mathbf{X}(t - \tau), \dot{\mathbf{X}}(t)), \quad i = 1, 2, \quad (3)$$

the  $\omega_i$  are the angular natural frequencies and  $\eta$  the reduced damping referred to as critical damping. Various mathematical or numerical methods can be used to solve this type of non-linear problem [6,7]. Here a direct numerical integration with the De Vogelaere-Fu method is used in a modified prediction–correction procedure in order to reach the limit cycle of vibration. This scheme, detailed in Ref. [8], gives very good accuracy (order 4), especially in non-linear applications where numerical damping may interfere with physical damping.

## 2. Aeroelastic force model

The structure is obviously a bluff body and neglecting shear stress, the force acting at a node on the surface of the chimney is the pressure force given by

$$\mathbf{F} = \frac{1}{2}\rho S V_a^2 C_p(\beta_a)\mathbf{n}_a, \quad (4)$$

where  $\rho$  is the air density,  $\mathbf{n}_a$  the unit vector normal to the surface corrected for structural deformation.  $S$  is the surface size corresponding to each node. For each node the pressure coefficients,  $C_p$ , are obtained from wind tunnel tests as a function of the wind direction which is referred to as the yaw angle  $\beta$  (stiff model with 576 steady pressure taps, Reynolds number approximately  $10^6$ ). The subscript “a” indicates the apparent wind velocity as seen by the structure. In other words, both the yaw angle and the velocity of the structure are taken into account.

The tests were carried out in one of the IAT’s large subsonic wind tunnels (test section  $5 \times 3$  m) on a 1/15 scale model of the chimneys and of the plant roof. The blockage ratio was less than 5% which is considered to be the limit for this type of tunnel and model, and no wall corrections were applied to the pressure coefficients. Some hot wire measurements were made for an unsteady investigation related to vortex shedding and bistable flow. No typical frequency was observed in the wake and, in particular, no bistable flow frequency was detected. The strong 3D nature of the geometry is considered to be the main reason for the absence of this effect.

For the computations, the wind velocities  $V$  were taken to have logarithmic atmospheric boundary layer profiles such as those presented in Fig. 2. The roughness

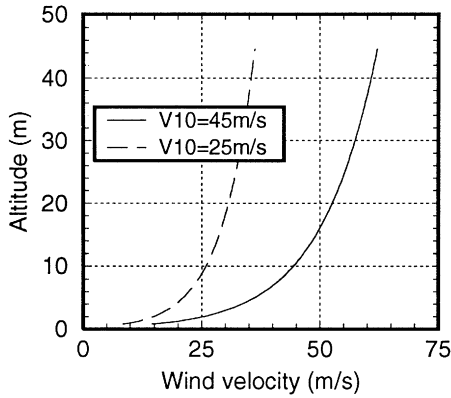


Fig. 2. Typical atmospheric boundary layer profiles.

length was taken to be 0.8 m. The wind velocity was kept constant and no turbulence effects were taken into account.

Moreover, both the  $V_a$  and  $\beta_a$  components are function of the structural deflection: when the downstream chimney has moved, the resulting aerodynamic forces have changed as if the wind direction had changed. Such a dependence may be written in a very simple geometric manner [8,9]. The resulting expressions are:

$$V_a^2 = V^2 + \dot{x}^2 + \dot{y}^2 - 2V\dot{x} \cos(\beta_p) + 2V\dot{y} \sin(\beta_p), \quad (5)$$

where  $x$  and  $y$  are the deflection components of a node in the east and north directions, respectively (the zero yaw angle corresponds to wind coming from the west) and (see Fig. 3):

$$\beta_p = \arctg\left(\frac{y}{L+x}\right) + \beta_0,$$

$$\beta_a = \arctg\left(\frac{-V \sin(\beta_p) - \dot{y}}{V \cos(\beta_p) - \dot{x}}\right). \quad (6)$$

The length  $L$  is the distance between the axes of the two chimneys. One may note that the angle  $\beta_p$  is due to the downstream chimney oscillations.  $\beta_0$  is the initial and constant wind direction. In these relationships, it is assumed that the wake of the upstream chimney is dragged along by the movement of the downstream structure, as shown in many experiments [10].

However, it is well known that such a model is insufficient to give reliable predictions because of the use of steady aerodynamic coefficients. Even when the reduced frequency is small, which means that the fluid is not influenced by the movement of the structure, there is a dynamic effect on the aerodynamic forces because the flow is mainly in stalled conditions. When the structure is moving, this stalled flow requires some time to re-establish itself resulting in a time lag between the fluid force and displacement [1,10–12].

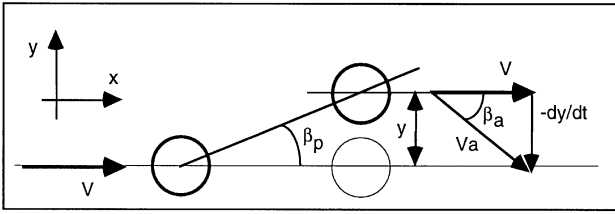


Fig. 3. Definition of velocity vectors and angles for two interfering chimneys.

In the following section, there is a discussion on strong quasi-steady theory where this time lag is neglected, (i.e. the fluid force is applied instantaneously to the moving structure) and on weak quasi-steady theory where the time lag is accounted for. In the latter, the name “quasi-steady” is kept since use is still made of the steady coefficients measured in the wind tunnel on the basis that the reduced frequency remains small.

### 3. Time lag interpretation and calculation

Some authors have reported experimental measurements of the delay between force and displacement [1,12]. In the case of 2D cylinders in subcritical flow, the time lag is found to be of the order of the period of the alternate vortex shedding [8,11].

In the present problem, the lag is the time taken for the fluid to adapt itself to the new configuration induced by the position variation. This variation is referred to the upstream chimney, as it is an interference phenomenon which is modelled relative to the angle  $\beta_p$ . The information that a position change has occurred will reach the downstream chimney at the mean flow velocity between them. In other words,  $\tau$  is the mean duration for the flow to cross the distance between chimneys. Therefore, the time lag is expressed as

$$\tau = \frac{\text{travelling length}}{\text{mean velocity in the wake}} = \frac{L}{V_w(\beta_p)} \tag{7}$$

where the velocity in the wake must be understood as the mean velocity between the two chimneys, that is, in the wake of the upstream one. This velocity is obviously a function of the wind direction.

This expression is the basis of the improved model for interference galloping. Its validity was verified in the 2D case of two cylinders in cross-flow [8,9] and the method was then applied to the chimneys. One major improvement of the weak quasi-steady model as compared to the strong one, is the good capability to compute the critical velocity and the amplitudes of steady oscillations. Fig. 4 shows the maximum amplitude of vibration versus wind velocity for a downstream cylinder in a 2D case ( $L/D = 3, \beta_o = 0$ ). The results of the present model [8,9] are compared to the well-known experimental results of Bokaian [13]. The critical velocity mentioned is obtained by linear analysis of the model.

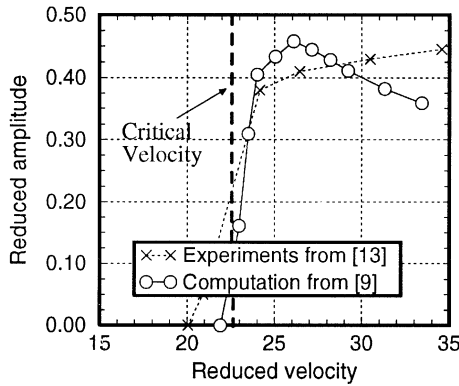


Fig. 4. Reduced amplitudes of oscillation versus reduced velocity for the downstream cylinder of 2 in-line cylinders, 2D case –  $L/D = 3$ .

However, the chimneys have a 3D shape and a rigorous calculation would require the estimation of this time lag for each node of the structure. In an industrial situation, the number of nodes is large and many parameters have to be studied such as the wind direction and its velocity. It is then necessary to make a compromise between precision and cost. Therefore, a mean time lag for the whole downstream chimney is estimated using the following procedure derived from the case of 2D cylinders.

The unknown is the mean flow velocity between chimneys. This could be measured in the wind tunnel but the number of tests would be large and besides, introducing a probe may create some perturbations in the flow. The idea is to find a simplified technique which could be used easily in many computation cases.

Let  $C_{xM}$  be the drag coefficient of one isolated chimney. This force is due to an incoming flow at velocity  $V$  and is proportional to  $V^2$ . A chimney in the wake of the first sees an incoming flow at velocity  $V_w$  which is the required unknown. This structure is subjected to a force which has two components: a drag force  $C_x$  and a lateral force  $C_y$ , which change with wind direction. Hence, one can obtain the velocity seen by the downstream chimney with

$$\left(\frac{V_w(\beta_p)}{V}\right)^2 = \frac{\sqrt{C_x^2(\beta_p) + C_y^2(\beta_p)}}{C_{xM}}. \quad (8)$$

The results of these expressions are plotted in Fig. 5 where the time lag has been reduced in the same way as a Strouhal number or a reduced frequency:

$$\tau_r = \frac{D}{V\tau}. \quad (9)$$

Fig. 5 also presents the resulting curve for equivalent 2D cylinders with diameter  $D$  and distance  $L$  taken as the mean value for the chimneys. It is interesting to note that for small yaw angles, the time lag is much larger than for the 3D structures, which

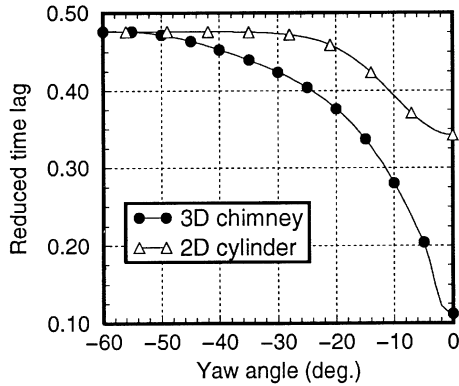


Fig. 5. Reduced time lag versus yaw angle.

corresponds to a smaller velocity between chimneys. It seems then that the velocity between structures, and consequently the forces on the downstream chimney, does not have a linear relationship with the  $L/D$  ratio.

This result is already well known and obvious for the bluff body aerodynamics community, but the relationship introduced here with the time lag in the aeroelastic formulation may explain the higher sensitivity to interference galloping of structures with small  $L/D$ . Indeed, a simple linear analysis would show that increasing time lag between force and displacement would increase the energy transfer from the flow toward the structure [1,8].

Although the aeroelastic force model presented here seems relatively simple, it contains some information and some complex effects:

- It was shown experimentally that the time lag is a function of reduced frequency and oscillation amplitude [1]. In the model, its dependence on the incoming velocity is naturally present. Moreover, the time lag changes continuously with displacement because it depends on the angle  $\beta_p$ .
- The experimental investigation of Ref. [12] shows that the time lag cannot be constant during a period of oscillation and that the force–displacement diagram (hysteresis curve) should be a function of the phase of the motion. This means that the force is dependent on whether the structural velocity is negative or positive, which is typically a dynamic stall phenomenon. The apparent wind in the present model effectively accounts for this.

#### 4. Results for the chimneys

Fig. 6 shows the amplitude of steady oscillations (reduced by a reference length which is the smallest diameter) of the top of the downstream chimney versus reduced velocity. The reference velocity is at a height of 10 m in the boundary layer profile. The

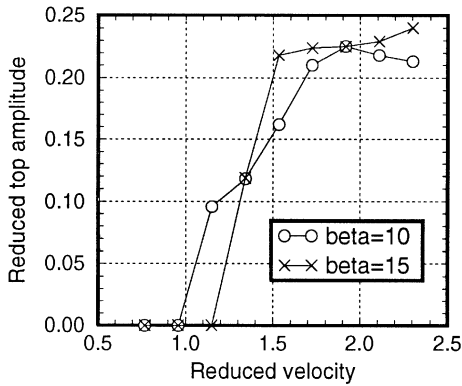


Fig. 6. Reduced amplitudes of oscillation versus reduced velocity of the downstream chimney 3D computation.

yaw angles are those which create the higher instability. These results were obtained for the strongest winds expected on the site.

The steady (limit cycle) amplitudes of oscillation shown are reached after a time which decreases as the wind velocity increases. This time can be relatively long when the velocity is small, typically 45 min, while for higher velocities the limit cycle can be reached after less than 10 min. During all this time, the wind is assumed constant in both speed and direction. These conditions are not very realistic, especially for the strongest winds, but they have the advantage of ensuring that the maximum load that could be applied to the structure is obtained. The critical velocity is well illustrated in the figure and obviously could be increased by increasing structural damping.

Fig. 7a presents the force–displacement at the top of the chimney (lateral force on the node at the top). A single clockwise loop is observed which characterises an energy transfer to the structure, i.e. aerodynamic excitation. In the classical definition of interference galloping, there would be two (or more) loops, one damped and the other amplified. The stationary vibration is then reached when the energy input is equal to the output. In the presented results, the single loop means that amplitude of motion is too small to reach a region where the aerodynamic forces create damping.

The amplitude of the movement is then limited by structural damping and the dynamic system is mostly linear. Fig. 7b shows the power spectrum of lateral displacement at the top. The fundamental natural frequency of the bending mode is approximately 3 Hz. There is little energy in first to fourth harmonics which confirms the almost linear behaviour.

Nevertheless, the improvement of the quasi-steady theory is significant because steady amplitudes of vibration can be calculated in cases where the strong quasi-steady theory would predict stability to galloping effects. In fact, the excitation of the motion is initiated by the delayed force and balanced by structural damping.



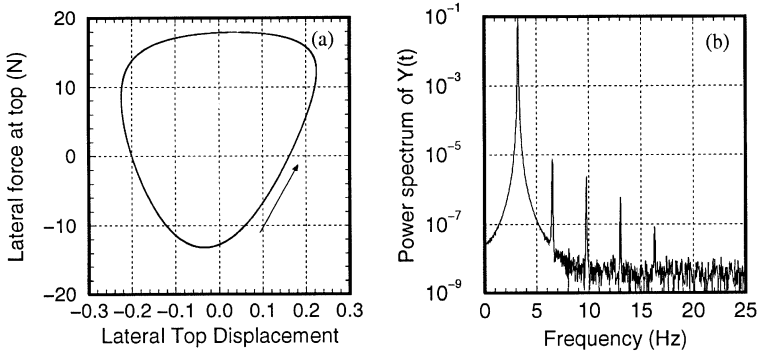


Fig. 7. Analysis of a steady solution; (a) Force–displacement diagram, (b) power spectrum of lateral displacement.

## 5. Conclusions

An aeroelastic force model was used to study the galloping oscillations of one chimney behind another subjected to atmospheric wind. The proposed force model is of the time-delayed quasi-steady type and an expression is given to calculate this time lag based on physical considerations of the flow–structure interaction. A next step could be a thorough investigation in relation with the Strouhal number, i.e. with vortex velocity, when vortex shedding occurs. Prediction of the coupled effects of galloping oscillations and of vortex induced oscillations remains an interesting objective.

It is to be pointed out that, the influence of the structural velocity will not be clearly identified as long as steady aerodynamic coefficients are used. Since stalled conditions are dominant, the quasi-steady assumption is only valid for very low reduced frequencies. One question is, for example, whether the effects of the apparent velocity and of the yaw angle are seen by the structure instantaneously. In other words, one could introduce a time lag between the fluid force and the velocity of the structure which is different from the one referred to position.

A final significant point is that the 3D nature of the structure was taken into account using steady local pressure coefficients measured in the wind tunnel. The nodal force proposed could be simplified into a sectional force and applied to 2D high rise structures. This was done previously [9] in order to compare it with experimental results taken from the literature.

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