NL2574 Spatio-temporal chaos

Spatio-temporal chaos is a dynamical regime developing in spatially distributed systems lacking long-time, large-distance coherence in spite of an organized regular behavior at the local scale. It is so to speak located in the middle of a triangle, the corners of which are temporal chaos, which is prevalent for a few spatially frozen degrees of freedom, spatial chaos, in disordered time-independent patterns, and turbulence, with cascading processes over a wide range of space and time scales.

Short-term local coherence is usually the result of some instability mechanism that generates dissipative structures of different kinds depending on whether or not a specific frequency ($\omega_c = 0$ or $\neq 0$) and/or a spatial periodicity ($k_c = 0$ or $\neq 0$) is introduced in the system (Cross & Hohenberg, 1993). Examples of spatio-temporal chaos may be found in every combination of these elementary cases: Rayleigh–Bénard convection produces a time-independent cellular pattern ($\omega_c = 0$, $k_c \neq 0$), the Belousov–Zhabotinsky (BZ) reaction-diffusion system is unstable against a homogeneous oscillatory mode ($\omega_c \neq 0$, $k_c = 0$), convection in binary fluid mixtures develops in the form of dissipative waves ($\omega_c \neq 0$, $k_c \neq 0$), and the same holds for hydrothermal waves. Transitions to spatio-temporal chaos have been observed and studied in many experimental systems, including parametrically excited surface waves, electro-hydrodynamic instabilities in nematic liquid crystals (Kramer & Pesch, 1996), and liquid films flowing down inclines (Chang, 1994). Figure 1 (left) illustrates the case of spiral defect chaos in convection.

In practice, confinement effects enter and compete with instability mechanisms in the ordering process. Their intensity can be appreciated through aspect ratios, measuring the physical size of the system in units of the instability wavelength. In small

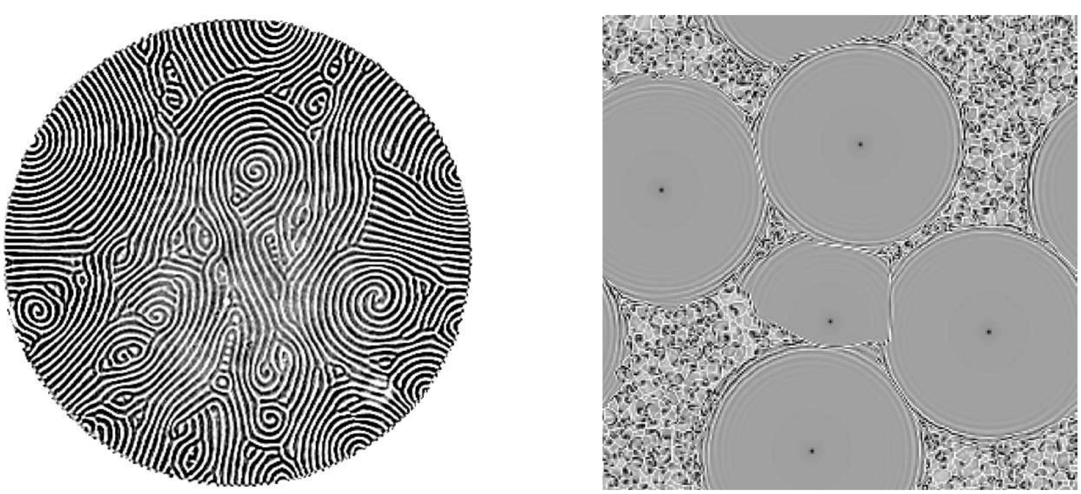

Figure 1. Left: Spiral defect chaos near threshold in Rayleigh–Bénard convection at $\mathcal{O}(1)$ Prandtl number. Courtesy Ahlers (1998). Right: coexistence of spirals and defect mediated chaos in the 2D CGL equation for $\alpha = -2$ and $\beta \simeq 0.75$ (Manneville, unpublished).

aspect-ratio experiments, confinement effects are effective in the three space directions and chaos is purely temporal. Spatio-temporal chaos develops when confinement is partially relaxed. Accordingly, phenomena developing at surfaces or in thin layers can be understood as (quasi-)two-dimensional (Gollub, 1994). In the same way, narrow channels or oriented media along a specific direction exemplify the case of (quasi-)one-dimensional systems (Daviaud, 1994).

A second important classification results from the nature of the bifurcation, either supercritical or subcritical (continuous or discontinuous), which implies either substitution or coexistence of bifurcating and bifurcated states. To a large extent, this feature dictates the type of theory most appropriate to understand the growth of spatio-temporal chaos.

Consider first the supercritical case. When confinement effects are unable to maintain order everywhere, reduced universal descriptions are generically obtained as "envelope equations" (Newell, 1974). Standing as a paradigm, the cubic complex Ginzburg–Landau (CGL3), reviewed by Aranson & Kramer (2002), spatially unfolds a local supercritical Hopf bifurcation signalling the emergence of uniform oscillations ($\omega_{\rm c} \neq 0$, $k_{\rm c} = 0$). In one dimension (1D), this equation reads

$$\partial_t A = A + (1 + {\rm i}\alpha)\partial_{xx} A - (1 + {\rm i}\beta)|A|^2 A\,, \qquad (1)$$

where the parameters α and β measure linear and nonlinear dispersion effects respectively. The CGL3 equation admits trivial exact solutions in the form of plane waves $A = A_q \exp[{\rm i}(qx - \omega_q t)]$, with amplitude $A_q = (1 - q^2)^{1/2}$ and angular frequency $\omega_q = \alpha q^2 + \beta(1 - q^2)$, which are stable or unstable depending on the value of α, β, and q. Other nonlinear solutions can exist; in one dimension, solitary waves called Bekki–Nozaki holes are the best known. In two dimensions (2D) they take the form spiral waves that are topological defects of the complex order parameter A. Figure 1 (right) illustrates the coexistence of spirals and defect-mediated chaos (see below) in the 2D CGL equation. Figure 2 displays the different possible steady-state regimes of the 1D CGL equation in the $(1/\beta, -\alpha)$ plane. In region I, plane waves attract most initial conditions. Phase turbulence is present in region IV slightly beyond the Benjamin–Feir instability (BF) line, as given by Newell's criterion for '$q = 0$' oscillations (Newell, 1974). In the vicinity of this line, the solution can be written as $A(x, t) = (1 + \varrho(x, t)) \exp({\rm i}\theta(x, t))$. The amplitude modulation ϱ, enslaved to the gradient of the phase perturbation θ, remains small, while θ is governed at lowest order by the Kuramoto–Sivashinsky equation (Kuramoto, 1978)

$$\partial_t \theta = D\partial_{xx}\theta - K\partial_{xxxx}\theta + g(\partial_x \theta)^2\,, \qquad (2)$$

where $D = 1 + \alpha\beta$ is an effective diffusion coefficient which is negative in the unstable range (Chaté & Manneville, 1994, a). Deeper in the unstable domain, in region V, a "revolt" of $|A|$ ends in the formation of defects (phase singularities at zeroes of $|A|$) and amplitude turbulence or defect-mediated turbulence sets in (Coullet *et al.*, 1989). Defects analogous to Bekki–Nozaki holes are observed to evolve in a spatio-temporal intermittent fashion in region II. As suggested by its position in the diagram, the "bi-chaos" regime in region III presents itself as a fluctuating mixture of states in regions IV and V.

By contrast, subcritical instabilities are characterized by the possibility of finding the system in one of several (usually two) states at a given point in space. Short-range coherence then implies the formation of homogeneous domains of each state, separated by fronts (Pomeau, 1986). In gradient systems, these fronts move regularly so as to decrease the potential, but in non-gradient systems they may have more complicated behaviors. A particularly interesting scenario develops when one of the competing local states is a chaotic transient while the other is regular. At a given time the whole system can be divided into so-called laminar and turbulent domains, and at a given point in space the system is alternatively laminar or turbulent, hence the name "spatio-temporal intermittency" (STI). According to Pomeau (1986) front propagation is then akin to a time-oriented stochastic process known as "directed percolation" (DP) used to model epidemic processes (Kinzel, 1983). Directed percolation defines a critical phenomenon, with an associated universality class (a specific set of scaling exponents governing the statistical behavior of the system as a function of the distance to threshold).

The existence of separated domains and sharp fronts supports the idea of modeling extended systems in terms of identical sub-systems arranged on a regular lattice, each with its own phase space, and coupled to its neighbors. In addition to space discretization, time discretization leads to the definition of coupled map lattices that have served to illustrate several transition scenarios such as cascades of spatial period doublings, defect-mediated regimes (Kaneko, 1993) or STI (Chaté & Manneville, 1994, b) that make explicit how local transient temporal chaos is converted into sustained spatio-temporal chaos. A last step can be taken by also discretizing the local phase

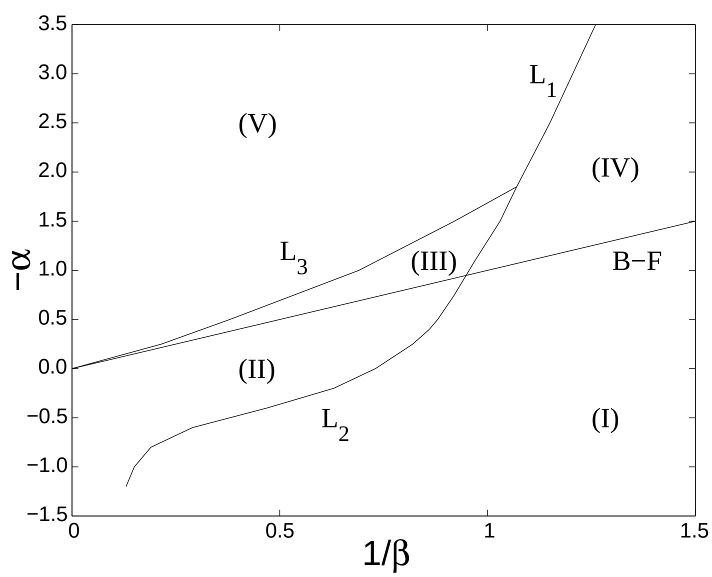

Figure 2. Bifurcation diagram of the 1D CGL equation. Data from Chaté (1994).

space, which yields cellular automata (Wolfram, 1986). Further randomization of the dynamics then points towards an understanding of the transition to turbulence in statistical physics terms (Kinzel, 1983). Such approaches should help in understanding plane Couette flow, the simplest shear flow produced by two parallel plates sliding in opposite directions. This configuration is a particularly intriguing example of a subcritical hydrodynamic system where domains of laminar flow, known to be stable for all flow conditions, coexist in a wide range of Reynolds numbers and in continuously varying proportions with domains of small-scale turbulence.

Spatio-temporal chaos definitely relates to the process of transition to turbulence when confinement effects are weak. It appears to occupy a central position at the crossroads of nonlinear dynamics, mathematical stability theory, and statistical physics of many-body systems and nonequilibrium processes, with a wide potential for applications (Rabinovich *et al.*, 2000).

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See also Bénard convection; Cellular automata; Chaos vs. turbulence; Coherence length; Coupled map lattice; Development of singularities; Gradient system; Hydrothermal waves; Kuramoto-Sivashinsky equation; Modulated waves; Multiple scale analysis; Navier–Stokes equations; Nonequilibrium statistical mechanics; Order parameter; Pattern formation; Phase dynamics; Reaction diffusion systems; Spiral waves; Surface waves; Topological defects; Turbulence.

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